R&D INCENTIVES UNDER BERTRAND COMPETITION: A DIFFERENTIAL GAME [1]

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Abstract: We investigate dynamic R&D for process innovation in an oligopoly where firms invest in cost-reducing activities. We focus on the relationship between R&D intensity and market structure, proving that the industry R&D investment monotonically increases in the number of firms. This result contradicts the established wisdom acquired from static games on the same topic. We also prove that, if competition is sufficiently tough, any increase in product substitutability reduces R&D efforts.

Keywords: Economic Design, Economic Systems, Optimal Control

1. INTRODUCTION

Strictly speaking, R&D activity consists in finding out basic innovations (be they pathbreaking or just incremental) and subsequently developing them into new technologies or components that can be embodied into final products and ultimately marketed. *Per se*, innovative activity is inherently an engineering one. However, the volume of funds devoted to R&D by both firms and governments indeed depends upon economic incentives. These,in turn, are shaped to a large extent by the structure of the market where the products incorporating innovations are sold to their final users, and the intensity of competition faced by those firms operating in such a market. Therefore, the allocation of resources to innovation, and the actual performance of R&D activity are both conditional upon economic factors. In order to investigate some aspects of this problem, we propose a dynamic analysis of the relationship between market power and R&D efforts, in order to reassess a well-known issue in the theory of industrial organization, that can be traced back to the debate between Schumpeter (1942) and Arrow (1962), about the bearings of the intensity of market competition on the pace of technical progress. The so-called Schumpeterian hypothesis maintains that there exists an inverse relationship between the intensity of competition and the pace of technical progress. That is, according to Schumpeter, monopoly is the market structure that should ensure the fastest and largest technical progress. This relies upon the idea that monopoly ensures the highest profit level and therefore the larger internal sources for funding R&D activities.

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Exactly the opposite view is expressed by Arrow, since he focuses upon the replacement effect, according to which a monopolist should be induced to rest on his laurels, while a firm operating in a competitive environment should strive for new technologies or new products, in order to throw her rivals out of business.While the Arrowian position measures the intensity of market competition in terms of market structure (i.e., the number of firms), the interpretation of the Schumpeterian hypothesis is a bit looser, and several versions have been alternatively investigated in the literature.[2]

In order to assess this issue, we consider an oligopoly where n firms sell a homogeneous product and compete in prices. Moreover, they also invest at each point in time in R&D for process innovation, i.e., reducing the marginal production cost of the good. R&D activity is characterized by positive externalities, i.e., each firm receives a positive spillover from the investments carried out by all other firms in the industry.

The game is *state-redundant* or *perfect*, so that the open-loop solution is a Markovian equilibrium. We proceed in two steps. First, we characterize the individually optimal path of R&D investment for a given level of marginal production cost. Second, we obtain the steady state levels of investment and marginal cost. With respect to both the optimal path and the steady-state level of R&D investment, the following conclusions hold. The individual effort is always decreasing in the number of firms while the opposite holds for the aggregate R&D investments. This result has an Arrowian flavour, since as the degree of competition becomes tougher, the aggregate investment becomes larger. This is in sharp contrast with the conclusions drawn from the static version of the same model (Hinloopen, 2000) where a non-monotone relationship exists between aggregate R&D investment and market structure. Under this perspective, our model highlights the value added of a properly dynamic analysis over the static approach based upon a multistage game. Then, we also evaluate the effect of product differentiation on R&D efforts. We find that (i) along the equilibrium path, the individual as well as the industry incentive to invest is increasing in the degree of product differentiation (provided that the number of firms is large enough), while (ii) the steady state R&D efforts are completely unaffected by product differentiation. Result (i) is clearly Schumpeterian in spirit, since any increase in product differentiation translates into a milder price competition on the market; hence, in such a case we may put forward a Schumpeterian argument according to which softening competition by reducing the degree of product substitutability ultimately induces firms to increase their R&D investments. This of course enhances technical progress.

The remainder of the paper is structured as follows. Section 2 illustrates the basic setup. The solution of the open-loop game is investigated in section 3, while section 4 contains comparative statics. Concluding remarks are in section 5.

2. THE SETUP

We consider an oligopoly with n single-product firms selling differentiated goods over continuous time, $t \in [0, \infty)$. At every t, firm i's inverse demand function is $p_i(t) = A - q_i(t) - s\sum_{j \neq i} q_j(t)$, so that the direct demand writes as follows (Spence, 1976):

$$q_i(t) = \frac{A}{1+s(n-1)} - \frac{(1+s(n-2))\,q_i(t)}{(1-s)\left[1+s(n-1)\right]} + \\ + \frac{s}{(1-s)\left[1+s(n-1)\right]} \sum_{j \neq i} p_j(t) \qquad (1)$$

where A is market size and $s \in [0, 1)$ measures the degree of substitutability between any two varieties: the higher is s, the lower is differentiation.[3] $p_i(t)$ is the market price chosen by firm i. Each firm produces at a constant marginal cost, c_i. Accordingly, her instantaneous cost function for the production of the final good is $C_i(c_i, q_i, t) = c_i(t)q_i(t)$. The marginal cost of firm i evolves over time according to the following equation:

$$\frac{dc_i(t)}{dt} \equiv \dot{c_i} = c_i(t)\left[-k_i(t) - \beta K_{-i}(t) + \delta\right] \qquad (2)$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t, while $K_{-i}(t)$ is the aggregate R&D effort of all other firms and parameter $\beta \in [0, 1]$ measures the positive technological spillover that firm i receives from the R&D activity of the rivals.[4] Parameter $\delta \in [0, 1]$ is a constant depreciation rate measuring the instantaneous decrease in productive efficiency due to the ageing of technology. The instantaneous R&D cost is:

$$\Gamma(k_i, t) = b\left[k_i(t)\right]^2, \qquad (3)$$

where b is a positive parameter. Throughout the game, firms discount future profits a the common and constant discount rate $\rho > 0$.

Firms adopt a strictly noncooperative behaviour in choosing both the output levels and the R&D

[2] Influential studies of the relationship between market structure and innovation are those of Flaherty (1980) and Spence (1984). For an exhaustive overview of the related literature, see Reinganum (1989) and Martin (2001).

[3] For a model where s is a state variable changing because of R&D for product innovation, see Cellini and Lambertini (2002, 2004).

[4] As in d'Aspremont and Jacquemin (1988).

efforts, each firm operating her own R&D division.[5] The objective of firm i consists in maximizing discounted profits:

$$\Pi_i = \int_0^\infty \left\{ [p_i(t) - c_i(t)] \left[\frac{A}{1+s(n-1)} + \right. \right. \tag{4}$$
$$- \frac{(1+s(n-2))\, q_i(t)}{(1-s)[1+s(n-1)]} +$$
$$\left. \left. \frac{s \sum_{j\neq i} p_j(t)}{(1-s)[1+s(n-1)]} \right] - b[k_i(t)]^2 \right\} e^{-\rho t} dt$$

subject to the set of dynamic constraints (2). The corresponding Hamiltonian function is:

$$\mathcal{H}_i(\mathbf{p},\mathbf{k},\mathbf{c}) = e^{-\rho t}\{[p_i(t) - c_i(t)]\, q_i(t) + \tag{5}$$
$$-b[k_i(t)]^2 - \lambda_{ii}(t) c_i(t)[k_i(t) + \beta K_{-i}(t) - \delta] +$$
$$- \sum_{j\neq i} \lambda_{ij}(t) c_j(t) [k_j(t) + \beta(k_i(t) + \sum_{l\neq i,j} k_l(t)) - \delta]\}$$

where $\lambda_{ij}(t) = \mu_{ij}(t) e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c_j(t)$, $q_i(t)$ is defined as in (1) and $\mathbf{p},\mathbf{k},\mathbf{c}$ are the vectors of control and state variables.

3. THE OPEN-LOOP SOLUTION

Here we characterize the Nash equilibrium under the open-loop information structure. As a first step, we prove the following result:

Lemma 1. The open-loop Nash equilibrium of the game is subgame (or Markov) perfect.

Proof. We are going to show that the present setup is a *perfect game* in the sense of Leitmann and Schmitendorf (1978) and Feichtinger (1983). In summary, a differential game is *perfect* whenever the closed-loop equilibrium collapses into the open-loop one, the latter being thus strongly time consistent, i.e., subgame perfect.[6] Consider the closed-loop information structure. The relevant first order conditions (FOCs) are:[7]

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = \frac{1}{(1-s)\Upsilon}\{[c_i - 2p_i][1+s(n-2)] +$$
$$+ A(1-s) + s\sum_{j\neq i} p_j\} = 0 \tag{6}$$

where:

$$\Upsilon \equiv 1 + s(n-1); \tag{7}$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = -2bk_i - \lambda_{ii} c_i - \beta \sum_{j\neq i} \lambda_{ij} c_j = 0. \tag{8}$$

As a first step, observe that (6) only contains firm i's state variable, so that in choosing the optimal output at any time during the game firm i may disregard the current efficiency of the rival. That is, there is no feedback effect in the output choice. Conversely, at first sight there seem to be a feedback between the R&D decisions, as (8) indeed contains all state variables, at least for any positive spillover effect.[8] The core of the proof consists in showing that no feedback effect are actually present, even for positive spillover levels.

Taking the above considerations into account, the adjoint or co-state equations are:

$$-\frac{\partial \mathcal{H}_i}{\partial c_i} - \sum_{j\neq i} \frac{\partial \mathcal{H}_i}{\partial k_j}\frac{\partial k_j^*}{\partial c_i} = \frac{\partial \lambda_{ii}}{\partial t} - \rho\lambda_{ii} \tag{9}$$

yielding:

$$\frac{\partial \lambda_{ii}}{\partial t} = q_i(t) + \lambda_{ii}(t)[k_i(t) + \beta K_{-i}(t) +$$
$$+ \rho - \delta] - \frac{\beta}{2b}\sum_{j\neq i} \lambda_{ji}(t)[\beta\lambda_{ii}(t)c_i(t) +$$
$$+ \lambda_{ij}(t)c_j(t) + \beta \sum_{l\neq i,j} \lambda_{il}(t) c_l(t)] \tag{10}$$

and:

$$-\frac{\partial \mathcal{H}_i}{\partial c_j} - \frac{\partial \mathcal{H}_i}{\partial k_i}\frac{\partial k_i^*}{\partial c_j} +$$
$$- \sum_{l\neq i,j} \frac{\partial \mathcal{H}_i}{\partial k_l}\frac{\partial k_l^*}{\partial c_j} = \frac{\partial \lambda_{ij}}{\partial t} - \rho\lambda_{ij} \tag{11}$$

where each term

$$\frac{\partial \mathcal{H}_i}{\partial k_j}\frac{\partial k_j^*}{\partial c_i} \tag{12}$$

captures the feedback effect from j to i, and partial derivatives $\partial k_j^*/\partial c_i$ are calculated using the optimal values of investments as from FOC (8), $k_j^* = -(\lambda_{jj}c_j + \beta\lambda_{ji}c_i)/(2b)$. Now note that $\partial\mathcal{H}_i/\partial k_i = 0$ by virtue of (8). Hence, (11) yields:

$$\frac{\partial \lambda_{ij}}{\partial t} = \lambda_{ij}\left(k_j + \beta k_i + \beta\sum_{l\neq i,j} k_l + \rho - \delta\right) +$$
$$-\frac{\beta}{2b}\sum_{l\neq i,j}\lambda_{lj}\left(\beta\lambda_{ii}c_i + \lambda_{il}c_l + \beta\sum_{j\neq i,l}\lambda_{ij}c_j\right) \tag{13}$$

These conditions must be evaluated along with the initial conditions $\{c_i(0)\} = \{c_{0,i}\}$ and the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{ij}c_j = 0\ ,\ i,j = 1,2. \tag{14}$$

Note that, on the basis of *ex ante* symmetry across firms, $\lambda_{lj} = \lambda_{ij}$ for all l. Accordingly, from (13),

[5] For a discussion of R&D cooperation in the same model, see Cellini and Lambertini (2003).

[6] The label 'perfect game' is due to Fershtman (1987), where one can find a general technique to identify any such games. Another class of games where open-loop equilibria are subgame perfect is investigated by Reinganum (1982). For further details, see Mehlmann (1988, ch. 4) and Dockner *et al.* (2000, ch. 7).

[7] Henceforth, the indication of time and exponential discountig is omitted for brevity.

[8] Intuitively, if $\beta = 0$, then the investment plans are completely independent and therefore it is apparent that no feedback effect operates.

we have that $\partial\lambda_{ij}/\partial t = 0$ admits $\lambda_{ij} = 0$ as a solution. Then, using this piece of information, we may rewrite the expression for the optimal investment of firm i as follows:

$$k_i^* = -\frac{\lambda_{ii}c_i}{2b}\,, \tag{15}$$

which entails that $\partial k_i^*/\partial c_j = 0$ for all $j \neq i$, i.e., feedback (cross-)effects are nil along the equilibrium path. Accordingly, the open-loop equilibrium is a degenerate closed-loop one, and it is strongly time consistent, or equivalently, subgame perfect. It is also worth observing that this procedure shows that FOCs are indeed unaffected by initial conditions as well. The property whereby the FOCs on controls are independent of states and initial conditions after replacing the optimal values of the co-state variables is known as *state-redundancy*, and the game itself as *state-redundant* or *perfect*.■

On the basis of Lemma 1, we can proceed with the characterization of the open-loop solution. The FOCs on controls as well as the transversality conditions are the same as above, while the co-state equations simplify as follows:

$$-\frac{\partial\mathcal{H}_i}{\partial c_i} = \frac{\partial\lambda_{ii}}{\partial t} - \rho\lambda_{ii} \Leftrightarrow \tag{16}$$

$$\frac{\partial\lambda_{ii}}{\partial t} = \lambda_{ii}\left[k_i + \beta K_{-i} + \rho - \delta\right] + $$
$$+\frac{A(1-s) - p_i\left[1 + s(n-2)\right] + s\sum_{j\neq i} p_j}{(1-s)\left[1 + s(n-1)\right]}$$

$$-\frac{\partial\mathcal{H}_i}{\partial c_j} = \frac{\partial\lambda_{ij}}{\partial t} - \rho\lambda_{ij} \Leftrightarrow \tag{17}$$

$$\frac{\partial\lambda_{ij}}{\partial t} = \lambda_{ij}\left[k_j + \beta K_{-j} + \rho - \delta\right] \tag{18}$$

From FOCs (6-8) we have, respectively:

$$p_i^* = \frac{A(1-s) + c_i\left[1 + s(n-2)\right] + s\sum_{j\neq i} p_j}{2\left[1 + s(n-2)\right]}\,, \tag{19}$$

$$k_i = -\frac{\lambda_{ii}c_i}{2b}\,, \tag{20}$$

since $\lambda_{ij} = 0$ for all $j \neq i$, at any $t \in [0,\infty)$. While (19) has the usual appearance of a standard Bertrand best reply function, the optimal R&D effort in (20) depends upon i's co-state variable. Such expression can be differentiated w.r.t. time to get the dynamic equation of $k_i(t)$:

$$\frac{dk_i}{dt} \equiv \dot{k}_i = -\frac{1}{2b}\left[c_i\dot{\lambda}_{ii} + \lambda_{ii}\dot{c}_i\right] \tag{21}$$

with $\dot{\lambda}_{ii}$ obtaining from (18). Then, (21) can be further simplified by using $\lambda_{ii} = -2bk_i/c_i$ which obtains from (8), and the Bertrand-Nash equilibrium price which obtains from (19) after imposing the obvious symmetry condition $c_j(t) = c_i(t)$, $k_j(t) = k_i(t)$ and $p_j(t) = p_i(t)$ for all j:[9]

$$p^N = \frac{A(1-s) + c\left[1 + s(n-2)\right]}{2 + s(n-3)}\,. \tag{22}$$

Using we may simplify the dynamics of the R&D effort of any single firm as follows:

$$\dot{k} = -\frac{c(A-c)\left[1 + s(n-2)\right] - 2b\rho k\Upsilon\Xi}{2b\Upsilon\Xi} \tag{23}$$

where Υ is defined as in (7) and $\Xi \equiv 2 + s(n-3)$. Imposing the stationarity condition $\dot{k} = 0$, we obtain the Nash equilibrium investment, given c:

$$k^N = \frac{c(A-c)\left[1 + s(n-2)\right]}{2b\rho\left[1 + s(n-1)\right]\left[2 + s(n-3)\right]}\,, \tag{24}$$

with $k^N \geq 0 \;\forall\; c \in (0,\, A)$. The steady state level of marginal cost c can be found by solving:

$$\dot{c} = -c\left[k^N\left(1 + \beta(n-1)\right) - \delta c\right] = 0 \tag{25}$$

which yields $c = 0$ and

$$c = \frac{A\Omega \pm \sqrt{\Omega\left(A^2\Omega - \Phi\Upsilon\Xi\right)}}{2\Omega} \tag{26}$$

where $\Omega \equiv \left[1 + \beta(n-1)\right]\left[1 + s(n-2)\right]$ and $\Phi \equiv 8b\delta\rho$. All solutions in (26) are real if and only if $A^2 \geq \Phi\Upsilon\Xi/\Omega$. If so, they also satisfy the requirement $c \in [0,\, A]$. By checking the stability conditions, we may prove the following:

Proposition 2. Provided that $A^2 \geq \Phi\Upsilon\Xi/\Omega$, the steady state point

$$c^{ss} = \frac{A\Omega - \sqrt{\Omega\left(A^2\Omega - \Phi\Upsilon\Xi\right)}}{2\Omega}$$
$$k^{ss} = \frac{\delta}{1 + \beta(n-1)}$$

is the unique saddle point equilibrium.

Proof. Under symmetry, the dynamics of control and state variables are written as in (23) and (25). Accordingly, the relevant Jacobian matrix is:

$$\mathfrak{J} = \begin{bmatrix} \dfrac{\partial\dot{c}}{\partial c} & \dfrac{\partial\dot{c}}{\partial k} \\ \dfrac{\partial\dot{k}}{\partial c} & \dfrac{\partial\dot{k}}{\partial k} \end{bmatrix} \tag{27}$$

whose trace and determinant are:

$$T(\mathfrak{J}) = \delta + \rho - k\left[1 + \beta(n-1)\right] \tag{28}$$

$$\Delta(\mathfrak{J}) = \rho\left[\delta - k\left(1 + \beta(n-1)\right)\right] - \frac{c(A-2c)\Omega}{2b\Upsilon\Xi}. \tag{29}$$

Then, it can be easily checked that the pair $(c^{ss},\ k^{ss})$ is the only solution yielding $\Delta(\mathfrak{J}) < 0$ always, while the other two steady state points are both unstable.■

[9] Note that $p^N = c$ if $s = 1$.

4. COMPARATIVE STATICS

Now we focus on the interplay between market structure (measured by the number of firms), product substitutability (measured by parameter s) and the incentive to invest in process R&D. To this aim, we examine effect of a change in n and s on individual and aggregate R&D efforts, both along the equilibrium path (expression (24)) and in steady state.

This discussion revisits the debate between Schumpeter (1942) and Arrow (1962). Their respective views can be summarized as follows. According to the Schumpeterian hypothesis, R&D investments and technical progress are positively related to the flow of profits and therefore we should expect to observe higher R&D efforts and a faster innovation process under monopoly than any other market form. Conversely, Arrow claims that the incentive to generate technical progress is negatively affected by market power, being then maximized under perfect competition. The Arrowian position relies upon the idea that innovation is more attractive for a competitive firm than for a monopolist who, by definition, can not improve his market power.

In order to assess this issue in the present model, we proceed as follows. The aggregate R&D investments along the equilibrium path and in steady state are, respectively:

$$K^N = \frac{c\,(A-c)\,[1+s\,(n-2)]\,n}{2b\rho\,[1+s\,(n-1)]\,[2+s\,(n-3)]}\,; \quad (30)$$

$$K^{ss} = \frac{\delta n}{1+\beta\,(n-1)}\,. \quad (31)$$

It is immediate to verify that, taking into account the integer constraint on n:

$$\frac{\partial K^N}{\partial n} \geq 0;\ \frac{\partial K^{ss}}{\partial n} \geq 0 \quad (32)$$

in the admissible range of parameters. The above properties prove the following result:

Proposition 3. The optimal R&D investment of the whole industry is non-decreasing in the number of firms. This holds both along the equilibrium path and in steady state.

That is, the industry behaviour is clearly Arrowian. If we examine the individual investment, we obtain $\partial k^N/\partial n, \partial k^{ss}/\partial n < 0$ everywhere. This entails that any increase in the number of firms brings about a decrease in individual R&D effort. The driving force is twofold: (i) tougher market competition reduces profits and therefore the funds available for financing R&D activity; (ii) a larger population of firms means a larger positive spillover that any firm receives from rivals. Overall, a scale effect prevails, so that the overall expenditure of the industry is monotonically increasing in n.

Hinloopen (2000) has solved the static Bertrand equilibrium with n firms, finding that both aggregate and individual R&D efforts are non-monotone w.r.t. n. Under this respect, the static approach proves to fall short of appropriately accounting for the inherently dynamic nature of R&D which is not captured by multistage game modelling.

Now examine the effect of s on optimal investments. First, note that steady state levels are independent of the degree of product substitutability.[10] Second, considering optimal R&D efforts along the equilibrium path, we have:

$$\frac{\partial k^N}{\partial s} \propto -\left[s^2\,(n-2)\,(s-3)+2s\,(n-3)+1\right] \quad (33)$$

and obviously $\partial K^N/\partial s = n\partial k^N/\partial s$. Derivative (33) is always negative, except at $n = 2$, where $\partial k^N/\partial s \propto 2s-1 > 0$ for all $s \in (1/2, 1]$. Hence, we may state:

Proposition 4. For all $n \geq 3$, the incentive to invest in R&D on the equilibrium path is decreasing in product substitutability. At $n = 2$, R&D efforts are decreasing in s for $s \in (0, 1/2)$, and conversely for $s \in (1/2, 1]$.

Any increase in substitutability, or decrease in differentiation, damages operative profits. Hence, the net effect on k^N and K^N is the balance of two opposite tendencies: (i) the decrease in operative profits lowers the funds for R&D activity; (ii) any increase in R&D for process innovation may allow firms to recover on the cost side what is being lost on the differentiation side. Proposition 4 says that, if n is sufficiently large, the first effect dominates the second because competition is too tough and the price is not worth the effort, while the opposite holds for $n = 2$. Contrary to Proposition 3, the flavour of Proposition 4 is Schumpeterian, at least for $n \geq 3$: any increase in product differentiation amounts to a decrease in the intensity of competition, and brings about an increase in R&D efforts.

5. CONCLUSIONS

We have analyzed dynamic R&D investments for cost-reducing innovation in a Bertrand oligopoly in order to evaluate the influence of market structure and product differentiation on R&D incentives.

Three features of our analysis are worth stressing. First, the game is perfect, or state-redundant,

[10] This is in sharp contrast with the static models on the same topic (see Delbono and Denicolò, 1990; Bester and Petrakis, 1993; Qiu, 1997; Hinloopen, 2000; and Lambertini and Mantovani, 2001).

so that the open-loop solution is Markovian, or subgame perfect. Second, if we look at the effects of market structure on innovation, an Arrowian conclusion obtains, since aggregate R&D effort is increasing in the number of firms, both along the equilibrium path and in steady state, for any degeree of product differentiation. This sharply differs from the ambiguous conclusions reached by the static models, where the smoothing of investment efforts over a long time horizon is ruled out by definition. Third, we have shown that the interplay between R&D incentives and product differentiation is ambiguous if $n = 2$, while individual and industry investments are monotonically decreasing in product substitutability if $n \geq 3$. This, in turn, is a Schumpeterian result. Therefore, as a final remark, we may say that, if the intensity of market competition is measured by market structure, *all else equal*, then the answer of the model is Arrowian; if instead the intensity of competition is measured by product substitutability for a given market structure, then the model points to a Schumpeterian conclusion.

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