A CLASS OF OPTIMAL DECENTRALIZED CONTROLLERS FOR OPEN LOOP STABLE PROCESSES

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Abstract: This work proposes a methodology to deal with full decentralized controller synthesis in the case of open loop stable plants in the discrete time domain. The results are based on a convenient characterization of the set of all stabilizing decentralized controllers and lead to a closed form one. Sufficient and necessary conditions for the stability of the resulting closed loop are established and the method is illustrated with examples.

Keywords: Control system synthesis, control system design, decentralized control, optimal control, optimization.

1. INTRODUCTION

Modern control design methods, when applied to multivariable (MIMO) plants usually lead to controllers of complex structure (Skogestad and Postlethwaite, 1996), (Zhou and Doyle, 1998), (Goodwin *et al.*, 2001). In those cases every manipulated plant input depends on more than two measurements, usually leading to a full MIMO controller; they are known as *centralized* controllers. From a practical viewpoint, those solutions have drawbacks related to the difficulty to build an insight, to tune the controller in the field and to achieve loop integrity. Moreover, there are situations when the complexity of a centralized control system design prevents it from being technically or economically feasible (Sandell *et al.*, 1978), (Yuz and Goodwin, 2003). Easy understandable and tunable strategies are preferred, as well as those which have proven useful in the past like simple local PID tuning (Yuz and Goodwin, 2003).

As a conceptual counterpart of centralized strategies, *decentralized* strategies have been developed. In the standard square decentralized structure, every manipulated plant input depends on only one measurement.

In the framework of nominal design, the performance achievable with centralized control is always superior to that achievable with decentralized control. However, decentralized controllers have advantages. Those advantages originate in their simplicity, their connection with classical single-input single-output control and in implementation and tuning issues (Conley, 2000), (Yuz and Goodwin, 2003). Nevertheless, that approach to control system design poses several fundamental questions (Sourlas and Manousiousthakis, 1995), (Goodwin *et al.*, 1999), (Salgado and Conley, 2004). Among those questions are, firstly, how to select those measurements that will be used to generate every plant input, i.e., how to pair inputs and outputs, and secondly, once the pairing has been decided, how to synthesize, if possible, a stabilizing controller.

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Recent references that compile classical results related to the first question can be found in (Conley, 2000), (Salgado and Conley, 2004) and in the references therein. The main drawback of classical interaction measures (RGA, DRGA, NI) is that they do not consider the dynamics of the process, since their either rely only on the frequency response at DC or, when the complete frequency axis is taken into account (as the DRGA does), they lead to results which are hard to analyze and to use. Therefore, most of that work is restricted to the 2×2 case and only in simple cases they help in the higher dimensional structure selection problem. A new tool called the participation matrix (PM) based on Gramians has been proposed in (Salgado and Conley, 2004). This interaction measure condenses information of the dynamics of the plant in a set of real numbers, allowing to easy identify the interaction structure of a given plant, thus providing useful information to decide on the input-output pairing.

With regard to the second question stated above, this paper proposes a strategy based on a $\mathcal{H}_2$ decentralized model matching problem. As in the centralized case (e.g. (Goodwin *et al.*, 2001)), an important first step is to parameterize all stabilizing controllers having certain structure. In (Gündes and Desoer, 1990) the set of all stabilizing decentralized controllers is characterized in terms of a number of stable parameters. It is also shown that the stability of the decentralized closed loop is equivalent to the unimodularity of some of those parameters. An equivalent formulation was presented in (Manousiousthakis, 1993), where the set of stabilizing decentralized controllers is described in terms of a stable parameter that must satisfy a finite set of quadratic constraints. In that work implications on optimal decentralized synthesis are briefly discussed.

In (Sourlas and Manousiousthakis, 1995) the parameterization introduced in (Manousiousthakis, 1993) is used to state the decentralized model matching problem. This approach is highly complex due to the non-convex nature of the associated optimization problem, as noted in (Sourlas and Manousiousthakis, 1995). In spite of the mentioned difficulties, in (Sourlas and Manousiousthakis, 1995) an approximation approach to solve the l_1 discrete time optimal control problem in the linear time invariant case is used. Due to the numerical nature of this solution, no closed form of the controller is given.

To avoid the high complexity of the mentioned optimization problem, a weighting function that makes the problem convex can be considered in the formulation of the functional (Goodwin *et al.*, 1999). However this approach does not guarantee a stabilizing solution, since the weighting factor depends explicitly upon the not known parameter. Moreover, the latter also implies that the minimization does not clearly reflects the desire to achieve a good fit in some frequency range: the weight is unknown.

In (Yuz and Goodwin, 2003) an optimal Youla parameter is found for a diagonal model of the process and, upon linear approximations, a minimization problem is proposed and solved using standard LQR theory that allows to find a *correcting* term that minimizes some quadratic measure of the achieved (*real*) sensibility. The issue of stability is not addressed in (Yuz and Goodwin, 2003).

In all the references mentioned above, the stability is guaranteed through methods which are quite involved and, therefore, they obscure conceptual aspects such as design trade-offs or relations to other synthesis methods. Moreover, in those cases there is no closed form for the controller, since the procedures rely on numerical methods. On the other hand, methodologies that lead to closed form controller and more conceptually significant solutions often oversimplify the analysis and stability must be tested separately.

This work proposes a methodology to deal with full decentralized controller synthesis in the case of open loop stable plants in the discrete time domain. Sufficient and necessary conditions for the stability of the resulting closed loop are established and the method is illustrated with examples.

This method solves a problem that is conceptually equivalent to the quadratic model matching problem, but restricted to the decentralized case subject to some additional constraints.

The paper is organized as follows: section §2 defines the decentralized optimal control problem and gives some preliminary results. Section §3 presents the proposed synthesis method and section §4 includes examples and a discussion of the results. Section §5 presents concluding remarks.

2. PRELIMINARIES AND PROBLEM DEFINITION

The space $(\mathcal{R})\mathcal{H}_2^{p\times p}$ is defined as the space of all (real rational) $p \times p$ transfer matrices which are functions of the complex variable z, are analytical $\forall |z| \geq 1$ and are strictly proper. $(\mathcal{R})\mathcal{H}_2^{*p\times p} \supset (\mathcal{R})\mathcal{H}_2^{p\times p}$ is the space of all (real rational) $p \times p$ transfer matrices that are stable and proper (biproper or strictly proper).

Due to the strictly proper nature of almost every real process, we will restrict ourselves to plant models $\mathbf{G_o}(z) \in \mathcal{RH}_2^{p\times p}$ [3]. In this paper, we will use the Youla parameterization of all stabilizing controllers for an open loop stable process. Using this, it is possible to write any proper stabilizing controller $\mathbf{C}(z)$ as (Goodwin *et al.*, 2001)

$$\mathbf{C}(z) = (\mathbf{I} - \mathbf{Q}(z)\mathbf{G_o}(z))^{-1}\mathbf{Q}(z) \qquad (1)$$

with $\mathbf{Q}(z) \in \mathcal{H}_2^*$ ($\mathbf{Q}(z)$ is called the Youla parameter).

Throughout this paper it will be assumed that $\mathbf{G_o}(z)$, $\mathbf{C}(z)$ and $\mathbf{Q}(z)$ are non singular for almost all z. Also, $[\mathbf{X}]_{ij} = X_{ij}$ will denote the ij entry of $\mathbf{X}$ (boldface will be used to denote matrices and vectors and normal face will be used for scalars).

***Lemma** 1.* Consider (1); if $\mathbf{G_o}(z) \in \mathcal{RH}_2$, then $\mathbf{C}(z)$ is a proper diagonal controller if and only if

$$[\mathbf{Q}^{-1}]_{ij}(z) = G_{o_{ij}}(z) \ \forall i \neq j, \qquad (2)$$

$$\text{and} \ \lim_{z\to\infty} [\mathbf{Q}^{-1}]_{ii}(z) \neq 0 \ \forall i. \qquad (3)$$

Proof: The results follow noting that (1) $\Leftrightarrow$ $\mathbf{C}^{-1}(z) = \mathbf{Q}^{-1}(z) - \mathbf{G_o}(z)$ and using the fact that $\lim_{z\to\infty} \mathbf{G_o}(z) = \mathbf{0}$.

For details see (Silva, 2004). □□□

Note that equation (3) is equivalent to require that $\mathbf{Q}^{-1}(z)$ is either biproper or improper.

The previous lemma states sufficient and necessary conditions to obtain a proper diagonal controller, but it does not address the issue of decentralized stabilization. In strictly theoretical terms, it is possible to address the decentralized stability problem in the way shown by the next observation, which follows immediately from lemma 1:

***Observation** 1.* If $\mathbf{G_o}(z) \in \mathcal{RH}_2$, then all decentralized stabilizing controllers can be written as

$$\mathbf{C}(z) = (\mathbf{I} - \mathbf{Q}(z)\mathbf{G_o}(z))^{-1}\mathbf{Q}(z) \qquad (4)$$

where $\mathbf{Q}(z)$ is in $\mathcal{H}_2^*$ and satisfies (2) (note that (3) is satisfied for all $\mathbf{Q}(z) \in \mathcal{H}_2^*$ that satisfies (2)).

Therefore, what is needed next is a procedure to find stable and proper Youla parameters that satisfy (2). Otherwise, the above observation does not have much practical interest. Nevertheless, observation 1 can be used to formulate the following optimal synthesis problem in $\mathcal{H}_2^*$:

***Problem** 1.* (Optimal decentralized control). Given a process model $\mathbf{G_o}(z) \in \mathcal{RH}_2$, find the Youla parameter $\mathbf{Q}^d_{opt}(z)$ that satisfies

$$\mathbf{Q}^d_{opt}(z) = \arg \min_{\mathbf{Q}(z)\in[\mathcal{H}_2^*]_d} ||F(\mathbf{Q}(z), \mathbf{G_o}(z))||_2^2 \quad (5)$$

[3] in the sequel we will drop the subindex $p \times p$ and, otherwise stated, we will assume that all involved matrices have this dimension.

where $[\mathcal{H}_2^*]_d$ denotes the set of all Youla parameters in $\mathcal{H}_2^*$ that satisfy (2). $F(\mathbf{Q}(z), \mathbf{G_o}(z))$ describes any functional whose 2 norm minimization is meaningful regarding control loop performance.

It is important to note that if one could solve the previous problem, the closed loop stability would be automatically guaranteed, since $\mathcal{H}_2^*$ contains only stable transfer functions.

A common choice for F is

$$F(\mathbf{Q}(z), \mathbf{G_o}(z)) = \frac{(\mathbf{I} - \mathbf{G_o}(z)\mathbf{Q}(z))\mathbf{v}}{z-1}; \ \mathbf{v} \in \mathbb{C}^{n\times 1} \qquad (6)$$

In this case, (5) leads to the minimization of the 2-norm of the control loop error with a step change in the reference. In the time domain, this is equivalent to minimize the sum of the quadratic error and it underlines the role of inversion in the solution of the control problem (Goodwin *et al.*, 2001).

Solving problem 1 subject to (6) is very hard due to the non convex nature of the constraints that (2) imposes on $\mathbf{Q}(z)$ (Sourlas and Manousiousthakis, 1995), (Goodwin *et al.*, 1999), (Yuz and Goodwin, 2003). In (Sourlas and Manousiousthakis, 1995) an approximate solution to the related l_1 version of problem 1 is derived. However, no closed form of the optimal Youla parameter or controller is given.

One way of circumventing the difficulties mentioned in the last paragraph, is to choose F as:

$$F(\mathbf{Q}(z), \mathbf{G_o}(z)) = \frac{\mathbf{C}(z)^{-1}}{z-1} = \frac{\mathbf{Q}^{-1}(z) - \mathbf{G_o}(z)}{z-1} \qquad (7)$$

This choice intends to obtain a controller with the highest possible gain at all frequencies and with infinite *DC* gain, i.e. with integration. This choice also facilitates the satisfaction of the constraints imposed by lemma 1.

3. DECENTRALIZED SYNTHESIS

This section presents a procedure to find Youla parameters to generate a close to optimal solution to problem 1 with F given by (7).

Consider $\mathbf{G_o}(z) \in \mathcal{RH}_2$. Using (7) and (2) in the functional subject to minimization in problem 1 (see (5)) we have that

$$J(\mathbf{Q}^{-1}(z)) = \sum_{i=1}^{p} \left|\left| \frac{[\mathbf{Q}^{-1}]_{ii}(z) - G_{o_{ii}}(z)}{z-1} \right|\right|_2^2 \qquad (8)$$

The main idea behind the minimization of (8) is to bring the diagonal terms of the inverse of the Youla parameter *as close as possible* to the diagonal terms of the process model. Considering (8) it is clear that the choice

$$[\mathbf{Q}^{-1}]_{ii}(z) = G_{o_{ii}}(z) \tag{9}$$

makes the functional equal to zero, but this would lead to an improper or even unstable (whenever the plant is non minimum (NMP) phase) Youla parameter.

An interesting observation is that, even if $\mathbf{G_o}(z)$ is stable and minimum phase, if one modifies slightly the choice (9) to make it proper, then the resulting Youla parameter would be unstable. This and other significant results are presented in the following lemma:

Lemma 2. Assume that (8) has to be minimized with $\mathbf{Q}^{-1}(z)$ satisfying (2), and such that $[\mathbf{Q}^{-1}]_{ii}(z)$ is either biproper or improper for $i = 1, \cdots, p$.

(1) If $\mathbf{Q}^{-1}(z)$ is chosen to be $\mathbf{Q}^{-1}(z) = \mathbf{G_o}(z) + \epsilon \mathbf{I}_{p\times p}$, $\epsilon \in \mathbb{R}$, then $|\epsilon|$ cannot be chosen arbitrarily small without making $\mathbf{Q}(z)$ unstable.
(2) If $\mathbf{Q}^{-1}(z) = \mathbf{X}(z) + \sum_{l=0}^{n_z} \mathbf{E}_l z^l$, where $\mathbf{X}(z)$ is strictly proper and stable and $\mathbf{E}_l$, $l = 0, \cdots, n_z$, are constant matrices, then

$$J(\mathbf{X}(z) + \mathbf{E}_o) \leq J\left(\mathbf{X}(z) + \sum_{l=0}^{n_z} \mathbf{E}_l z^l\right) \tag{10}$$

(3) Given $\mathbf{Q}^{-1}(z)$ analytical for $|z| = 1$, then always exists a stable and proper $\mathbf{Q}_{st}^{-1}(z)$ such that

$$J(\mathbf{Q}_{st}^{-1}(z)) \leq J(\mathbf{Q}^{-1}(z)) \tag{11}$$

Proof: Part 1 follows form a root locus argumentation. Parts 2 and 3 follow from appropriate use of the orthogonality of $\mathcal{H}_2$ and $\mathcal{H}_2^{\perp}$. See (Silva, 2004) for details. □□□

The previous lemma states that the search for $\mathbf{Q}_{opt}^d(z)$ should be restricted to those Youla parameters that are biproper and have a stable inverse.

Lemma 2 defines boundaries for the solution. However, it is not possible to significatively minimize the functional J, using standard $\mathcal{H}_2$ analytic techniques. This motivates a conceptual reformulation of the functional J as follows:

$$\bar{J}(\mathbf{Q}^{-1}(z)) = \sum_{j=1}^{p} \left\| \frac{1 - G_{o_{ii}}(z)([\mathbf{Q}^{-1}]_{ii}(z))^{-1}}{z-1} \right\|_2^2 \tag{12}$$

This is justified in the fact that minimizing the distance between A and B is conceptually equivalent to minimize the distance between AB^{-1} and the identity.

Under the *additional assumption* that the optimal $[\mathbf{Q}^{-1}]_{ii}(z)$ are minimum phase, i.e. $([\mathbf{Q}^{-1}]_{ii}(z))^{-1}$ is stable, it is possible to find closed forms for the optimal $[\mathbf{Q}^{-1}]_{ii}(z)$ using Blashke products and some additional considerations (Silva and Salgado, 2005). Specifically, we have that

$$([\mathbf{Q}^{-1}]_{ii}(z))_{opt*}(z) = \xi_i(z) G_{o_{ii}}(z) \tag{13}$$

where $\xi_i(z)$ is a scalar transfer function of the form

$$\xi_i(z) = z^{reld\{G_{o_{ii}}(z)\}} \prod_{j=1}^{n_{z_i}} \frac{1 - c_{ij}}{1 - \bar{c}_{ij}} \frac{1 - z\bar{c}_{ij}}{z - c_{ij}} \tag{14}$$

where $reld\{X\}$ denotes the relative degree of X, $\bar{z}$ the complex conjugate of z and $\{c_{ij}\}_{j=1\cdots n_{z_i}}$ denotes the set of non-minimum phase zeros of $G_{o_{ii}}(z)$.

Note that we have added an asterisk as subscript of the optimal values of $[\mathbf{Q}^{-1}]_{ii}(z)$ to emphasize that the optimality is subjected to the assumption that the optimal $[\mathbf{Q}^{-1}]_{ii}(z)$, $i = 1, 2, ..., p$, are minimum phase. Also note that $\xi_i(z)$ is such that $\xi_i(z) G_{o_{ii}}(z)$ is stable, minimum phase, biproper and has the same DC gain as $G_{o_{ii}}(z)$.

Using the procedure sketched above, the Youla parameter $\mathbf{Q}_{opt*}^d(z)$ is given by

$$\mathbf{Q}_{opt*}^d(z) = \left[diag\{([\mathbf{Q}^{-1}]_{ii}(z))_{opt*}\} + \mathbf{G_o}(z) - \mathbf{G_d}(z)\right]^{-1} \tag{15}$$

where $\mathbf{G_d}(z) = diag\{G_{o_{ii}}(z)\}$, and is such that conceptually minimizes J. Note that the properties of $\xi_i(z)$ imply that $\mathbf{Q}_{opt*}^d(z)$ is biproper and has a stable inverse.

Lemma 3. Consider $\mathbf{G_o} \in \mathcal{RH}_2$ and the synthesis procedure described previously. Then,

(1) $\mathbf{Q}_{opt*}^d(z)$ given in (15), defines a proper decentralized controller with infinite DC gain.
(2) The resulting control loop is internally stable if and only the image of the function $[\det\{\mathbf{Q}_{opt*}^d(e^{j\omega})\}]^{-1}$, $\omega \in [-\pi, \pi]$, does not encircle the origin.

Proof: Part 1 follows direct from the definitions. Part 2 follows upon using the principle of the argument.

For details see (Silva, 2004). □□□

The previous result provides sufficient and necessary conditions for decentralized stabilization for the class of controllers defined by (15). However there is no guarantee that the proposed procedure yields a controller which satisfies that condition. It can be shown (Silva, 2004) that if the process model satisfies a kind of diagonal dominance condition, the proposed procedure yields a stabilizing Youla parameter.

4. EXAMPLES

This section presents examples to illustrate the proposed synthesis procedure. In order to evaluate the performance of the proposed controller, a step change is applied in the reference and the following index evaluated:

$$SQE = \sum_{k=0}^{\infty} \mathbf{e}^T(k)\mathbf{e}(k) \qquad (16)$$

where $\mathbf{e}(k)$ is the control error. The SQE of the proposed loop is then compared with the minimum achievable SQE in a full MIMO framework, SQE_{opt}, which is evaluated using the results reported in (Silva and Salgado, 2005).

***Example** 1.* (3 × 3 process model). Consider a process modelled by

$$\mathbf{G_o}(z) = \begin{bmatrix} \frac{5k_1}{z^7(5z-4)} & 0 & \frac{4}{z^5(5z-4)} \\ \frac{2.5}{z^6(5z-4)} & \frac{20k_2}{5z-4} & \frac{6}{z^4(5z-4)} \\ 0 & \frac{2.5}{5z-4} & \frac{3k_3}{z^4(5z-4)} \end{bmatrix}$$

where k_i are constants. Let $\mathbf{G_{o(k_1,k_2,k_3)}}$ denote the transfer matrices resulting from particular choices for k_1, k_2, k_3. Also the sum of the diagonal terms of the PM in each case is denoted by $D_{(k_1,k_2,k_3)}$.

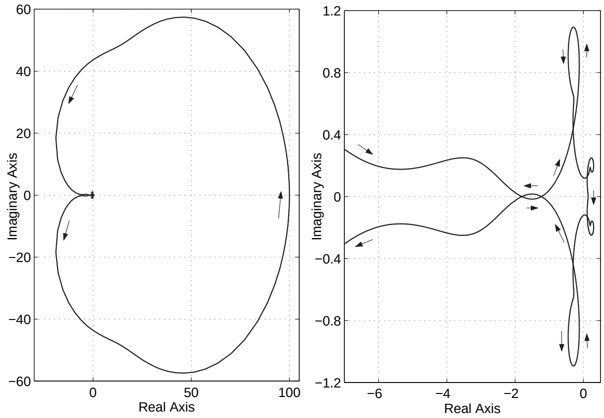
After applying the methodology proposed in the previous sections, the indexes SQE and SQE_{opt} were computed in each case considering $\mathbf{r}(k) = \frac{1}{3}\begin{bmatrix}2 & -2 & 1\end{bmatrix}^T \mu(k)$ and the results, as well as the sum of diagonal terms of the corresponding PM, presented in table 1 (see (Salgado and Conley, 2004) for complete interpretation guidelines of the PM).

Table 1. Sum of the diagonal terms of the PM and performance indexes SQE and SQE_{opt} for each choice of the parameters (k_1, k_2, k_3) in example 1.

(k_1, k_2, k_3)	D	SQE	SQE_{opt}
$(1, 1, 2)$	0.78	5.08	4.06
$(1, 0.5, 1)$	0.57	11.42	4.27
$(0.5, 0.5, 1)$	0.48	13.99	4.27
$(0.5, 0.35, 1)$	0.37	∞ (unstable loop)	4.69

In the first three cases the polar plot of $[\det\{\mathbf{Q}^d_{opt*}(z)\}]^{-1}$ does not encircle the origin and, therefore, the resulting closed loop is stable as part 2 of lemma 3 guarantees. As an illustration, figure 1 shows the polar plot for the choice $(k_1, k_2, k_3) = (1, 0.5, 1)$ and no encirclements of the origin occur. In the fourth case, $(k_1, k_2, k_3) = (0.5, 0.35, 1)$, the polar plot encircles 4 times the origin which explains the instability of the resulting closed control loop in this case.

The results show that in those cases where the PM suggests a diagonal control scheme ($D_{(k_1,k_2,k_3)} >$ 70%), the performance of the decentralized control scheme is *not too distinct* to the optimal centralized one. This is the case of $\mathbf{G_{o(1,1,1)}}$ illustrated in figure 2, where the evolution of the control error is plotted for each channel considering both the proposed controller and the full MIMO optimal controller.



(a) (b)

Fig. 1. (a) Polar plot of $[det\{\mathbf{Q}^d_{opt*}(z)\}^{-1}$ in the case $(k_1, k_2, k_3) = (1, 0.5, 1)$; (b) detail.

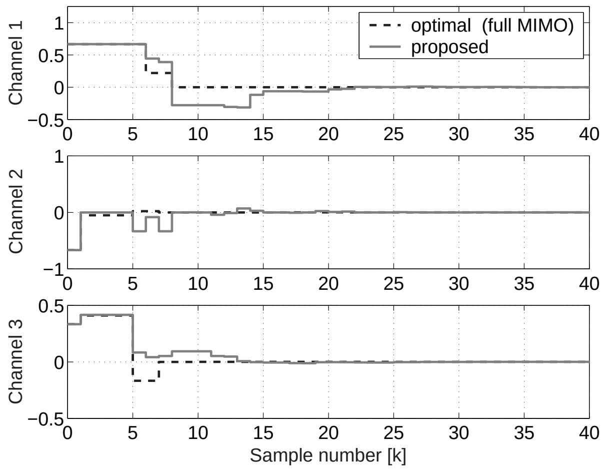


Fig. 2. Error evolution considering the proposed controller (solid) and optimal full MIMO controller (dashed). The reference is $\mathbf{r}(k) = \frac{1}{3}[2 \quad -2 \quad 1]^T$.

On the other hand, when the PM does not recommend a decentralized control strategy ($D_{(k_1,k_2,k_3)} <$ 40%), as in the case of $\mathbf{G_{o(0.5,0.35,1)}}$, the decentralized loop that results applying the proposed synthesis strategy may be unstable.

For relatively ambiguous PM diagonal sums ($D_{(k_1,k_2,k_3)} \sim 50\%$), the proposed methodology achieves closed loop stability, but the performance, as measured by SQE, is significantly deteriorated in comparison with the centralized optimum.

Example 2. (Unstable closed loop). Consider a process having the transfer function

$$\mathbf{G_o}(z) = \begin{bmatrix} (z-1/5)^{-1} & \frac{z-3}{z(z-1/2)} \\ z^{-2} & \frac{-2+z}{(z-1/2)(z-3/5)} \end{bmatrix}.$$

In this case the proposed methodology leads to an unstable decentralized closed loop, as predicted by the polar plot of $[\det\{\mathbf{Q^{-d}_{opt*}}(z)\}]^{-1}$, which in this case encircles three times the origin.

The PM in this case is given by

$$\boldsymbol{\phi} = \begin{bmatrix} 0.0242 & 0.4553 \\ 0.0445 & 0.4760 \end{bmatrix}$$

which suggests that it is unwise to attempt a decentralized control strategy to control this plant. Therefore, not surprisingly, the proposed strategy in this example fails to stabilize the loop.

5. CONCLUSIONS

A synthesis procedure for a class of decentralized controllers has been presented. The proposed method allows to find, under certain conditions, a stabilizing decentralized controller for open loop stable plants. The main idea behind the results is that a choice of a suitable cost function for a $\mathcal{H}_2$ optimization procedure, leads to a significant simplification of the problem and, at least, an approximate optimum can be established.

As appreciated in the examples, the performance deterioration of the proposed decentralized control loop compared to the *best* centralized one is not as significant as one would suppose at fist glance. This can be explained considering structure selection tools such as the PM, which help to identify the interaction structure of the plant and to establish whether a decentralized control scheme is advisable.

Future work in this area should cover the unstable plant case and the development of an analytical procedure to guarantee the stability of the resulting control loop, without imposing too hard to satisfy constraints such as diagonal dominance.

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