THE HIDDEN RISKS OF OPTIMIZING BOND PORTFOLIOS UNDER VAR

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Abstract: Value at risk (VaR) has become a standard measure of portfolio risk over the last decade. It even became one of the corner stones in the Basel II accord about banks' equity requirements. Nevertheless, the practical application of the VaR concept suffers from two problems: how to estimate VaR and how to optimize a portfolio for a given level of VaR? This application to bond portfolios shows that a solution to the two aforementioned problems gives raise to a third one: the actual VaR of bond portfolios optimized under a VaR constraint might exceed its nominal level to a large extent. Thus, optimizing bond portfolios under a VaR constraint might increase risk. This finding is of relevance not only for investors, but even more so for bank regulation authorities. *Copyright© 2005 IFAC*

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1. MOTIVATION

Value at risk (VaR) has become a standard measure of portfolio risk over the last decade. Given a portfolio with an initial value of V_0, the VaR for a given probability α until time τ are the losses which will not be exceeded until τ with probability α. With respect to the distribution of the future wealth, the VaR limits the α quantile that covers the worst outcomes. Alternatively, it can be interpreted as the loss limit that is likely to be exceeded only in a fraction of α of the following τ periods. While other risk measures (as, e.g., volatility) include both positive and negative deviations from the expected value, VaR captures only on losses, i.e., the common notion of risk that "things can go (utterly) wrong." The success of this concept might be attributed to three causes. First, the idea of VaR is highly intuitive and closely related to investors' goals. Second, VaR does not depend on any specific assumptions about return distributions or risk aversion. Third, and this might have been the crucial factor, VaR has been imposed on banks and other financial institutions by the Basel II accord about banks' equity requirements. [2] Consequently, VaR can be considered as a standard instrument in assessing portfolio risk and credit risk. Unfortunately, the implementation of VaR is hampered by two major problems. First, though easy to interpret, it turns out to be at least as difficult to estimate as any other risk measure. Second, when used as a constraint in portfolio optimization, the resulting

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[2] See (Basel Committee on Banking Supervision, 2003). For a critique of the Basel II proposals, see e.g., (Daníelsson *et al.*, 2001).

optimization problem cannot be dealt with using standard routines.

In order to deal with the first problem, three different approaches have been suggested and are used in practice: [3] (i) using parametric models by assuming certain distributional properties of asset returns, (ii) historic simulation, i.e. using an empirical distribution of asset returns based on past returns, and (iii) Monte Carlo approaches which typically combine (i) and (ii). These three methods are also explicitly sanctioned by the rules imposed in the Basel II accord, [4] i.e. banks are free to choose and implement one of the methods for assessing the VaR of their asset portfolios. Since parametric distributions (including the normal) have difficulties in describing financial data, the use of empirical distributions is an accepted or even favored alternative both in theory [5] and practice. [6]

The second problem results from the functional form of the risk constraint when using VaR in an optimization context. [7] This problem arises naturally from requirements imposed by the Basel II accord on institutional investors such as banks. According to the minimum capital requirements, banks have to underlie each investment with a certain amount of equity depending on the risk class of the assets considered. Therefore, banks might want to (i) lock as little equity as possible, or (ii) construct a yield maximizing portfolio that (just) meets the VaR limit given the existing equity. This paper focuses on the second case assuming that banks' equity is fixed in the short run.

After providing a solution approach to the two problems of portfolio optimization under VaR, a third problem emerges with important implications for investors and bank regulation. When choice of the assumed distribution is left to the portfolio manager / investor, she will have an incentive to choose the method that allows highest returns, especially when this method is supposedly more reliable than other methods and is used for ex post evaluation. [8] In the given setting this method turns out to be the historic simulation. While this method is useful to provide an ex post assessment of historic value at risk and also expected VaR for a given portfolio, it fails in a portfolio optimization setting. In fact, the optimization procedure results in high return portfolios just meeting the VaR constraint on the historic data. However, the actual VaR of these portfolios out of sample turns out to be much higher than its nominal level. In fact, optimizing portfolios under a VaR constraint typically results in portfolios with a VaR much higher than the defined constraint. This effect can be described as the hidden risk of optimizing portfolios under VaR. To our knowledge, this is the first paper to provide empirical evidence on this issue based on optimized portfolios.

The rest of this paper is organized as follows. Section 2 introduces the optimization problem, the data used for the empirical analysis, and the optimization heuristic to solve the complex portfolio optimization problem under VaR. In Section 3, some statistics on the distribution of asset returns and their stability are presented, before turning to the actual VaR of the optimized portfolios and summarizing the main findings. Section 4 concludes.

2. MODEL

2.1 The Optimization Problem

The investor for the chosen problem has an initial endowment of V_0 that can be either invested in bonds or kept as cash; without loss of generality, the rate of return of the latter is assumed to be zero. Given that the losses until time τ must not exceed a (fixed) value of $\underline{\delta}^{VaR} \cdot V_0$ with a given probability of α, and that this VaR constraint is the only constraint, a manager of a bond portfolio will be inclined to find a combination that has maximum expected yield that does not violate this VaR constraint.

The optimization model can therefore be written as

$$\max_{n_i} E\left(r_P\right) = \sum_i \frac{n_i \cdot L_i \cdot D_{i,0}}{V_0} \cdot r_i$$

s.t.

$$\sum_i n_i \cdot L_i \cdot D_{i,0} \leq V_0, \quad n_i \in \mathbb{N}_0^+ \forall i$$

$$prob\left(V_\tau \leq V_0 \cdot \left(1 - \underline{\delta}^{VaR}\right)\right) = \alpha$$

where L_i and $D_{i,0}$ are the lot size (in CHF) and current clean price (in per cent), respectively, of bond i, and r_i is its yield to maturity. n_i is the number of lots kept in the portfolio which has to be non-negative. Moreover, the cash position must also be non-negative. V_τ is the value of the

[3] See, e.g., (Jorion, 2000).
[4] (Basel Committee on Banking Supervision, 2003), §490 (c) states: "No particular type of VaR model (e.g. variance-covariance, historical simulation, or Monte Carlo) is prescribed. However, the model used must be able to capture adequately all of the matrial risks exposure of the institution's equity portfolio."
[5] See, e.g. (Jorion, 2000), (Pritsker, 1997) or (Lucas and Klaasen, 1998).
[6] cf. footnote 4.
[7] In this context, see also (Basak and Shapiro, 2001) and (Alexander and Baptista, 2003).
[8] cf. (Basel Committee on Banking Supervision, 2003) §§149 and 151 – 152.

portfolio at time τ (consisting of the value of the bonds including accrued interest) plus cash. For estimating V_τ, the following methods are applied:

- Assuming *normal distribution*, the VaR constraint can be rewritten as
$$E(V_\tau) - |u_\alpha| \cdot \sigma_{V_\tau} \geq V_0 \cdot \left(1 - \underline{\delta}^{VaR}\right)$$
where u_α is the respective quantile of the standard normal distribution s.t. $N(u_\alpha) = \alpha$. The expected value for V_τ and its volatility are alternatively estimated from past observations either in a standard way ("plain vanilla" or "pv" henceforth) or with weighted values where more recent observations contribute stronger. The latter version turned out advantageous for stock portfolios in a similar setting (see (Maringer, 2005)) with decay factor of 0.99 which is applied here, too. The weights are therefore $w_s = \frac{0.99^{(S+1)-s}}{\sum_{t=1}^{S} 0.99^t}$ where the simulations are ordered chronologically and $s = 1$ is the simulation based on the oldest, $s = S$ on the most recent of the S observations.
- Assuming *empirical distribution*, the VaR constraint can be rewritten as $\sum_s b_s \leq \alpha$ where $b_s = 0$ if the VaR limit is not exceeded, otherwise $b_s = 1/S$ ($b_s = w_s$ as defined above) for the plain vanilla (weighted average) version.

For the main computational study presented in the following sections, the investor will be endowed with V_0 = CHF 1,000,000, and the VaR constraint will be that the next day's wealth will not be below 990,000 (i.e., $\underline{\delta}^{VaR} = 0.01$) with a probability of $\alpha = [0.025; 0.05; 0.1]$. With respect to the available data for the empirical study, these default probabilities α are higher than those usually applied in practice; lower values for α would demand longer time series and are prone to unwanted data fitting. In preliminary studies, longer data series were used and alternative values for α and $\underline{\delta}^{VaR}$ were investigated. The findings confirmed the qualitative results reported for the main study and are therefore omitted in the sense of brevity.

2.2 Data

The computational study is based on the fixed coupon bonds quoted on the Swiss stock exchange in local currency, i.e. CHF. From all quoted bonds, 42 Swiss and 113 foreign issuers are chosen randomly, though it was sought that no industry sector or issued volume is over- or under represented. For these bonds, daily (clean) closing prices (when traded) for the period January 1999 through June 2003 are available. All included bonds have a time to maturity of at least two years (typically five years) and the median issued amount is CHF 100,000,000 and CHF 200,000,000 for domestic and international bonds, respectively.

From this data set, random selections of bonds were drawn by first choosing a random date and then selecting $N = 10$ (20) different bonds. Any of these selections was accepted only if a minimum number of different quotes within the in sample as well as the out of sample time frames were observed (in sample frame: chosen date plus 200 in sample days; out of sample frame: the subsequent 100 trading days). For both values of N, 250 of such case sets were generated independently.

2.3 Optimization Method

Due to the type of the risk constraint combined with the integer constraint on the number of traded lots and the non-negativity constraint, the optimization problem cannot be solved analytically, but it can be approached with heuristic optimization techniques (HO). The recent literature holds several examples for successful applications of HO to portfolio optimization, including optimization under different risk measures (e.g., (Dueck and Winker, 1992)), cardinality constraints and integer constraints (e.g., (Chang *et al.*, 2000) or (Maringer and Kellerer, 2003)), index tracking (e.g. (Gilli and Këllezi, 2002*a*)) or optimization under VaR constraints (e.g. (Gilli and Këllezi, 2002*b*) or (Maringer, 2005)). For the given optimization problem, a modified version of *Memetic Algorithms*[9] is used where principles of heuristic local search are combined with evolutionary search strategies. This approach has proofed useful and reliable in (Maringer and Winker, 2003) where stock portfolios are optimized under Value at Risk and where the algorithm and its characteristics are presented in more detail. The implementation was done on two standard Pentium IV computers using Matlab 6. The different values for the shortfall probability α, the considered methods for estimating a portfolio's VaR, and the number of different case sets resulted in 6,000 different optimization problems for the main computational study. Each of these was solved repeatedly and independently, and the best found solution of any of the runs was used for the subsequent analyses. Depending on the problem size and distributional assumptions, the computational time ranged approximately from 10 to 20 seconds per run.

[9] See (Moscato, 1989) and (Maringer and Winker, 2003).

3. RESULTS

3.1 Distribution

The decision of whether to estimate the VaR with the normal (or any other parametric) rather than the empirical distribution depends on how well the main properties of the observed data for the assets (or at least, via the CLT, the resulting portfolios) can be captured with the parametric distribution. For the given data set, the portfolio values appear far from normally distributed: regardless of the method for VaR estimation, there is hardly any optimized portfolio where the null hypothesis of normal distributed price changes cannot be rejected at the usual 5% level of significance both based on a standard Jarque-Bera test. Using a Kolmogorov-Smirnow test, or the Selector statistics (see (Schmid and Trede, 2003)) the picture remains more or less unchanged. Evaluating the in sample data, only for some 98% of the optimized portfolios, the H_0 of normally distributed returns has to be rejected, and for the first 100 out of sample days, the rejection rate is still 90% and higher. At the same time, a Kolmogorov-Smirnow test suggests that for 80% of the portfolios, the same in sample and out of sample distributions are the same. It appears remarkable that these rates are the virtually the same regardless of the number of different assets in the portfolio, N, the confidence level, α, – and the four distribution assumption, i.e., parametric and empirical in their"plain vanilla" and "weighted average" versions. Also, testing the distributions of non-optimized portfolios and of the individual assets leads to similar results. At first sight, this seems to confirm the view that the normality assumption in the optimization process might be inadequate and that the use of empirical distributions might be the better choice.

To test whether the distributions are stable and allow reliable estimates of the VaR, random weights for any portfolio in the two case sets are repeatedly generated where the integer and the budget constraints are the only restrictions. Then, the share of portfolios with out of sample losses higher than the expected VaR is determined. As can be seen in the upper part of Table 1 for the first out of sample day, the use of the empirical distributions allows for estimations of the VaR such that the frequency of larger losses corresponds more or less to the respective confidence level. Under the normality assumption, higher values for α result in overly cautious estimations of the VaR – violations of which occur less often than expected. In particular for higher values of α, the empirical distribution produces more reliable results than the normal distribution. This relative advantage remains unaffected when longer out of sample periods are used for evaluation; the respective statistics are therefore omitted in the sense of brevity.

3.2 The Hidden Risks in Optimized Portfolios

Unlike portfolios without optimization, the value of portfolios that are optimized under the empirical distribution will fall significantly more often below $V_0 \cdot E\left(\delta^{VaR}\right)$, the expected VaR [10], than the chosen confidence level α. Out of sample the actual frequency of excessive shortfalls will be 1.5 to three times the frequency originally expected (depending on α and case set; see Table 1). When the same portfolios are optimized under the normal distribution, however, the frequency will be underestimated only for small α's, for high confidence levels, on the other hand, the frequency will be overestimated, i.e., the VaR is estimated too cautiously. The assumption of the normal distribution leads (for both optimized and random portfolios) to more cautious estimates of the VaR when α is high. The extreme leptokurtosis of the actual distributions cannot be captured by the normal distribution, and as a result it is hardly possible to get reliable estimates for the VaR limit: For large values of α, the VaR limit is estimated too far away from the expected value, for lower values, however, the actually realized values are within the bandwidth of their accepted value. [11] The empirical distribution virtually always leads to highly significant deviations between the accepted and the actually encoutered shortfall probability, whereas under the normal, virtually all of the actually realized shortfall probabilities are within the accepted range.

The advantage of the empirical over the normal distribution that had been identified for non-optimized portfolios and the statistical properties of the actual distribution, seems therefore lost and in some cases even reverted into the opposite when a VaR constraint is used in the optimization process. Despite its specification errors, the normal distribution seems to cause less problems than empirical distribution that has been shown to be closer to reality for the single assets and non-optimized portfolios.

The major reason for this is that VaR is a quantile risk measure and therefore focuses on the number

[10] Due to the specification and the chosen assets, the critical VaR, the out of sample data were compared to, is set to $E\left(\delta^{VaR}\right) \leq \underline{\delta}^{VaR}$, the loss actually expected with the planed probability of α. Statistical significance tested with the statistic by (Kupiec, 1995).

[11] For very small values of α the opposite can be observed: the VaR is underestimated, and the limit is violated too often. With respect to the data set, however, tests with smaller values of α than the ones presented were not possible, a more detailed discussion of these effects has therefore be left to future research.

variant / method	$N=10, \alpha=...$ 2.5%	5.0%	10.0%	$N=20, \alpha=...$ 2.5%	5.0%	10.0%
random weights, $T_{oos}=1$						
empirical	2.6 %	4.2%	9.1%	3.2%	5.6%	10.2%
empirical, weighted	2.4%	4.1%	8.2%	3.0%	5.4%	9.9%
normal	2.9%	4.1%	6.4%*	3.2%	5.3%	8.1%
normal, weighted	2.8%	4.0%	6.2%*	3.1%	5.1%	8.0%
optimized weights, $T_{oos}=1$						
empirical	5.2%**	7.6%	16.8%***	8.8%***	10.8%***	16.9%***
empirical weighted	6.0%***	8.0%*	15.2%***	8.4%***	10.8%***	16.1%***
normal	3.2%	3.6%	7.2%	4.4%	6.8%	8.4%
normal weighted	3.2%	4.0%	6.0%**	3.6%	5.6%	6.8%
optimized weights, $T_{oos}=100$						
empirical	7.8%***	10.6%***	18.6%***	10.4%***	14.0%***	20.1%***
empirical weighted	7.8%***	10.5%***	17.2%***	10.1%***	13.4%***	19.2%***
normal	6.2%***	7.5%	9.6%	8.0%***	9.6%***	12.1%
normal weighted	6.0%***	7.2%	9.3%	7.7%	9.2%***	11.6%

Fig. 1. Shortfall ratios on the first T_{oos} out of sample days (difference statistically significant at the 5% (*), 2.5% (**) and 1%(***))

of shortfalls rather than their magnitude.[12] This can be exploited when empirical distributions are used. When optimizing under an empirical distribution, a number of excessive losses beyond the specified VaR limit will contribute equally to the confidence level α as would the same number of small losses; the optimization process will therefore "favor" those losses that come with high yields. Since it is usually the high yield bonds that exhibit massive losses in the past, these bonds will be given high weights. The problems arising from this effect are reinforced when the high yield of a bond comes from a small number of high losses rather than several small losses: a loss beyond the specified VaR limit will be considered a rare event, and the loss limit estimated with the confidence level α will be distinctly below the accepted limit, i.e., $E\left(\delta^{VaR}\right) \ll \underline{\delta}^{VaR}$. Out of sample, this expected limit might turn out to be too optimistic and is therefore violated too often, hence the actual out of sample shortfall rate is distinctly higher than the originally accepted level of α.

In addition, there is a hidden danger of data fitting for the empirical distribution: Slight in sample violations of the specified VaR limit of $\underline{\delta}^{VaR}$ can (and will) sometimes be avoided by slight changes in the combination of assets' weights that have only a minor effect on the portfolio yield. As a consequence, there might be more cases close to the specified VaR than the investor is aware of since they are just slightly above the limit and therefore do not count towards the level α; out of sample, however, this hidden risk causes more shortfalls than expected.[13]

The consequences of these effects are twofold: First, the "empirical" optimizer underestimates the chances for exceeding the VaR limit since the scenarios where the limit is narrowly not exceeded in sample have a fair chance of actually exceeding it out of sample – hence the percentage of cases or days with excessive losses is higher than α, i.e., the expected percentage. Second, since the "empirical" optimizer does accept extreme losses in sample, she has a good chance of facing them out of sample as well. The "empirical" investor will therefore not only encounter losses exceeding the estimated VaR limit more frequently than the "normal" investor, the "empirical" investor's losses will also be higher. It is noteworthy that these "hidden risks" are not the result from including securities with some fancy return distribution that obviously exploit VaR's properties.

To what extent the deficiencies of empirical distribution are exploited in the optimization process depends on several aspects where the number of the in sample observations or simulations certainly is a very crucial one. Long time series, however, are not always available nor can they be reliably generated,[14] in addition the stability of the distribution becomes a major issue, and including more historic data might bring only diminishing contributions when weighted values (or alternative prediction models such as GARCH models) are used. Detailed tests of these aspects, however, were not possible with the available data and have therefore to be left to future research.

[12] See also (Artzner *et al.*, 1999).

[13] Because of the peakedness and the discussed effect, that $E\left(\delta^{VaR}\right) < \underline{\delta}^{VaR}$ for larger values of α, this effect of data fitting does not show as often as for assets with other empirical distributions (see (Maringer, 2005)).

[14] The problem of small sample sizes becomes even more apparent when, e.g., credit portfolios are considered instead of publicly traded assets.

4. CONCLUSION

During the last years, Value at Risk has become an industry standard and has been imposed by the Basel II accords on equity requirements. Meanwhile, the literature has pointed out several shortfalls and theoretical caveats that are associated with the nature of this risk measure. This paper adds another aspect to this discussion: the pitfalls that might come when VaR is used not only for evaluation purposes of assets or portfolios, but already as an explicit constraint on the risk in the portfolio optimization process itself. Meant as answer to the problem of estimating the amount of capital that is at stake with a given confidence level, the suggested solution causes serious new problems.

The findings from an empirical study are that exact methods for estimating the risk (such as the use of empirical distributions) favor portfolios that actually have serious hidden risk by exploiting the nature and definition of the risk constraint; on the other side, inexact methods (such as the normality assumption) have less hidden risk, but are error prone because of their specification errors. Hence, neither approach appears capable of justifying VaR as a sole risk measure in the context of portfolio optimization. The identified effects are neither in the interest of the investor nor the regulator. One central issue for future research is therefore to find a way where either the advantages of the (originally reliable and flexible) empirical distributions can be preserved for optimization or the accuracy of parametric can be improved.

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