A NEW APPROACH FOR OPTIMAL CONTROL OF MULTIPLE- ARM ROBOTIC SYSTEMS

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Abstract: Optimal control frameworks are very important in optimization of multiple-arm robotic systems, although complexity, non-linearity and having large scales usually cause some computational problems in the capacity and time needed. In this paper, it is shown how by conducting the study of multiple-arm robotic systems, within a decomposition-coordination framework, the overall optimum can be achieved in only a few iterations. By using this methodology, the overall problem is considered as optimization of a two-level large-scale system. So with the aim of optimization, the problem is first decomposed into m sub-problems at the first level, where each sub-problem is solved using a typical gradient optimization method. Then by using the proposed methodology, which is based on the gradient of the interaction prediction errors, at the second level, the coordination of the overall system is done.

Keywords: Multiple-arm robotic systems, large-scale systems, interaction prediction principle, coordination, optimal control.

1. INTRODUCTION

Optimization and performance are the primary concern in most real world problems. Comparing the performance of a single human hand with that of two shows that the heavier tasks which are very hard to be done with only one hand, can be performed easily by using two hands. Similarly in robotic systems, the robot manipulators become more effective when two or more of them work together in an assembly line or in transportation. Several methods for coordination of multiple(dual)-arm robotic systems have been proposed, see (Uchiyama and Dauchez, 1998; Kokkinis, 1988; Hayati, et al., 1988; Kosuge, 1991).

Optimal control has great importance in multiple-arm robotic systems. Some optimal control problems are issued by Nakamura (1991). The dynamic redundancy is used in various optimal load distribution schemes that minimize the squared of the force magnitude with the object partitioned, see (Hayati, 1986). An arbitrary quadratic function of the forces aimed to penalize the use of moments; the internal force (Nakamura, et al., 1987) or the strain energy of the object has also been modeled as a collection of springs (Nakamura, 1988). Furthermore, several time-optimal controls have been proposed, see (Bobrow (1988)).

In this paper, a multiple–arm robotic system has been considered in a large–scale system framework. The notion of large-scale systems may be described as a complex system composed of a number of constituents or smaller sub-systems serving particular functions, share resources and governed by interrelated goals and constraints (Mahmoud, 1977; Jamshidi, 1983; Singh and Titly, 1978). Hierarchical control is an important approach in control of large-scale systems. The basic notion behind hierarchical control is decomposition of a given large-scale system into a number of small-scale systems, and then coordinating the resulted sub-systems (Mesarovic et al., 1970a, b).

Coordination of large-scale systems is mainly based on two principles; Interaction Prediction Principle (IPP) and Interaction Balance Principle (IBP). In this paper, a new formulation using the two-level gradient based coordination of large-scale systems is used to compute the optimal control of a multiple-arm robotic system. This new coordination scheme was first developed by Sadati (1997). For using this approach, the multiple-arm robotic system has first been decomposed into m sub-systems; arm1 , arm 2 , … , arm m-1, and a manipulated object as the mth one. So with the aim of optimization, the control problem is first decomposed into m sub-problems, corresponding to m sub-systems, at the first level, where each sub-problem is solved by using a gradient optimization method. Then by using the new methodology which is based on the gradient of the interaction prediction errors related to the predicted interactions, at the second level, the coordination of the overall system is done.

2. STATEMENT OF THE PROBLEM

For conducting the study of multiple-arm robotic systems, within a decomposition-coordination framework, first the concept of coordination is introduced and then, the Interaction Prediction Principle as a kind of coordinating the overall two-level large-scale systems is considered and finally, it is shown how the control problem of multiple-arm robotic systems can be solved using the Model Coordination and the IPP.

2.1 The Overall Control Problem

Let there be given an overall process $P : \boldsymbol{U} \rightarrow \boldsymbol{X}$ and a performance function $G : \boldsymbol{U} \times \boldsymbol{X} \rightarrow \boldsymbol{V}$ with $\boldsymbol{U}$ as the set of controls, $\boldsymbol{X}$ as the set of states, and $\boldsymbol{V}$ *as* the set of performance values. Let also g be defined on $\boldsymbol{U}$ by $g(U) = G[U , P(U)]$. Now, the goal of the overall control problem, which is denoted by D, is to find a control action $\hat{U}$ in $\boldsymbol{U}$ which minimizes g over $\boldsymbol{U}$. Such a control action will be referred to as the overall optimum.

2.2 The Infimal Control Problems

Let $\boldsymbol{U} = \boldsymbol{U}_1 \times ... \times \boldsymbol{U}_m$ and $\boldsymbol{X} = \boldsymbol{X}_1 \times ... \times \boldsymbol{X}_m$. For each i = 1, … ,m, let there be given the sub-systems $P_i : \boldsymbol{U_i} \times \boldsymbol{Z}_i \rightarrow \boldsymbol{X}_i$, with $\boldsymbol{Z}_i$ as the set of interface inputs, such that when inter-coupled as shown in Fig. 1, they form the overall process. For each i = 1 , ...,m, let the mapping $H_i : \boldsymbol{U} \times \boldsymbol{X} \rightarrow \boldsymbol{Z}_i$ gives the interface input appearing at the i-th sub-system, in the coupled system. The i-th infimal control problem is then formulated in terms of an objective function g_i given on $\boldsymbol{U}_i \times \boldsymbol{Z}_i$ in terms of the i-th sub-system and a performance function $G_i : \boldsymbol{U}_i \times \boldsymbol{Z}_i \times \boldsymbol{X}_i \rightarrow \mathbf{V}$, by

$$g_i (U_i , Z_i) = G_i [U_i , Z_i , P_i (U_i , Z_i)] \tag{1}$$

Now, two cases arise as to how the coordination might be effected and the infimal control problems can be defined. These two cases are called the Model Coordination and the Goal Coordination. Accordingly, two coordination principles, namely IPP and IBP are used.

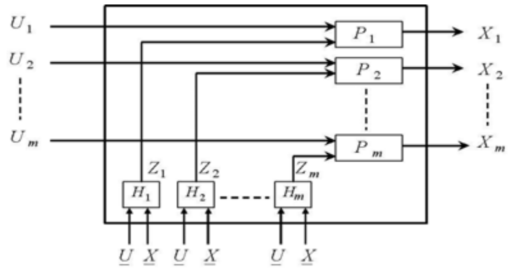

Fig. 1. Decomposition of the overall process P.

2.3 Model Coordination and Interaction Prediction Principle

Let $\boldsymbol{Z} = \boldsymbol{Z}_1 \times .... \times \boldsymbol{Z}_m$. Each $Z_{\boldsymbol{p}} = (Z_{p1} , ..., Z_{pm})$ in $\boldsymbol{Z}$ gives, for each $i = 1$, ... , m , the sub-system model $P_{iz_p}(U_i) = P_i(U_i, Z_{p_i})$. Then for each Z_p in $\boldsymbol{Z}$, the infimal control problem $D_i(Z_p)$ is to find a control $\hat{U}_i$ in $\boldsymbol{Ui}$ such that

$$g_i(\hat{U}_i, Z_{pi}) = \min_{\boldsymbol{U}_i} \; g_i(U_i, Z_{pi}) \tag{2}$$

where minimization is only over the set $\boldsymbol{U_i}$ of local controls. Let $Z_p = (Z_{p1}, ... , Z_{pm})$ be the predicted interface inputs and let Z_1 , ... , Z_m be the actual interface inputs occurring when the control $\hat{U}(Z_p) = [\hat{U}_1(Z_p), ..., \hat{U}_m(Z_p)]$ is implemented. The overall optimum is then achieved if the predicted interface inputs are correct (IPP) i.e. $Z_{pi} = Z_i$ *for all* i = *1* , ... ,*m*. If the IPP applies, it immediately yields that the supremal control problem D_o is to find $Z_p = (Z_{p1}, ... , Z_{pm})$ in $\boldsymbol{Z}$ such that $e_i = Z_i - Z_{pi} = 0$, for each i = 1,…, m. Alternatively, if e_i can not be made to be zero, the supremal control problem can be defined as minimization of an appropriate function of the errors e_1, …, e_m.

3. MODELLING THE MULTIPLE-ARM ROBOTIC SYSTEM

Consider m-1 arms firmly holding a single object; since the object is rigidly grasped by the end effectors, the elastic restrain motion of the object relative to the two grippers should be included in the dynamic model similar to (Kokkinis, 1988). The dynamic of each robot arm is given by (Kokkinis, 1988; Spong and Vidyasagar, 1989)

$$M_k(\underline{q}_k)\ddot{\underline{q}}_k + V_k(\underline{q}_k, \dot{\underline{q}}_k) + F_k(\dot{\underline{q}}_k) + G_k(\underline{q}) = \underline{U}_k + J_k^T \underline{f}_k \tag{3}$$

where $\underline{q}_k$ consists of the joint variables, $M_k(\underline{q}_k)$ is the inertia matrix, $V_k(\underline{q}_k, \dot{\underline{q}}_k)$ represents the coriolis/ centripetal vector, $G_k(\underline{q}_k)$ is the gravity vector, $F_k(\dot{\underline{q}}_k)$ is the friction vector, J_k is Jacobian Matrix, $\underline{f}_k$ is the external forces which are applied to the end effector by the object and finally, $\underline{U}_k$ represents the generalized forces. Now, the motion of the rigid object can be given by

$$M_0(\underline{X}_0)\ddot{\underline{X}}_0+V_0(\underline{X}_0,\dot{\underline{X}}_0)+F_0(\dot{\underline{X}}_0)+G_0(\underline{X}_0)=\sum_{i=1}^{m-1}\underline{f}_i=\underline{f}_{ext} \tag{4}$$

where $\underline{X}_0$ is the object position and $\underline{f}'_i$s are the forces applied by the arms to the object, described by (M_o,V_o,F_o,G_o), defined similar to (M_k,V_k,F_k,G_k) .

Now, for each arm, the forward kinematic mapping can be written as (Kokkinis, 1988)

$$\underline{X}_{ek}=P_k(\underline{q}_k) \tag{5a}$$

$$\dot{\underline{X}}_{ek}=J_k(\underline{q}_k)\dot{\underline{q}}_k \quad ; \; k=1,2,\ldots,m-1 \tag{5b}$$

On the basis of the assumption that the contact deformation is small, the contact of each end-effecter to the object can be modeled as a linear spring with strain energy given as

$$W_k=\frac{1}{2}(\underline{X}_0-\underline{X}_{ek})^T K_k(\underline{X}_0-\underline{X}_{ek}) \quad ; \; k=1,2,\ldots,m-1$$

where K_k is a positive definite weighting matrix describing the stiffness of springs. Now, by differentiating the above relation, we have

$$\underline{f}_k=-K_k(\underline{X}_o-\underline{X}_{ek}) \quad ; \quad k=1,2,\ldots,m-1$$

Now, let us assume that $\underline{X}_1=[\,\underline{q}_1^T \;\; \underline{q}_2^T \;\; \underline{X}_0^T]^T$, $\underline{X}_2=\frac{d\underline{X}_1}{dt}$

Thus, the state vector can be defined as $\underline{X}=\left[\underline{X}_1^T \;\; \underline{X}_2^T\right]^T$ and we can show that the discrete state space equation of the overall system can be given as

$$\dot{\underline{X}}=\underline{F}(\underline{X},\underline{U}) \tag{6a}$$

$$\underline{X}[0]=\underline{X}_0 \tag{6b}$$

where $\underline{X}$ is the state vector and $\underline{U}$ is the control vector. Also the initial state $\underline{X}_0$ is assumed to be known.

The control problem is now to find U which minimizes the cost function given by

$$J=G_{n+1}(\underline{X}[n+1])+\sum_{k=0}^{n}G_k(\underline{X}[k],\underline{U}[k]) \tag{7}$$

where G_k is in general a scalar non-linear function of its arguments.

4. THE TWO-LEVEL GRADIENT BASED METHOD

Let us assume that we have a general non-liner dynamic system described by the state space equations (6), where $\underline{F}$ is continuously double differentiable analytical function. The problem is to find $\underline{U}$ which minimizes the cost function given by (7).

4.1 Decomposition of the Overall Problem into M Sub-Problems

Suppose the overall system comprises of m sub-systems which are interconnected, and also each sub-system described by non-linear state space equations of the following form

$$\begin{cases}\underline{X}_i[k+1]=F_i(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_i[k]) \\ \underline{X}_i[0]=\underline{X}_{i0} \\ \underline{Z}_i[k]=H_i(\underline{X}[k])=H_i(\underline{X}_1[k],\ldots,\underline{X}_m[k])\end{cases} \tag{8}$$

where $\underline{X}_i$ is the state, $\underline{U}_i$ is the control and $\underline{Z}_i$ is the interaction input to the ith sub-system. The cost function can also be decomposed as

$$J=\sum_{i=1}^{m}J_i$$

So the overall problem can now be decomposed into m first-level sub-problems, described as

$$\min_{X_i,U_i} J_i=G_{i_{n+1}}\left(\underline{X}_i[n+1],\underline{Z}_i[n+1]\right)+\sum_{k=0}^{n}G_{ik}(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_i[k]) \tag{9}$$

$$s.t.\;\begin{cases}\underline{X}_i[k+1]=F_i(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_i[k]) & (10a)\\ \underline{X}_i[0]=\underline{X}_{i0} & (10b)\end{cases}$$

The second level problem can also be expressed as: Predicting the interactions in such a way that the real values of interactions; $\underline{Z}_i[k]=H_i(\underline{X}_1[k],\ldots,\underline{X}_m[k])$ become equal to the predicted interaction values; $\underline{Z}_{pi}[k]$ (IPP) .

4.2 Optimizing the First-Level Sub-Problems with Gradient Method

In the first level, the sub-problems can be defined as minimizing equation (9) subject to (10). Now assuming constant predicted values for interactions, from last iteration, we have $\underline{Z}_1[k]=\underline{Z}_{pi}[k]$. The necessary conditions for optimality can be written in terms of the Lagrangian L_i , in the following form

$$L_i=G_{i_{n+1}}\left(\underline{X}_i[n+1],\underline{Z}_i[n+1]\right)+\sum_{k=0}^{n}G_{ik}(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_i[k])$$
$$+\sum_{k=0}^{n}\underline{\lambda}_i^T[k]\left(\underline{X}_i[k+1]-F_i(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_i[k])\right)$$

so that the first order necessary conditions become

$$\frac{\partial L_i}{\partial \underline{\lambda}_i[k]}=\underline{X}_i[k+1]-F_i(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_i[k])=0$$

$$\frac{\partial L_i}{\partial \underline{X}_i[n+1]}=\frac{\partial G_{i_{n+1}}(\underline{X}_i[n+1],\underline{Z}_{Pi}[n+1])}{\partial \underline{X}_i[n+1]}+\underline{\lambda}_i[n]=0$$

$$\frac{\partial L_i}{\partial \underline{X}_i[k]}=\frac{\partial G_{ik}(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_{pi}[k])}{\partial \underline{X}_i[k]}$$
$$-\underline{\lambda}_i^T[k]\frac{\partial F_i(\underline{X}_i[k],\underline{U}_i[k],\underline{Z}_{pi}[k])}{\partial \underline{X}_i[k]}+\underline{\lambda}_i[k-1]=0$$

$$\frac{\partial L_i}{\partial \underline{U}_i[k]} = \frac{\partial G_{ik}(\underline{X}_i[k], \underline{U}_i[k], \underline{Z}_{pi}[k])}{\partial \underline{U}_i[k]} - \underline{\lambda}_i^T[k] \frac{\partial F_i(\underline{X}_i[k], \underline{U}_i[k], \underline{Z}_{pi}[k])}{\partial \underline{U}_i[k]} = 0$$

Now, in order to find the states and the controls that satisfy the above necessary conditions, an algorithm based on the gradient method, has been presented:

1) Choose initial values for $\underline{U}_i[0], \underline{U}_i[1], \ldots, \underline{U}_i[n]$.

2) Use $\underline{X}_{i0}$ and the initial values for $\underline{U}_i[0], \underline{U}_i[1], \ldots, \underline{U}_i[n]$, to find the values of $\underline{X}_i[1], \underline{X}_i[2], \ldots, \underline{X}_i[n+1]$ by equation (10a).

3) Calculate $\underline{\lambda}_i[k]$ for $k = n, n-1, \ldots, 0$ by using the necessary conditions, backward in time

$$\underline{\lambda}_i[n] = -\frac{\partial G_{i_{n+1}}(\underline{X}_i[n+1], \underline{Z}_{Pi}[n+1])}{\partial \underline{X}_i[n+1]}$$

$$\underline{\lambda}_i[k-1] = \underline{\lambda}_i^T[k] \frac{\partial F_i(\underline{X}_i[k], \underline{U}_i[k], \underline{Z}_{pi}[k])}{\partial \underline{X}_i[k]} - \frac{\partial G_{ik}(\underline{X}_i[k], \underline{U}_i[k], \underline{Z}_{pi}[k])}{\partial \underline{X}_i[k]}$$

4) Calculate $\frac{\partial L_i}{\partial \underline{U}_i[k]}$ for $k = n, n-1, \ldots, 0$ using $\underline{\lambda}_i[k]$

$$\frac{\partial L_i}{\partial \underline{U}_i[k]} = \frac{\partial G_{ik}(\underline{X}_i[k], \underline{U}_i[k], \underline{Z}_{pi}[k])}{\partial \underline{U}_i[k]} - \underline{\lambda}_i^T[k] \frac{\partial F_i(\underline{X}_i[k], \underline{U}_i[k], \underline{Z}_{pi}[k])}{\partial \underline{U}_i[k]}$$

5) Update $\underline{U}_i[k]$ by adding $\Delta \underline{U}_i[k] = -\eta \frac{\partial L_i}{\partial \underline{U}_i[k]}$ to prior values of $\underline{U}_i[k]$.

6) If $\sum_{k=0}^{n} \left\| \frac{\partial L_i}{\partial \underline{U}_i[k]} \right\|^2 < \varepsilon$ stop the algorithm, else go to step (2).

4.3 Solving the Second-Level Problem with the New Gradient Approach

In the first level, the cost functions J_i's are minimized, assuming $\underline{Z}_{pi}$ predicted by the second level using a gradient algorithm. In the second level, the values of $\underline{Z}_{pi}$ are updated in a direction which are getting closer to the real values of $\underline{Z}_i$ in every iteration using the new gradient algorithm with the following updating equation (Sadati, 1997; Sadati, 2000)

$$\underline{Z}_p^{new} = \underline{Z}_p^{old} - \eta P^T \underline{e}$$

where, $P = \frac{\partial \underline{e}}{\partial \underline{Z}_p}$ and $\underline{e} \triangleq \underline{Z} - \underline{Z}_p$.

4.4 Solution of the Multiple-Arm Robotic System Using the Two-Level Gradient Based Approach

Now let us decompose the multiple-arm robotic system into m sub systems; arm1, arm2, …, arm m-1 in addition to one object. Since there are interactions between sub-systems, the new equations describing the dynamic of each robot arm and the object can be given as

$$M_k(\underline{q}_k)\underline{\ddot{q}}_k + V_k(\underline{q}_k, \underline{q}_k) + F_k(\underline{\dot{q}}_k) + G_k(\underline{q}_k) = \underline{U}_k + J_k^T K_k(\underline{Z}_k - \underline{X}_{ek}); \quad k = 1, 2, \ldots, m-1$$

where $\underline{Z}_k = \underline{X}_0$, and

$$M_0(\underline{X}_0)\underline{\ddot{X}}_0 + V_0(\underline{X}_0, \underline{\dot{X}}_0) + F_0(\underline{\dot{X}}_0) + G_0(\underline{X}_0) = -\sum_{k=1}^{m-1} K_i(\underline{X}_0 - \underline{Z}_{mk})$$

where $\underline{Z}_{mk} = P_k(\underline{q}_k) = \underline{X}_{ek}$; $k = 1, 2, \ldots, m-1$.

Also lets assume that $\underline{X}_{k1} = \underline{q}_k$ and $\underline{X}_{m1} = \underline{X}_0$. Thus, the state space equations of each sub-system can be written as

$$\begin{cases} \underline{\dot{X}}_{k1} = \underline{X}_{k2} \\ \underline{\dot{X}}_{k2} = M_k^{-1}(\underline{X}_{k1})\left[-V_k(\underline{X}_{k1}, \underline{X}_{k2}) - F_k(\underline{X}_{k2}) - G_k(\underline{X}_{k1})\right] \\ \qquad + M_k^{-1}(\underline{X}_{k1})\left[\underline{U}_k + J_k^T K_k(\underline{Z}_k - P_k(\underline{X}_{k1}))\right] \end{cases}$$

k=1,2,..,m-1

$$\begin{cases} \underline{\dot{X}}_{m1} = \underline{X}_{m2} \\ \underline{\dot{X}}_{m2} = M_0^{-1}(\underline{X}_{m1})\left[-V_0(\underline{X}_{m1}, \underline{X}_{m2}) - F_0(\underline{X}_{m2}) - G_0(\underline{X}_{m1})\right] \\ \qquad + M_0^{-1}(\underline{X}_{m1})\left(-\sum_{k=1}^{m-1} K_i(\underline{X}_{m1} - \underline{Z}_{mk})\right) \end{cases}$$

where, after discretizing the above equations, the sub-system equations can be written as (9), where Z_i is of the following forms

$$\underline{Z}_i = \begin{cases} \underline{X}_{m1} & ; \quad i = 1, 2, \ldots, m \\ \left[Z_{m1}^T \quad Z_{m2}^T \quad \cdots \quad Z_{mm-1}^T\right]^T & ; \quad i = m \end{cases}$$

Now we can apply the two-level gradient based method to this decomposed system, where the local control problems can be described as in equation (9), subject to (10).

5. SIMULATION RESULTS

In this section, the proposed strategy is applied to a dual-arm robotic system. This system is shown in Fig. 2, and the corresponding parameters of the robot are listed in Tables 1, (Laroussi, et al., 1988).

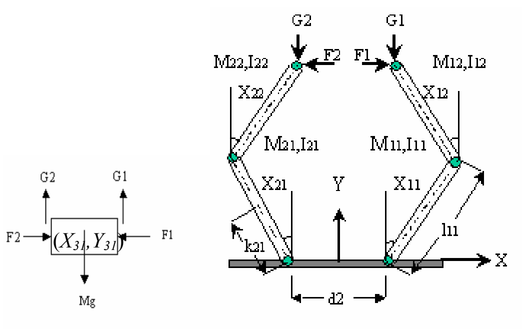

Fig. 2. Dual-arm robotic system with a load.

The dynamic equation of each sub-system follows equations described before. where $\underline{q}_1 = [X_{11} \quad X_{12}]^T$,

$\underline{q}_2 = [X_{21} \quad X_{22}]^T$, $\underline{X}_0 = [X_{31} \quad Y_{31}]^T$, also X_{11}, X_{12}, X_{21}, X_{22} are the joint variables of the robots, and X_{31}, Y_{31} are the position coordinates of the object. Moreover, the dynamic matrices are given by

$$M_i = \begin{bmatrix} I_{i1} + m_{i1}k_{i1}^2 + m_{i2}l_{i1}^2 & m_{i2}l_{i1}k_{i2}\cos(X_{i1} - X_{i2}) \\ m_{i2}l_{i1}k_{i2}\cos(X_{i1} - X_{i2}) & I_{i2} + m_{i2}k_{i2}^2 \end{bmatrix}$$

$$M_0 = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}$$

$$V_i = \begin{bmatrix} m_{i2}l_{i1}k_{i2}\sin(X_{i1} - X_{i2}).\dot{X}_{i2} \\ -m_{i2}l_{i1}k_{i2}\sin(X_{i1} - X_{i2}).\dot{X}_{i1} \end{bmatrix}$$

$$G_i = \begin{bmatrix} -(m_{i1}gk_{i1} + m_{i2}gl_{i1})\sin X_{i1} \\ -m_{i2}gk_{i2}\sin X_{i2} \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 \\ Mg \end{bmatrix}$$

$$J_i = \begin{bmatrix} l_{i1}\sin X_{i1} & (-1)^{i+1}l_{i1}\cos X_{i1} \\ l_{i2}\sin X_{i2} & (-1)^{i+1}l_{i2}\cos X_{i2} \end{bmatrix}$$

$$\underline{U}_i = [U_{i1} - U_{i2} \quad U_{i2}]^T$$

$$f_i = [F_i \quad G_i]^T$$

where G_i, F_i are the forces which are modeled by springs and are inserted by the object to the end effectors (i=1,2).

Table 1 Parameters of the Two-Arm Robot

Parameter	Link 11	Link 12	Link 21	Link 22
M (kg)	1	1	1	1
l (m)	0.25	0.25	0.25	0.25
k (m)	0.125	0.125	0.125	0.125
I (kg.m^2)	0.005	0.005	0.005	0.005

Also the load characteristics are given as; M = 0.25(kg) and d1= 0.7(m).

Now, we can present the above equations in state space forms, as described in equation (9).

Also a cost function can be assigned to the general two-arm robotic system, given by

$$J = \sum_{k=0}^{9} \left\| (f_{ext}[k] - f_m[k]) \right\|^2 + \left\| \underline{U}[k] \right\|^2$$

where f_m is a reference force applied to the object. The simulation results using T = 0.1 sec, and the stiffness of each spring taken as 100 N/m, with both the centralized approach (In this approach, the whole problem has been solved in one shot, using a typical gradient optimization method) and also the two-level gradient based method, are shown in Fig. 3 and Fig. 4, respectively.

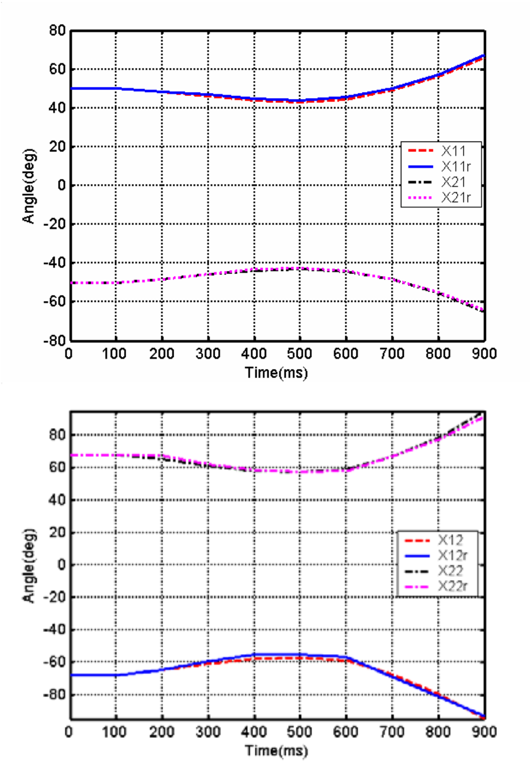

Fig. 3. Optimal state trajectory of the angles (centralized(r) and hierarchical).

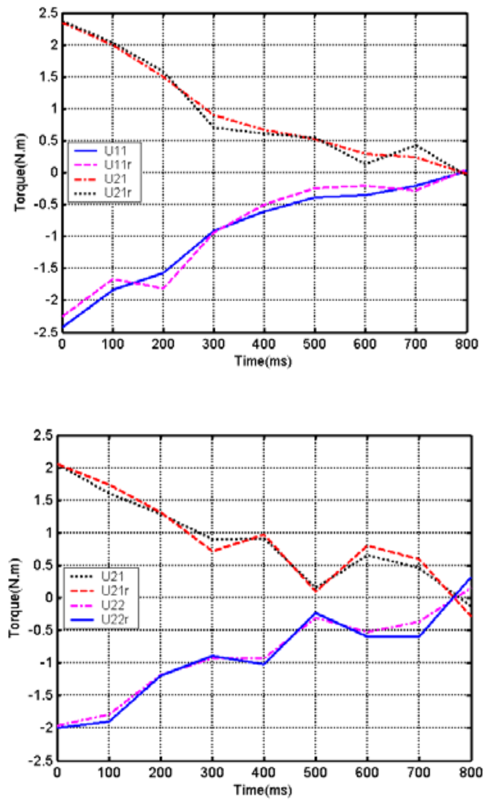

Fig.4. Optimal torques (centralized(r) and hierarchical).

It is shown that the two-level gradient based approach can converge to an small prescribed level of interaction errors in only 6 iterations, as is shown in Fig. 5. It should be noted that if the number of iterations increases, the real optimal solution can be achieved in only several iterations.

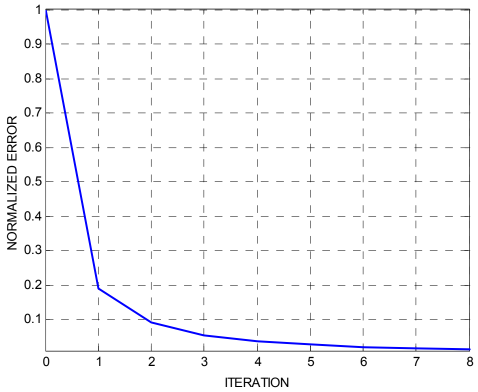

Fig.5. Normalized sum-squared error.

6. CONCLUSION

In this paper, a new approach for optimal control of multiple-arm robotic systems has been presented. Based on the approach taken in this study and the analysis of the results, the main features of this paper are reviewed briefly. First, a new control scheme for the multiple-arm robotic systems, based on the previously developed two-level gradient based coordination method for coordinating the two-level large-scale systems, is developed. Second, the simulation results show that the proposed approach can produce results close to the real optimal solution, as the number of iterations increase to several steps. Third, the resulting coordination strategy produces significant computational savings, because all calculations are realized using low-order sub-systems, where each uses the local information at sub-system level and the prediction of the interactions, from the second level, where by coordinating these solutions, the overall optimum can be achieved. Fourth, the proposed approach allows an efficient multiprocessor parallel implementation of the overall strategy. Fifth, the approach is extendable to solving the problems within the frameworks of hierarchical multi-level large-scale systems.

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