ADAPTIVE PREDICTIVE FUNCTIONAL CONTROL OF A CLASS OF NONLINEAR SYSTEMS BASED ON DYNAMICAL LINEARIZATION

ZHANG Bin, ZHANG Wei-dong

Department of Automation, Shanghai Jiaotong University, Shanghai 200030, China

Abstract: This paper employs a concept of partial derivative called Pseudo-Partial-Derivative (PPD) to dynamically linearize nonlinear system, and the aggregation method is applied to deal with the future predictive PPD, then adaptive predictive functional control algorithm of nonlinear system is presented. The design demands low requirement for the model, which is based directly on PPD derived online from the input output data, and the given algorithm can also provide the bounded input output sequence and track setpoint without steady-state error. Simulations for the time-delay process and the pH neutralization experiment of the chemical reaction process show that the proposed method is efficient for the system parameters perturbation and external disturbance.

Keywords: predictive functional control, nonlinear system, aggregation, dynamic linearization, pseudo-partial-derivative.

1. INTRODUCTION

The area of nonlinear systems control has been swarmed by researches in the past few years. This is motivated by the fact that the nonlinear systems are difficult to control. Usually a discrete SISO nonlinear system can be described by the following equation

$$y_P(k+1) = f(y_P(k),\ldots,y_P(k-n_y),u(k),\ldots,u(k-n_u)) \quad (1)$$

where n_y and n_u are the orders of the outputs y_P and the inputs u, respectively, f denotes a nonlinear mapping function.

There are many literatures researching the different nonlinear control algorithms on the basis of special models such as Hammerstein model (Fruzzetti, *et al.*, 1997), Wiener model (Norquay, *et al.*, 1999), bilinear model (Daniel-Berhe and Unbehauzn, 1998) for the nonlinear systems, which has greatly push ahead the development of the nonlinear systems research. However it is difficult to find an appropriate nonlinear model f to describe actual processes. At the same time, a nonlinear optimization problem is rarely convex which also makes the online computation very difficult. Therefore researchers turn to neural networks (Zhang, 1999) which can satisfactorily map the bounded nonlinear function, however there also exist difficulty in realizing the rapid, reliable solution of the control algorithm in real time. Note that tons of data exist in the process control, a model-free learning adaptive algorithm is proposed by Hou and Huang (1997) based on the input and output information of the system only, where a new concept of partial derivative called pseudo partial derivative (PPD) is used to linearize the nonlinear systems online. The same idea is applied to give an adaptive-predictive PI controller (Tan *et al.*, 1999).

Predictive functional control (PFC) (Richalet, 1993; Ernst *et al.*, 1996; Zhang, 2000) is a new model predictive control. It is such an algorithm that achieves computational simplicity by using simpler but more intuitive design guidelines (Rossiter and Richalet, 2002) and has achieved wide success in industrial applications. The advantages of less calculation on line, simpler algorithm and higher control precision can also be observed in PFC. In this paper, a pseudo partial derivative (PPD) concept is initially introduced to dynamically linearize nonlinear system, the PPD re-linearized the nonlinear model as the plant moves from one operating point to another, and to use the latest linear model as the internal model at each step, as a result a QP problem is the one requiring solution at each step, although the model varies from time to time. A method of the aggregation is adopted for dealing with the future value of PPD. Then PFC technique is used to design the nonlinear adaptive controller. The resulting controller has simpler structure and the tuning is easier, the proposed algorithm can provide the bounded inputs outputs sequences and track the setpoint without steady-state error. At last, simulations for the time delay process and experiment for the measurement of the acidity or alkalinity of the chemical reaction process show that the proposed algorithm has high precision and better robustness for the parameters perturbation.

2. NONLINEAR ADAPTATIVE FUNCTIONAL CONTROL ALGORITHM

2.1 The dynamic linearized internal model.

For the nonlinear system (1), the following two assumptions are given.

Assumption 1 (A1): The partial derivative of $f(\bullet)$ with respect to control input $u(k)$ is continuous.

Assumption 2 (A2): The system is generalized Lipschitz, that is, satisfying $|\Delta y_P(k+1) \le C|\Delta u(k)|$, for $\forall k$ and $\Delta u(k) \ne 0$,where $\Delta y_P(k+1) = y_P(k+1) - y_P(k)$, $\Delta u(k) = u(k) - u(k-1)$, C is a constant.

By the above assumptions, the following result can be obtained.

Theorem 1 (Hou and Huang, 1997) For the nonlinear system (1), we assume that (A1) and (A2) hold. Then there must exsit $G(k)$, called pseudo-partial-derivative (PPD), when $\Delta u(k) \ne 0$, it can derive

$$\Delta y_P(k+1) = G(k)\Delta u(k) \tag{2}$$

where

$$|G(k)| \le C \tag{3}$$

With Theorem 1, equation (2) can be served as an internal model to predict future process output.

$$y(k+1) = y(k) + \hat{G}(k)\Delta u(k) \tag{4}$$

where

$$|\hat{G}(k)| \le C \tag{5}$$

$y(k)$ is the model output, $\hat{G}(k)$ is an estimate of $G(k)$.

2.2 Predictive output and structured control variables.

Using (4), at sampling time $k+H_i$ inside the optimization horizon, the output can be predicted by the following equation

$$y(k+H_i) = y(k) + \sum_{j=1}^{H_i} \hat{G}(k+j-1)\Delta u(k+j-1) \tag{6}$$

Predictive functional control is different from other model predictive controls. Instead of calculating control signal without restrictions, which may result in a wild control signal, PFC adopts structured future manipulated variables, it considers that the future manipulated variables are parameterized by n_B base functions u_{Bj} previously known.

$$u(k+n) = \sum_{j=1}^{n_B} \mu_j(k) u_{Bj}(n) \quad n = 0,1,\cdots,H_i - 1 \tag{7}$$

where $\mu_j\ (k)(j \in [1, n_B])$ are the unknown coefficients.

The choice of the base functions is driven by the character of the setpoint and the process, it is usually selected as polynomial, sine and exponential. For many applications it is sufficient to describe process input with the form of $u(k+i) = \mu_1(k) + \mu_2(k)i$, thus we have

$$\Delta u(k+H_i-1) = \Delta u(k+H_i-2) \ldots = \Delta u(k) = \mu_2(k) \tag{8}$$

Substitute (8) into (6), it is derived that

$$y(k+H_i) = y(k) + \sum_{j=1}^{H_i} \hat{G}(k+j-1)\Delta u(k) \tag{9}$$

2.3 Optimization and control law equation.

PFC computes future process input so that the predicted process output can follow the reference trajectory. For many applications description of the first-order exponential reference trajectory is sufficient

$$y_{ref}(k+i) = w(k+i) - \eta^i (w(k) - y_P(k)) \tag{10}$$

where $\eta = \exp(-T_s / T_{ref})$, T_{ref} is the desired response time of the closed loop system, y_P is the process output, w is the setpoint.

PFC algorithm adopts an online optimizing method. A quadratic performance index may be adopted, process inputs are calculated by minimizing the sum of the quadratic difference between the predicted process output and reference trajectory at all coincidence points. The criterion takes the following form

$$\begin{aligned} min\ J_P = &\sum_{i=H_1}^{H_2} [(y_{ref}(k+i) - y(k+i) - e(k+i)]^2 \\ &+ r\sum_{i=1}^{M} u^2(k+i-1) \end{aligned} \tag{11}$$

where r is a weighting efficient, and $e(k+i)$ is the future error which can be given by $e(k+i) = e(k) = y_P(k) - y(k)$.

Substitute (9) into (11), the calculation of the process input $\Delta u(k)$ is easy if $\hat{G}(k+i)$ is known. Note that for $\forall\ k$, $|\hat{G}(k)| < C$, it is required that PPD should be bounded. Hereafter the idea of aggregation is adopted.

Assume $\hat{G}(k) = \lambda$, then $\hat{G}(k+i) = \lambda^{i+1}$, where λ is an unknown coefficient which must satisfy the constraint of (5). Since $\hat{G}(k)$ is bounded, we can choose $0 < \lambda < 1$ so that $\hat{G}(k+i)$ can automatically meet the requirement of the constraint of (5). However, the constraint for $\hat{G}(k)$ may cause wild overshoot of $u(k)$, thus a weight of the control input $u(k)$ is introduced in the index (11).

Under the above assumptions, $\mu_1(k)$ and $\mu_2(k)$ are unknown coefficients in index (11), which requires that at least two coincidence points H_1 and H_2 should be selected. Then the index (11) is rewritten as

$$min\ J_P = [y_{ref}(k+H_1) - y(k+H_1) - e(k+H_1)]^2 +$$
$$[y_{ref}(k+H_2) - y(k+H_2) - e(k+H_2)]^2 + r\sum_{i=1}^{M} u^2(k+i-1) \tag{12}$$

Substitute (9) and (10) into (11), let

$$\frac{\partial J_P}{\partial \mu_1(k)} = 0\ , \frac{\partial J_P}{\partial \mu_2(k)} = 0 \tag{13}$$

Manipulated variable can be obtained by solving (13)

$$u(k) = u(k-1) + M(A_1S_1 + A_2S_2) /$$
$$(M(S_1^2 + S_2^2 + r\sum_{i=1}^{M}(i-1)^2) - r(\sum_{i=1}^{M}(i-1))^2) \tag{14}$$

where $A_i = y_{ref}(k+H_i) - y(k) - e(k)$, $S_i = \sum_{j=1}^{H_i} \hat{G}(k)^j$, $i = 1,2$.

Subject to equation (4), online searching $\hat{G}(k)$ is required. With equation (2), many algorithms to estimate $\hat{G}(k)$ can be obtained, here the following convergent adaptive algorithm of $\hat{G}(k)$ is adopted (Hou and Huang, 1997).

$$\hat{G}(k) = \hat{G}(k-1)$$
$$+ \frac{\Delta u(k-1)}{\gamma + \Delta u^2(k-1)}(\Delta y_P(k) - \hat{G}(k-1)\Delta u(k-1)) \tag{15}$$

where $0 < \gamma < 1$, and the initial value λ of $\hat{G}(0)$ can be taken in range of $[0,1]$.

Explicit control input constraints is not directly handled in this paper, when input and/or state – related constraints need to be considered, the technique proposed by Abu el Ata-Doss (1991) can be adopted.

3. PERFORMANCE ANALYSIS

It is known that the stability is key to the designed control system. In order to prove the convergence of the closed loop system, the following assumption is given.

Assumption 3 (A3): The PPD satisfies $G(k) \geq 0$ for $\forall k$, and $G(k) = 0$ exists only at finite instant k.

Theorem 2 Subject to Assumption (A1, A2, A3), the algorithm (14) for the nonlinear system (1) is applied to track the setpoint w, then H_1 and H_2 exist so that

$$\lim_{k\to\infty}(y_P(k+1) - w) = 0 \tag{16}$$

and $\{y_P(k)\}, \{u(k)\}$ are the bounded sequences.

Proof: For tracking constant setpoint signal, the error between output y_P and constant setpoint w

$$E(k+1) = |\, y_P(k+1) - w \,|$$
$$= |\, y_P(k) - w + G(k)\Delta u(k) \,| \leq |1-\rho| E(k) \tag{17}$$

where $\rho = \dfrac{MG(k)((1-\eta^{H_1})S_1 + (1-\eta^{H_2})S_2)}{M(S_1^2 + S_2^2 + r\sum_{j=1}^{M}(j-1)^2) - r(\sum_{j=1}^{M}(j-1))^2}$

From Assumption 3, $G(k)S_i(i=1,2) > 0$. It is easy to tell that $M\sum_{j=1}^{M}(j-1)^2 > (\sum_{j=1}^{M}(j-1))^2$ and $0 < (1-\eta^{H_i}) < 1$, then $H_i(i=1,2)$ exist making $S_i^2 > G(k)(1-\eta^{H_i})S_i\ (i=1,2)$, that is to say $0 < 1-\rho < 1$.

By (17) naturally the result of (16) holds.
Moreover from (14) we know

$$\Delta u(k) = \frac{\rho}{G(k)} E(k) \tag{18}$$

Then

$$|\, \Delta u(k) \,| \leq bE(k) \tag{19}$$

where $b = \rho_{\max} / \lambda$, and $\rho_{\max}$ is the upper bound of ρ.

Apply the triangular inequality property to (8), the following relationship exists

$$|\, u(k) \,| \leq |\, u(k) - u(1) \,| + |\, u(1) \,| \leq \ldots$$
$$|\, \Delta u(k) \,| + |\, \Delta u(k-1) \,| + \ldots + |\, \Delta u(2) \,| + |\, u(1) \,| \tag{20}$$

Thus $\{y_P(k)\}, \{u(k)\}$ are bounded sequences. □

Note that the parameter estimation of $\hat{G}(k)$ is convergent, the tuning parameters in the controller (14) are only coincidence point H_1, H_2, input horizon M and reference trajectory time T_{ref}, and the design method has no requirement on the structure of the plant, therefore the control system has strong robustness.

4. SIMULATIONS

The following process with large time delay and pH measuring of the acidity or alkalinity process are adopted to show the effectiveness of the proposed method.

(1) Consider the process with large time delay

$$P(s) = \frac{1}{(s+1)^3} e^{-15s}$$

The controller parameters in this paper are hereby assigned $H_1 = 18$, $H_2 = 22$, $\hat{G}(0) = 0.99$, $M = 5$, $T_s = 1$, $T_r = 1$, $\gamma = 0.98$, $r = 40$. The method of Astrom-Hagglund PI tuning is applied to make comparison with the method proposed in this paper. With Astrom-Hagglund PI tuning method (Astrom and Hagglund, 1991), the controller is derived by $G_c(s) = 0.2115 + 0.0286 / s$, and a load disturbance with magnitude 0.2 is introduced at $t = 250$. The step responses of the closed loop control system are shown in Fig.1, it indicates that the proposed method exhibits superior performance over the method of Astrom-Hagglund's.

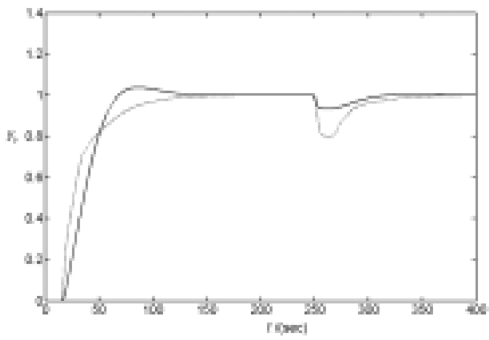

Fig. 1. Step responses of the closed loop (Proposed method, solid line; A-H method, dotted line).

(2) Consider the chemical reaction dealing with the measurement of the acidity or alkalinity, where pH is an important parameter. Generally a pH neutralization experiment can be expressed by the following equation (Zhang, 1999).

$$x(z) = f_1(u(z))$$
$$= u(z) - (1.207 + r_1)u^2(z) + 1.15u^3(z)$$
$$G(z) = \frac{y(z)}{x(z)} =$$
$$\frac{(0.0185 + r_2)z^{-2} + (0.0173 + r_3)z^{-3} + 0.00248z^{-4}}{1 - (1.558 + r_4)z^{-1} + 0.597z^{-2}}$$

where r_1, r_2, r_3, r_4 are time-varying parameters of the process with their initial value being set zero. Select the initial value of PPD with $\hat{G}(0) = 0.91$, coincidence point $H_1 = 1$, $H_2 = 18$, and input horizon $M = 6$, sampling time $T_s = 1$, the desired time of the closed loop $T_r = 1$, and $\gamma = 0.95$, $r = 50$. The step responses for tracking different setpoints are given in Fig. 2, which shows that the proposed method can achieve excellent control performance. Next, the disturbance with magnitude 0.2 is added on the output at the time $t = 250$, the controller parameters stay the same as that of the Fig. 2, the step response given in Fig. 3 shows that the control system can eliminate the external disturbance.

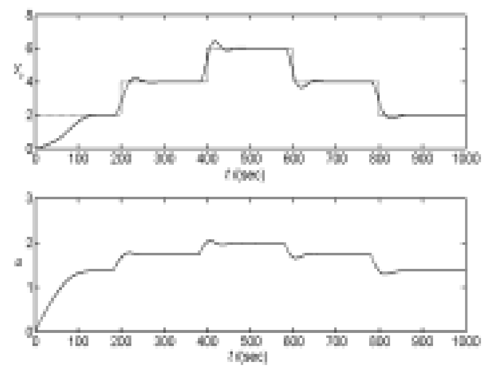

Fig. 2. Output (top) and input (bottom) of the closed-loop step responses for tracking the different setpoint

Consider the case that process parameters perturbation occur at different times $t = 200(r_1 = 0.1, r_2 = 0.01)$ and $t = 400$ ($r_3 = 0.001$, $r_4 = -0.008$), respectively. In this case, the process parameters perturbation has already become very violent. The step response is given in Fig.4, where the controller stays the same as previous simulations. It is shown that the design algorithm has strong robustness for the parameters perturbation.

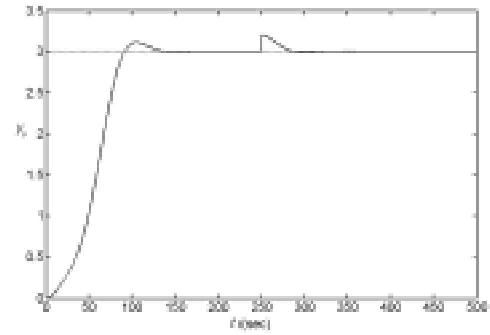

Fig. 3. Step response with disturbance of the closed-loop system.

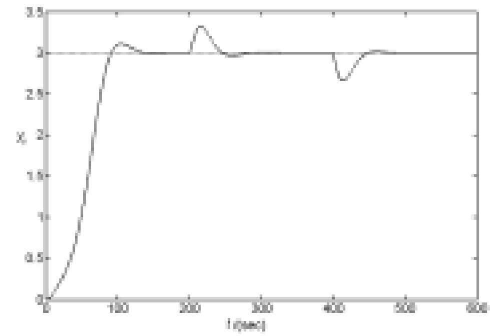

Fig. 4. Step response of the closed-loop with time varying parameters.

5. CONCLUSIONS

A novel concept so called PPD is used to dynamically linearize nonlinear process, an idea of aggregation is adopted to deal with future PPD, then the adopted predictive functional control of nonlinear process is implemented. A theorem, which illustrates that a specifically designed control system can track the setpoint with zero error and the inputs and outputs sequences are bounded, is derived in this paper. What's more, the design method requires neither the structure of the plant nor any external testing signal. All the results in this paper can be extended easily to the MIMO nonlinear processes.

REFERENCES

Abu el Ata-Doss S., Finai P. and J. Richalet (1991). Handing input and state constraints in predictive functional control. *Proceeding of the 30th conference on decision and control,* Brighton, England, 985-990.

Astrom K.J, and T. Hagglund (1991). Industrial adaptive controller based on frequency response techniques. *Automatica*, 27(4):599-600.

Daniel-Berhe S, and H. Unbehauzn (1998). Bilinear continuous-time systems identification via hartley-based modulating functions. *Automatica,*34(4):499-503.

Ernst T.E.F.H.C., I.H.B. Verbruggen, I.A. J. J. Boom and J. Richalet (1996). *First principle modeling and predictive functional control of enthalptic processes*. Delft University of Technology (France),

Fruzzetti K.P., A. Palazoglu and K.A. McDonald (1997). Nonlinear model predictive control using Hammerstein models. *Journal of Process Control*, 7(1):31-41.

Hou Z.S. and W.H. Huang (1997). The model-free learning adaptive control of a class of SISO nonlinear systems. *Proceeding of the American Control Conference*. New Mexico, 343-344.

Norquay S.J., A. Palazoglu and J.A. Romagnoli (1999). Application of wiener model predictive control (WMPC) to a pH neutralization experiment. *IEEE Transaction on Control System Technology*, 7(4):437-445.

Richalet J. (1993). Industrial application of model based predictive control. *Automatica*, 29(5):1251-1270.

Rossiter J.A, and J. Richalet (2002). Handing constraints with predictive functional control of unstable processes *Proc of American Control Conference,* Anchorage, AK, 4746-4751.

Tan K.K., S.N. Huang, T.H. Lee and F.M. Leu (1999). Adaptive-predictive PI control of a class of SISO systems. *Proceeding of the American Control Conference*. California, 3848-3852.

Zhang J. (1999). Developing robust non-linear models through bootstrap aggregated neural networks. *Neurocomputing*, 25:93-113.

Zhang Q.L. (1999). *Predictive functional control and its applications*. PH.D. dissertation, University of Zhejiang, Hangzhou, China.