OBSERVER DESIGN FOR T-S FUZZY SYSTEMS WITH MEASUREMENT OUTPUT NOISES

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Abstract: In this paper, new observer design approaches are presented for Takagi-Sugeno (T-S) fuzzy models with measurement output noises. In order to decouple the measurement noise, an augmented fuzzy descriptor model is first constructed. Then a fuzzy descriptor proportional and derivative (PD) observer is developed. When the fuzzy subsystems have the same output matrices, another fuzzy descriptor observer is designed, which can be transformed into a normal fuzzy observer. Via the proposed two kinds of fuzzy observer design, the state and the output noise can be estimated at the same time. The simulation result shows the satisfactory tracking performance.

Keywords: Descriptor system, fuzzy systems, measurement noises, observer, TS model.

1. INTRODUCTION

T-S fuzzy model was developed successfully to investigate non-linear control systems (Takagi and Sugeno, 1985). For the observer design of fuzzy T-S systems, there are some results reported. In the reference (Yoneyama et al, 2000), the full-order fuzzy observer design was presented and the separation principle was also discussed. The reduced-dimensional fuzzy observer was proposed by Ma et al (2001). The sliding model observer was investigated by Tong et al (2000). By using linear matrix inequalities (LMIs), an alternative fuzzy observer was designed by Bergsten et al (2002).

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The robust fuzzy observer was discussed for T-S fuzzy systems with parameter uncertainties (Ma, 2002). However no efforts were made for fuzzy T-S systems with measurement noises.

For crisp systems with measurement output disturbances, the conventional observer cannot be utilized to obtain satisfactory estimation performance (Busawon et al, 2001). Recently, new descriptor observer approach was developed for systems with measurement output disturbance (Gao et al, 2003). The proportional gain and the derivative gain were used simultaneously in this observer design, i.e. PD observer design. In the present paper, motivated by the work by Gao et al (2003), a fuzzy PD observer is designed for T-S fuzzy models.

2. PRELIMINARIES

T-S fuzzy model is described by the following fuzzy IF-THEN rules:

Rules ^{i}: IF z_1 is M_{1i} and … and z_p is M_{pi}, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y = C_i x(t) \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^p$ is the measurement output vector, $z = \begin{bmatrix} z_1 & \dots & z_p \end{bmatrix}$ are the premise variables and $M_{1i} \dots M_{pi}$ are fuzzy sets.

Then the final fuzzy system is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z)(A_i x(t) + B_i u(t)) \\ y = \sum_{i=1}^{r} h_i C_i(z) x(t) \end{cases} \quad (2)$$

where

$$h_i(z) = w_i(z) \Big/ \sum_{j=1}^{r} w_i(z),$$

$$w_i(z) = \prod_{j=1}^{p} M_{ji}(z). \quad (3)$$

Hence $h_i(z)$ satisfies

$$h_i(z) > 0, \ \sum_{i=1}^{r} h_i(z) = 1. \quad (4)$$

When a measurement noise occurs, the T-S fuzzy system (2) becomes

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(A_i x(t) + B_i u(t)) \\ y = \sum_{i=1}^{r} h_i C_i x(t) + \omega(t) \end{cases} \quad (5)$$

where $\omega(t) \in R^p$ is the measurement output noise vector. The conventional fuzzy observer is constructed as

$$\dot{\hat{x}} = \sum_{i=1}^{r} h_i(A_i \hat{x} + B_i u - K_i(y - \sum_{j=1}^{r} h_j C_j \hat{x})) \quad (6)$$

where $\hat{x}(t) \in R^n$ is the estimation of the state vector, $K_i \in R^{n \times p}$ is the gain matrix such that there exists a common symmetric positive definite matrix P suc h that $(A_i + K_i C_j)^T P + P(A_i + K_i C_j) < 0$ for all i, j.

Let $e(t) = x(t) - \hat{x}(t)$, the estimation error dynamics is governed by

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j((A_i + K_i C_j)e(t) + K_i \omega(t)). \quad (7)$$

It is impossible to find a set of suitable matrices K_i $(i = 1, 2, \dots, r)$ such that $\lim_{t \to \infty} e(t) = 0$, because the measurement output noise is amplified by the gain matrix K_i $(i = 1, 2, \dots, r)$. Therefore, this motivates us to develop new fuzzy observer design approaches.

2. FUZZY PD OBSERVER

In this section, a fuzzy descriptor PD observer is designed to estimate the system state and the measurement output noise at the same time. Since there exist time-varying weights in the output equation, the PD observer design approach for linear crisp systems developed by Gao et al (2003) cannot be utilized directly in our fuzzy observer design. Now we make the following investigation.

Denote $\bar{x}(t) = \begin{bmatrix} x(t)^T & y(t)^T & \omega(t)^T \end{bmatrix}^T$, an augmented fuzzy system can be obtained from (1):

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = \sum_{i=1}^{r} h_i(\bar{A}_i \bar{x}(t) + \bar{B}_i u(t) + \bar{N}\omega(t)) \\ y = \bar{C}\bar{x}(t) = \sum_{i=1}^{r} h_i C_i^0 \bar{x}(t) + \omega(t) \end{cases} \quad (8)$$

where

$$\bar{E} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}, \ \bar{N} = \begin{bmatrix} 0 \\ 0 \\ I_p \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0 & 0 \\ C_i & -I_p & I_p \\ 0 & 0 & -I_p \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & I_p & 0 \end{bmatrix},$$

$$C_i^0 = \begin{bmatrix} C_i & 0 & 0 \end{bmatrix}.$$

The system in the form of (8) is called descriptor fuzzy model, which was first investigated by Taniguchi et al (2000). Consider the following fuzzy descriptor system:

$$\begin{cases} E' \dot{\eta}(t) = \sum_{i=1}^{r} h_i(A_i' \eta(t) + \bar{B}_i u(t)) \\ \hat{\bar{x}}(t) = \eta(t) + Ky(t) \end{cases} \quad (9)$$

where $\eta(t) = \hat{\bar{x}}(t) - Ky(t)$. Substituting $\eta(t)$ into (9), one has

$$E' \dot{\hat{\bar{x}}}(t) - E' K\bar{C}\dot{\bar{x}}(t)$$

$$= \sum_{i=1}^{r} h_i [A_i'(\hat{\bar{x}} - K\sum_{j=1}^{r} h_j C_j^0 \bar{x} - K\omega) + \bar{B}_i u]$$

$$= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j [A_i'(\hat{\bar{x}} - KC_j^0 \bar{x} - K\omega) + \bar{B}_i u]. \quad (10)$$

Subtracting (10) from the differential equation of (8) yields

$$(\bar{E} + E' K\bar{C})\dot{\bar{x}}(t) - E'\dot{\hat{\bar{x}}}(t)$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [(\bar{A}_i + A'_i K C_j^0)\bar{x} - A'_i \hat{\bar{x}}$$
$$+ \bar{N}\omega + A'_i K\omega]. \quad (11)$$

It can be concluded that when

$$\bar{N} = -\sum_{i=1}^{r} h_i A'_i K, \quad (12)$$

the effect of measurement output noise $\omega(t)$ can be eliminated. Clearly, Eq. (12) is equivalent to

$$\bar{N} = -A'_i K \quad \text{for all } i. \quad (13)$$

The equation (11) can be rewritten as follows

$$(\bar{E} + E' K\bar{C})\dot{\bar{x}} - E'\dot{\hat{\bar{x}}}$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j ((\bar{A}_i - \bar{N}C_j^0)\bar{x} - A'_i \hat{\bar{x}})$$
$$= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \bar{A}_i\bar{x} - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \bar{N}C_j^0\bar{x} - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j A'_i\hat{\bar{x}}$$
$$= \sum_{i=1}^{r} h_i\bar{A}_i\bar{x} - \sum_{j=1}^{r} h_j\bar{N}C_j^0\bar{x} - \sum_{i=1}^{r} h_i A'_i\hat{\bar{x}}$$
$$= \sum_{i=1}^{r} h_i ((\bar{A}_i - \bar{N}C_i^0)\bar{x} - A'_i \hat{\bar{x}}). \quad (14)$$

Let $e(t) = \bar{x}(t) - \hat{\bar{x}}(t)$, then (14) becomes

$$E'\dot{e}(t) = \sum_{i=1}^{r} h_i A'_i e(t), \quad (15)$$

provided that

$$\bar{A}_i - \bar{N}C_i^0 = A'_i, \quad (16)$$
$$\bar{E} + E'K\bar{C} = E'. \quad (17)$$

In this case, the error $e(t) \to 0$ when $t \to \infty$ if there exists a common matrix X which satisfies the following conditions (Taniguchi et al, 2000)

$$E'^T X = X^T E' \geq 0 \quad (18)$$
$$A_i'^T X + X^T A'_i < 0 \quad (19)$$

Noticing that

$$\bar{A}_i = \begin{bmatrix} A_i & 0 & 0 \\ C_i & -I_p & I_p \\ 0 & 0 & -I_p \end{bmatrix}, \ \bar{N} = \begin{bmatrix} 0 \\ 0 \\ I_p \end{bmatrix},$$
$$C_i^0 = \begin{bmatrix} C_i & 0 & 0 \end{bmatrix}, \quad (20)$$

and by solving (16) we have

$$A'_i = \begin{bmatrix} A_i & 0 & 0 \\ C_i & -I_p & I_p \\ -C_i & 0 & -I_p \end{bmatrix}. \quad (21)$$

Furthermore from (12), the gain K can be achieved as follows

$$K = \begin{bmatrix} 0 & I_p & I_p \end{bmatrix}^T. \quad (22)$$

Substituting (22) into (17), one has

$$E' = \begin{bmatrix} I_n & E_{12} & 0 \\ 0 & E_{22} & 0 \\ 0 & E_{32} & 0 \end{bmatrix} \quad (23)$$

where E_{12}, E_{22}, E_{32} are any matrices with appropriate dimensions. Now the following theorem can be concluded.

Theorem 1. The PD fuzzy observer (9) can be used to asymptotically estimate the state and measurement output noise of (5) if Eqs. (12), (16) and (17) hold, and there exists a common symmetric positive definite matrix P such that

$$A_i^T P + PA_i < 0, \ i \in \underline{r} = \{1, 2, \cdots, r\} \quad (24)$$

Proof. Construct a fuzzy descriptor PD observer in the form (9), where

$$E' = \begin{bmatrix} I_n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A'_i = \begin{bmatrix} A_i & 0 & 0 \\ C_i & -I_p & I_p \\ -C_i & 0 & -I_p \end{bmatrix}$$
$$K = \begin{bmatrix} 0 \\ I_p \\ I_p \end{bmatrix}, \ X = \begin{bmatrix} P & 0 & 0 \\ 0 & \alpha I_p & 0 \\ 0 & \alpha I_p & \alpha I_p \end{bmatrix} \quad (25)$$

and α is any positive real number. Thus, Eqs. (12), (16) and (17) hold and

$$E'\dot{e}(t) = \sum_{i=1}^{r} h_i A'_i e(t) \quad (26)$$

where $e(t) = \bar{x}(t) - \hat{\bar{x}}(t)$. Moreover, it can easily be shown that $E'^T X = X^T E' \geq 0$, i.e. (18) holds, if P is a symmetric positive definite matrix. Notice that

$$A_i'^T X + X^T A'_i$$
$$= \begin{bmatrix} PA_i + A_i^T P & 0 & -\alpha C_i^T \\ 0 & -2\alpha I_p & 0 \\ -\alpha C_i & 0 & -2\alpha I_p \end{bmatrix} < 0 \quad (27)$$

which is equivalent to

$$\begin{cases} \begin{bmatrix} -2\alpha I_p & 0 \\ 0 & -2\alpha I_p \end{bmatrix} < 0, \\ PA_i + A_i^T P - \begin{bmatrix} 0 & -\alpha C_i^T \end{bmatrix} \\ \times \begin{bmatrix} -2\alpha I_p & 0 \\ 0 & -2\alpha I_p \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -\alpha C_i \end{bmatrix} < 0. \end{cases} \quad (28)$$

From (28) one has equivalently

$$A_i^T P + PA_i + \frac{\alpha}{2} C_i^T C_i < 0. \quad (29)$$

If there exists a common symmetric positive definite matrix P such that $A_i^T P + PA_i < 0$, there must exist a small enough positive number α such that (29) holds. Therefore (27) or (19) holds. As a result, the estimation error $e(t)$ asymptotically tends to zero as the time goes, namely $\lim_{n\to\infty} e(t) = 0$. This completes the proof.

Remark 1. The proposed PD descriptor fuzzy observer (9) can ensure the tracking error to be impulse-free and asymptotically stable. The estimates of the state and noise can be obtained simultaneously.

3. T-S NORMAL FUZZY OBSERVER

Now we consider the particular cases, i.e.

$$C_1 = C_2 = \cdots = C_r = C . \tag{30}$$

In this case, the system (5) can be rewritten as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i (A_i x(t) + B_i u(t)) \\ y = Cx(t) + \omega(t) \ . \end{cases} \tag{31}$$

Construct an augmented fuzzy descriptor system as following

$$\begin{cases} \overline{E}\dot{\overline{x}} = \sum_{i=1}^{r} h_i (\overline{A}_i \overline{x} + \overline{B}_i u + \overline{N}\omega) \\ y = \overline{C}\overline{x} = C^0 \overline{x} + \omega \end{cases} \tag{32}$$

where

$\overline{x}(t) = \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}$, $\overline{E} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}$, $\overline{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & -I_p \end{bmatrix}$,

$\overline{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$, $\overline{N} = \begin{bmatrix} 0 \\ I_p \end{bmatrix}$, $\overline{C} = \begin{bmatrix} C & I_p \end{bmatrix}$,

$C^0 = \begin{bmatrix} C & 0 \end{bmatrix}$.

Consider a descriptor fuzzy system as follows

$$\begin{cases} E' \dot{\eta}(t) = \sum_{i=1}^{r} h_i (A_i' \eta(t) + \overline{B}_i u(t)) \\ \hat{\overline{x}}(t) = \eta(t) + Ky(t) \ . \end{cases} \tag{33}$$

Substituting $\eta(t) = \hat{\overline{x}}(t) - Ky(t)$ into the differential equation, one has

$$E' \dot{\hat{\overline{x}}}(t) - E' K\overline{C}\dot{\overline{x}}(t)$$

$$= \sum_{i=1}^{r} h_i (A_i' (\hat{\overline{x}} - KC^0 \overline{x} - K\omega) + \overline{B}_i u) . \tag{34}$$

Subtracting (35) from (32) yields

$$(\overline{E} + E'K\overline{C})\dot{\overline{x}}(t) - E'\dot{\hat{\overline{x}}}(t)$$

$$= \sum_{i=1}^{r} h_i [(\overline{A}_i + A_i'KC^0)\overline{x} - A_i'\hat{\overline{x}} + \overline{N}\omega + A_i'K\omega] . \tag{35}$$

Let $e(t) = \overline{x}(t) - \hat{\overline{x}}(t)$, and suppose

$$\overline{A}_i + A_i'KC^0 = A_i' , \tag{36}$$

$$\overline{N} = -A_i'K , \tag{37}$$

$$\overline{E} + E'K\overline{C} = E' . \tag{38}$$

One has the error dynamic

$$E'\dot{e}(t) = \sum_{i=1}^{r} h_i A_i' e(t) . \tag{39}$$

From (36)-(38), it can be computed that

$A_i' = \begin{bmatrix} A_i & 0 \\ -C & -I_p \end{bmatrix}$, $K = \begin{bmatrix} 0 \\ I_p \end{bmatrix}$,

$$E' = \begin{bmatrix} I & 0 \\ MC & M \end{bmatrix} , \tag{40}$$

where M is a full-rank matrix with appropriate dimension. From (40), the error dynamic equation (39) becomes

$$\begin{bmatrix} I & 0 \\ MC & M \end{bmatrix} \dot{e}(t) = \sum_{i=1}^{r} h_i \begin{bmatrix} A_i & 0 \\ -C & -I_p \end{bmatrix} e(t) \tag{41}$$

or equivalently

$$\dot{e}(t) = \sum_{i=1}^{r} h_i \begin{bmatrix} I & 0 \\ MC & M \end{bmatrix}^{-1} \begin{bmatrix} A_i & 0 \\ -C & -I_p \end{bmatrix} e(t)$$

$$= \sum_{i=1}^{r} h_i \begin{bmatrix} I & 0 \\ -C & M^{-1} \end{bmatrix} \begin{bmatrix} A_i & 0 \\ -C & -I_p \end{bmatrix} e(t)$$

$$= \sum_{i=1}^{r} h_i \begin{bmatrix} A_i & 0 \\ -CA_i - M^{-1}C & -M^{-1} \end{bmatrix} e(t) . \tag{42}$$

Eq. (42) implies that $\lim_{n\to\infty} e(t) = 0$ if there exists a common symmetric positive definite matrix X such that

$$A_i^{*T} X + X^T A_i^* < 0 \tag{43}$$

where

$$A_i^* = \begin{bmatrix} A_i & 0 \\ -CA_i - M^{-1}C & -M^{-1} \end{bmatrix}.$$

To summarize, one has the following theorem.

Theorem 2. The PD observer in the form of (33) can be used to asymptotically estimate the state and measurement output noise of (31) if Eqs. (36)-(38) hold with $(-M^{-1})$ being a stable matrix, and there exists a common symmetric positive definite matrix P such that

$$A_i^T P + PA_i < 0 \ , \ i \in \underline{r} = \{1, 2, \cdots, r\} . \tag{44}$$

Proof. Let $X = \begin{bmatrix} P & 0 \\ 0 & \alpha P_0 \end{bmatrix}$, where α is any positive real number and $P_0 > 0$, so $X > 0$. Notice that

$$A_i^{*T} X + X^T A_i^*$$
$$= \begin{bmatrix} A_i & 0 \\ -CA_i - M^{-1}C & -M^{-1} \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \alpha P_0 \end{bmatrix} +$$
$$\begin{bmatrix} P & 0 \\ 0 & \alpha P_0 \end{bmatrix} \begin{bmatrix} A_i & 0 \\ CA_i - M^{-1}C & -M^{-1} \end{bmatrix}$$
$$= \begin{bmatrix} A_i^T P + PA_i & -\alpha(CA_i + M^{-1}C)^T P_0 \\ * & -\alpha((M^{-1})^T P_0 + P_0 M^{-1}) \end{bmatrix}. \quad (45)$$

Thus, $A_i^{*T} X + X^T A_i^* < 0$ if and only if
$$\alpha((-M^{-1})^T P_0 + P_0(-M^{-1})) < 0, \quad (46)$$
and
$$PA_i + A_i^T P + \alpha[CA_i + M^{-1}C]^T P_0$$
$$\times [M^{-T} P_0 + P_0 M^{-1}]^{-1} P_0 (CA_i + M^{-1}C) < 0. \quad (47)$$

If $(-M^{-1})$ is a stable matrix, a positive definite matrix P_0 can be chosen such that (46) holds. Furthermore, if there exists a common symmetric positive definite matrix P such that (44) holds, there must exist a small enough positive number α such that (47) holds. Therefore, the common symmetric positive definite matrix $X = \begin{bmatrix} P & 0 \\ 0 & \alpha P_0 \end{bmatrix}$ satisfies $A_i^{*T} X + X^T A_i^* < 0$. As a result, one has immediately $\lim_{n \to \infty} e(t) = 0$. This completes the proof.

Remark 2. The proposed observer (33) can be transformed into a normal observer directly. If the output matrices of the subsystems do not satisfy $C_1 = C_2 = \cdots = C_r = C$, the T-S fuzzy system can be rewritten as
$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i (A_i x + B_i u) \\ y = Cx + \sum_{i=1}^{r} h_i (C_i - C)x + \omega \end{cases} \quad (48)$$
where C is the output matrix chosen from $C_1, C_2, \cdots, C_r$. Let
$$\omega_0 = \sum_{i=1}^{r} h_i (C_i - C)x(t) + \omega(t),$$
the system above can be described as
$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i (A_i x(t) + B_i u(t)) \\ y = Cx(t) + \omega_0(t). \end{cases} \quad (49)$$

Thus we can design the T-S fuzzy PD observer to estimate $x(t)$ and $\omega_0(t)$ of system (49) by using the approach given by Theorem 2. It is obvious that
$$\begin{bmatrix} I & 0 \\ \sum_{i=1}^{r} h_i (C_i - C) & I \end{bmatrix} \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \omega_0(t) \end{bmatrix}. \quad (50)$$

Therefore the following T-S fuzzy PD observer can be used to estimate $x(t)$ and $\omega(t)$ for system (5)
$$\begin{cases} E'\dot{\eta}(t) = \sum_{i=1}^{r} h_i (A_i' \eta + \overline{B}_i u) \\ \hat{\bar{x}} = \begin{bmatrix} I & 0 \\ \sum_{i=1}^{r} h_i (C_i - C) & I \end{bmatrix}^{-1} (\eta + Ky). \end{cases} \quad (51)$$

Now we have the following theorem immediately.

Theorem 3. The PD observer (51) can be used to asymptotically estimate the state and measurement output noise of (5) if the observer coefficients are defined in (40) with $(-M^{-1})$ being a stable matrix, and there exists a common symmetric positive definite matrix P such that
$$A_i^T P + PA_i < 0, \; i \in \underline{r} = \{1, 2, \cdots, r\}. \quad (52)$$

4. SIMULATION EXAMPLE

Consider a mass-spring-damper system (Tanaka et al, 1996)
$$G\ddot{x} + g(x, \dot{x}) + f(x) = \phi(\dot{x})u \quad (53)$$
where G is the mass, $g(x, \dot{x}) = D(c_1 x + c_2 \dot{x})$ is the nonlinear term with respect to the damper, $f(x) = c_3 x + c_4 x^3$ is the nonlinear term with respect to the spring, and $\phi(\dot{x}) = 1 + c_5 \dot{x}^3$ is the nonlinear term with respect to input force. In this example, the parameters are as follows:
$$G = 1, \; D = 1, \; c_1 = 0.5, \; c_2 = 1.726,$$
$$c_3 = 0.5, \; c_4 = 0.67, \; c_5 = 0 \quad (54)$$

By assuming $x \in [-1, 1]$ and $\dot{x} \in [-1, 1]$, we can get the following T-S fuzzy system
$$\begin{cases} \dot{\xi}(t) = \sum_{i=1}^{2} h_i (A_i \xi(t) + Bu(t)) \\ y = C\xi(t) + \omega(t) \end{cases} \quad (55)$$
where ω is the measurement output noise, $\xi(t) = (x \;\; \dot{x})^T$, $h_1 = 1 - x^2$, $h_2 = x^2$,
$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1.726 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0 & 1 \\ -1.67 & -1.726 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (56)$$

From A_1 and A_2 above, it can be computed that
$$P = \begin{bmatrix} 1.726 & 0.990 \\ 0.990 & 1.726 \end{bmatrix} \quad (57)$$
such that

$$A_i^T P + PA_i < 0\,,\quad i = 1, 2\,. \tag{58}$$

By setting $M = 1$, and from (40), it can be computed that

$$A_1' = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1.726 & 0 \\ -1 & 0 & -1 \end{bmatrix},\ E' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \overline{B} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$K = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ A_2' = \begin{bmatrix} 0 & 1 & 0 \\ -1.67 & -1.726 & 0 \\ -1 & 0 & -1 \end{bmatrix}. \tag{59}$$

Then the PD observer in the following form can be obtained immediately. Thus we construct the following PD observer

$$\begin{cases} E'\dot{\eta}(t) = \sum_{i=1}^{r} h_i\,(A_i'\eta(t) + \overline{B}u(t)) \\ \hat{\overline{x}}(t) = \eta(t) + Ky(t) \end{cases} \tag{60}$$

where $\hat{\overline{x}}(t)$ is the estimation of $\begin{bmatrix} x & \dot{x} & \omega \end{bmatrix}^T$.

Figures 1 and 2 respectively show the trajectories of x, $\dot{x}$, ω and their estimates with the initial conditions $x(0) = 0.1$, $\dot{x}(0) = -0.1$ and $\eta(0) = \begin{bmatrix} 0.5 & 0.1 & 0 \end{bmatrix}^T$. It can be seen that the properties of the tracking and convergence are desirable.

5. CONCLUSIONS

By transforming a T-S fuzzy system with measurement output noise into an augmented fuzzy descriptor systems, two kinds of new observer design approaches have been formulated. Using the proposed design techniques, the measurement output noise can be decoupled completely. Moreover, the system state and the noise can be asymptotically estimated directly. The present fuzzy observer design provides tools in many control topics such as fault detection and fault diagnosis etc.

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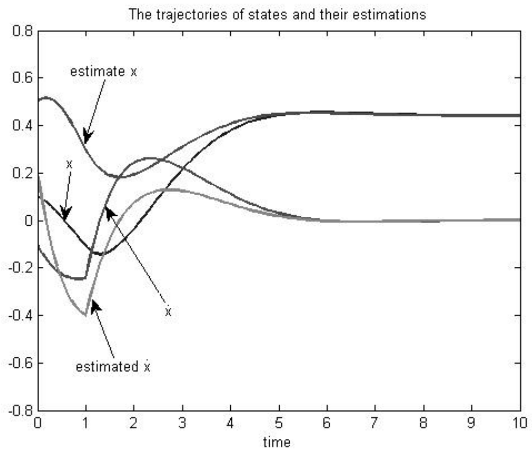

Fig. 1. The states and their estimates

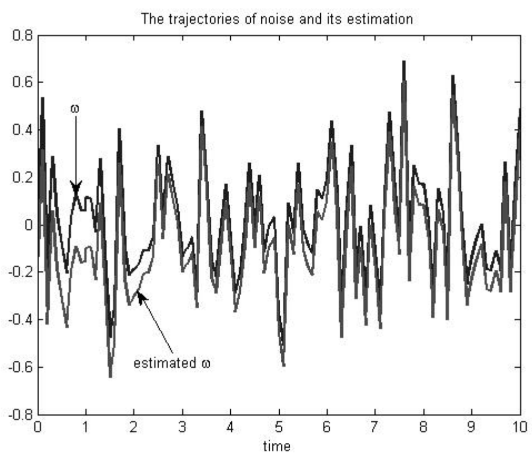

Fig. 2. The noise and its estimate