ON IDENTIFICATION OF A FLEXIBLE MECHANICAL SYSTEM USING DECIMATED DATA

Svante Gunnarsson [1]

Department of Electrical Engineering, Linköping University, SE-581 83 Linköping, Sweden

Abstract: System identification of a flexible mechanical system using decimated data is studied. It is illustrated how the use of decimated data can give erroneous results due to the inter-sample behavior of the signals, and an intuitive explanation to this phenomenon is proposed. The possible improvement by using alternative assumptions for the inter-sample behavior is investigated.

Keywords: Identification, physical models, continuous time systems, flexible arms, sampling

1. INTRODUCTION

The aim of this paper is to study identification of grey-box (physically parameterized) models of continuous time systems using discrete time data. Of particular interest is to study the consequences of decimation of the data before identification. This issue is of general importance, but the presentation in this paper will emphasize identification of flexible mechanical systems. There are several possible approaches to the grey-box identification problem. One idea is to start by identifying a continuous time black-box model and then compute the physical parameters from the coefficients of the continuous-time. The first step can be done in the time domain directly or via a frequency domain model. Direct time domain identification of black-box continuous time models using discrete time data has been been studied extensively by several authors. See (Unbehauen and Rao, 1998) for a thorough survey and e.g. (Rao and Garnier, 2002) and (Garnier *et al.*, 2003) for recent contributions. An alternative method is to first estimate the frequency response of the system using some frequency domain method and then fit a parametric frequency response curve to the initial estimate. See e.g. (Pintelon and Schoukens, 2001) and (Suthasun *et al.*, 2003). Irrespectively of how the continuous-time black-box model has been obtain, the second step, i.e. to determine the physical parameters from the coefficients of the black-box model, can be very difficult, in particular for high order models. Due to this difficulty this paper will primarily deal with methods which aim at identifying the physical parameters directly. Also here several alternatives exist. One possibility is to use a sequence of specially designed experiments where individual or subsets of the unknown parameters are estimated in each experiment. See e.g. (Isaksson *et al.*, 2003). Another approach, which is the one that will be applied here, is to estimate the parameters directly by using a time domain prediction error approach. See e.g. (Östring *et al.*, 2003) and (N.R.Kristenssen *et al.*, 2004) One important application area is motion control systems in general, see e.g. (Chou *et al.*, 2003), and industrial robots, see e.g. (Rostgaard *et al.*, 2001), (Daniel-Berhe and Unbehauen, 1997), in particular.

One important aspect of identification of continuous time systems using discrete time data is how to handle the inter-sample behavior of the data. In e.g. (Schoukens *et al.*, 1994) it is shown how the violation of the assumed inter-sample behavior may lead to erroneous results. The problem is also treated in (Andersson *et al.*, 1994). In some applications the sampling interval used during the data collection is

[1] Supported by the Swedish Research Council.

determined by the hardware, and when the sampling frequency appears to be unnecessarily high it is appealing to decimate the input and output signals to a slower sampling rate. The example that will be discussed in Section 3 is a realistic description of the movements around axis one of an industrial robot. The sampling rate, determined by the robot control system, is 2 kHz. In order to catch the low frequency behavior of the system a data collection experiment of $10-20$ seconds is desirable. This implies large data sets and, especially for higher order models, heavy computations. From that reason decimation of data would be useful.

The purpose of this paper is to illustrate and explain some phenomena that can occur when system identification is carried out using decimated data. The phenomena will be explained heuristically and the basic observations are first illustrated in Section 2 using a simple first order example. Using these observations a two-mass flexible mechanical system is studied in Section 3. Finally some conclusions are given in Section 4.

2. FIRST ORDER SYSTEM

2.1 Problem description

Consider a linear continuous time system with input $u(t)$, output $y(t)$, and transfer function $G(s)$. Assume that the input signal is applied to the continuous time system using zero order hold. The relationship between the input and output signals, in the sampling points, is given by the discrete time frequency response function $G_T(e^{i\omega T})$. In general, for a given ω the frequency function will have larger negative phase shift for larger T, due to the delay caused by the hold function.

As an illustration consider a first order system with transfer function

$$G(s) = \frac{100}{s+100} \tag{1}$$

and assume that the input is a sinusoid with angular frequency 50 rad/s. The input is generated using two different sampling intervals, $T_1 = 0.5 \cdot 10^{-3}$ s and $T_2 = 5 \cdot 10^{-3}$ s respectively. Figure 2.1 shows the frequency functions $G_{T_1}(e^{i\omega T_1})$ and $G_{T_2}(e^{i\omega T_2})$ respectively. At $\omega = 50$ rad/s the phase difference between the two frequency functions is approximately $7°$.

The continuous time system, equation (1), is simulated using zero-order hold input and the sampling intervals T_1 and T_2 respectively. Figures 2 and 3 show the input and output signals, and the difference in phase shift is clearly seen in Figure 3.

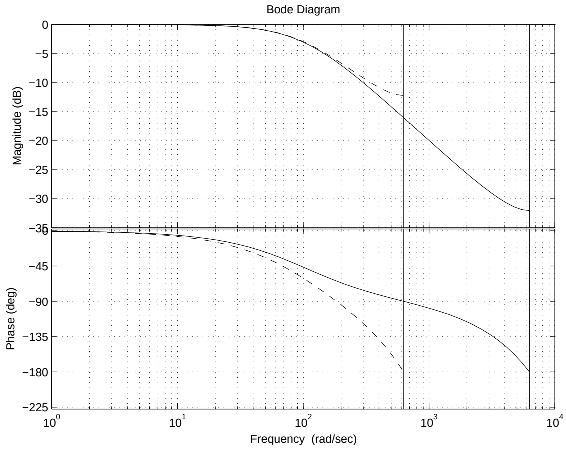

Fig. 1. Discrete time frequency function of the system given by equation (1). Solid: $T = T_1$. Dashed: $T = T_2$.

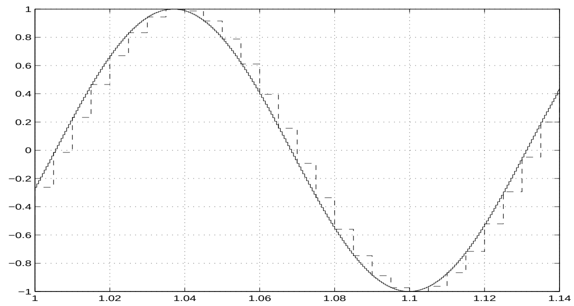

Fig. 2. Input signal. Solid: $T = T_1$. Dashed: $T = T_2$

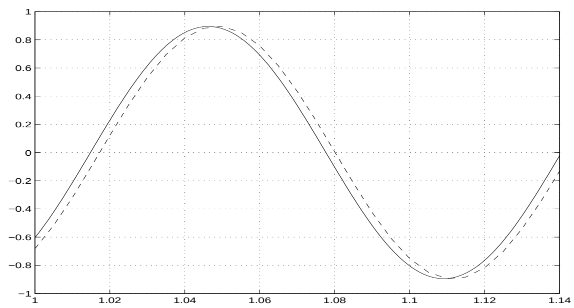

Fig. 3. Output signal. Solid: $T = T_1$. Dashed: $T = T_2$

2.2 Black-box identification

Assume now that the signals from the simulation using sampling interval T_1 (solid lines in Figures 2 and 3) are decimated by a factor ten. Since there are no disturbances present the decimation can be done by simply picking every tenth sample of the input and output vectors. Using the decimated signals in discrete time black-box identification implies that zero order hold input is assumed, and for the input this corresponds to the dashed curve in Figure 2. This input signal is combined with the decimated version of the solid curve in 3, and the resulting data set is used for identification. For sampling interval T_2 the true discrete time transfer function G_{T_2} describes the relationship between the dashed input in Figure 2 and dashed output signal in Figure 3. The system identification, on the other hand, tries to find a model that describes the relationship between the dashed input in Figure 2 and the solid output in Figure 3. Due to the phase difference between the solid and dashed output signals the true input-output relationship will not belong to the model class, and the resulting model will be biased. The model fit can be characterized by the bias integral presented in e.g.(Ljung, 1999). Assume that the true relationship between the input and output signal is given by the transfer operator $G_0(q)$ and the model structure is given by $G(q, \theta)$. The asymptotic model, as the number of data tends to infinity, is given by

$$\theta^* = \arg\min_\theta \int_{-\pi/T}^{\pi/T} \mid G_0(e^{i\omega T}) - G(e^{i\omega T}, \theta) \mid^2 \\ \times \Phi_u(\omega) d\omega \qquad (2)$$

where $\Phi_u(\omega)$ denotes the spectrum of the input signal. The model fit will depend on the properties of the excitation signal used for identification. Here the input is a a single frequency sinusoid and it implies that the identified model will have correct amplification and phase shift at the frequency of the sinusoid. A first order output error (OE) model is identified using the decimated data set. Transforming back the estimated model to continuous time results in the model

$$\hat{G}_{zoh}(s) = \frac{135}{s + 143} \qquad (3)$$

i.e. a substantial error in both time constant and static gain. The frequency functions of the true and estimated models are shown in Figure 4. The model tries to match the negative phase shift between the dashed input and the solid output, which is less than for the true transfer function for sampling interval T_2. It is therefore natural that the pole of the model is moved to higher frequency in order to match the phase shift.

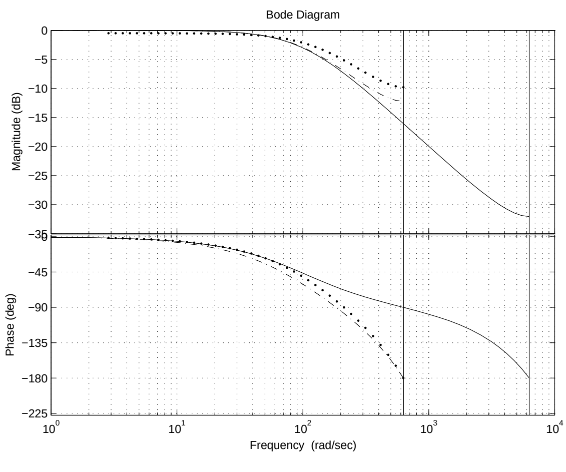

Fig. 4. Discrete time frequency functions. Solid: True system for $T = T_1$. Dashed: True system for $T = T_2$. Dashed-dotted: Estimated model

2.3 Grey-box identification

An alternative to the approach used above above is to use the `idgrey` model structure in the System Identification Tool-box, see (Ljung, 2000). The continuous time model is defined as a state space model

$$\dot{x}(t) = A(\theta)x(t) + Bu(t) + Ke(t) \qquad (4)$$
$$y(t) = C(\theta)x(t) + e(t) \qquad (5)$$

and the matrices $A(\theta), B(\theta), C(\theta)$, and $K(\theta)$ are specified in an m-file. The user of the tool-box can, by using the appropriate options, control how the inter-sample properties, specified in the data object, will affect the identification. By specifying that the m-file always delivers the continuous time system matrices the inter-sample property determines if zero order hold or first order hold is used when computing the discrete time predictor. For the first order example this gives that zero-order hold inter-sample behavior of the decimated data set yields the same model as in (3). First order hold character of the input will give a better, but of course not perfect, description of the input character. In this case the estimated continuous time model is given by

$$\hat{G}_{foh}(s) = \frac{98}{s + 97} \qquad (6)$$

The example illustrates that the assumption that the decimated data set has zero order hold inter-sample properties can give erroneous result due the phase difference of the output at different sampling rates. It also shows that the error will depend on the frequency contents of the input signal. Considerably better results are obtained by identifying the continuous time model directly and specifying first order hold input character of the decimated data set. An alternative approach would be to convert the discrete time black-box model to continuous time using a first order hold assumption.

3. TWO-MASS MECHANICAL SYSTEM

3.1 Problem description

Consider now a two-mass flexible mechanical system shown in Figure 3.1

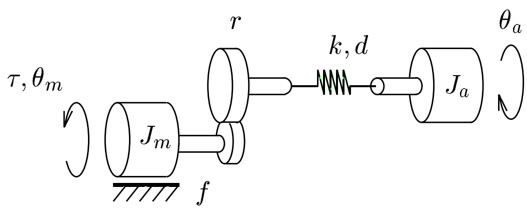

Fig. 5. Two-mass model

Here J_m and J_a denote the moments of inertia of the first and second mass respectively. The parameters k and d denote the stiffness and damping of the spring respectively, and f and r denote the viscous friction of the first mass and the gear ratio respectively. Torque balances of the two masses give

$$J_m\ddot{\theta}_m = -f\dot{\theta}_m - rk(r\theta_m - \theta_a) - rd(r\dot{\theta}_m - \dot{\theta}_a) + u \qquad (7)$$

and

$$J_a\ddot{\theta}_a = k(r\theta_m - \theta_a) + d(r\dot{\theta}_m - \dot{\theta}_a) \qquad (8)$$

respectively. Considering the torque as input signal, the angular velocity of the first mass as output signal, and using the state variables $x_1 = r\theta_m - \theta_a, x_2 = \dot{\theta}_m, x_3 = \dot{\theta}_a$ the equations (7) and (8) give

$$\dot{x} = Ax + Bu \quad y = Cx \qquad (9)$$

where

$$A = \begin{pmatrix} 0 & r & -1 \\ -\frac{rk}{J_m} & -\frac{f + r^2 d}{J_m} & \frac{rd}{J_m} \\ \frac{k}{J_a} & \frac{dr}{J_a} & -\frac{d}{J_a} \end{pmatrix} \qquad (10)$$

$$B = \begin{pmatrix} 0 & \frac{1}{J_m} & 0 \end{pmatrix}^T \quad C = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \qquad (11)$$

In the sequel the parameter values given in Table 1 will be used. The gear ratio $r = 1/118$ is known a priori.

Table 1. Nominal parameter values

Parameter	Nominal value
J_a	11
k	$1.5 \cdot 10^5$
J_m	$9 \cdot 10^{-4}$
f_m	$1 \cdot 10^{-3}$
d	10

The model defined by (10) and (11) and Table 1 represents a realistic description of the dynamics of an industrial robot when moving around axis one. See e.g. (Östring *et al.*, 2001)

The transfer function of the system is given by

$$G(s) = \frac{B(s)}{A(s)} \qquad (12)$$

where

$$B(s) = J_a s^2 + ds + k \qquad (13)$$

$$A(s) = J_a J_m s^3 + s^2(J_a f_m + d(J_m + r^2 J_a)) + s(k(J_m + r^2 J_a) + df_m) + kf_m \qquad (14)$$

Assuming zero order hold the corresponding discrete time transfer functions are computed, and the corresponding frequency functions are shown in Figure 6. The figure shows that the discrete time frequency function corresponding to $T = 5 \cdot 10^{-3}$ has larger negative phase shift.

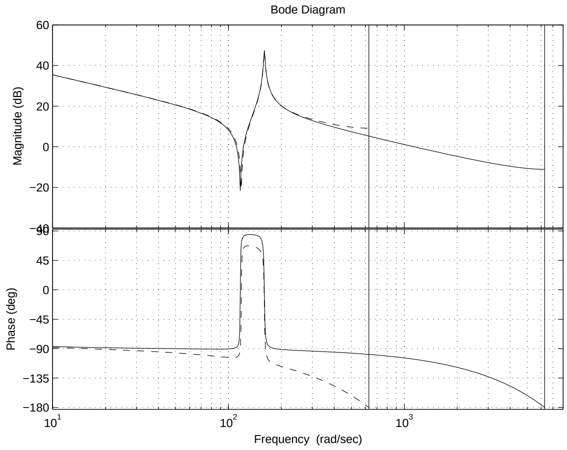

Fig. 6. Discrete time frequency function of the system given by equation (12). Solid: $T = T_1$. Dashed: $T = T_2$.

3.2 Black-box identification

The input is chosen as a chirp signal, i.e. a sinusoid where the frequency changes from 10 to 80 Hz during the experiment. The experiment lasts for 10 seconds, and the sampling interval is, like in the first order case, $T = T_1 = 0.5 \cdot 10^{-3}$ seconds. The continuous time system is simulated using zero order hold input, and the input and output signals are decimated by a factor ten before the identification is carried out. A third order output error model is identified, and the resulting model is shown in Figure 7. It is clearly seen that the identified model differs substantially from true discrete time system. The notch frequency is essentially lower for the identified model and the phase curve has a different behavior.

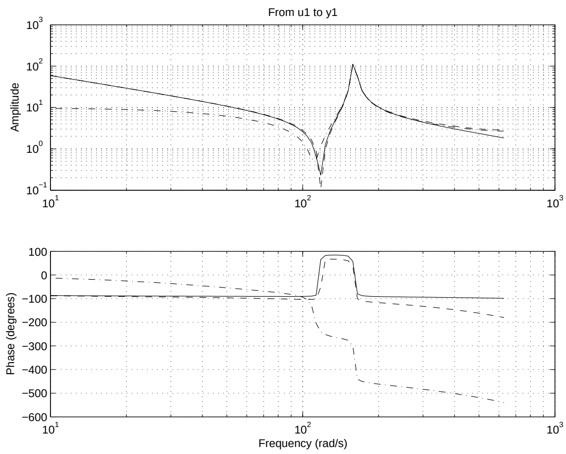

Fig. 7. Discrete time frequency functions. Solid: True system for $T = T_1$. Dashed: True system for $T = T_2$. Dashed-dotted: Estimated model

The properties of the identified model are also revealed by studying the Nyquist curves of the frequency functions as shown in Figure 8. The figure shows that the Nyquist curve of the estimated model, with sampling interval T_2, follows the Nyquist curve of the true frequency function corresponding the T_1, i.e. the shorter sampling interval. This is in agreement with the observations from the first order example. The identified model tries to model, within the frequency range of the input, the relationship between the input signal and the output signal corresponding to the shorter sampling interval. See also the bias integral (2). It has to be noted that the achieved model depends on where the energy of the input is located.

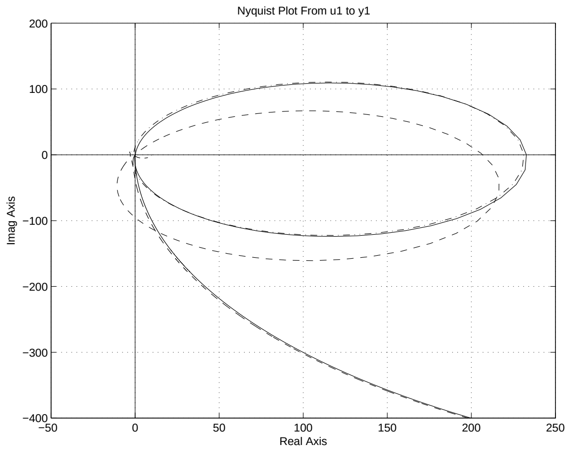

Fig. 8. Discrete time frequency functions. Solid: True system for $T = T_1$. Dashed: True system for $T = T_2$. Dashed-dotted: Estimated model

An important property of the identified model is that the zeros of the model are located outside the unit circle, and when the discrete time model is converted to continuous time the zeros are located in the right half plane. The estimated numerator polynomial is given by

$$N(s) = 1.0 \cdot 10^3 s^2 - 5.1 \cdot 10^3 s + 1.3 \cdot 10^6 \quad (15)$$

A comparison of this polynomial and the numerator $B(s)$ of equation (12) shows that the estimated damping d is negative, which is a non-physical results. The relationship between the zeros of continuous-time systems and the discrete-time counterparts has been studied in e.g. (Wahlberg, 1990) and (Åström *et al.*, 1980), but these results provide limited insight into this behavior.

3.3 Grey-box identification

The starting point is an m-file defining the structure of the physically parameterized model. This structure follows from the state space model given by (10) and (11). In order to improve the behavior of the identification procedure it has been found to useful to scale the physical parameters such that they all are of the same order of magnitude. The system is simulated for 10 seconds, using zero order hold input and sampling interval $T = T_1$. The input and output data vectors are decimated by a factor ten. Using the decimated data set two sets are defined. In the first set the inter-sample behavior is set to be zero order hold and in the second set it is set to be first order hold. The results from identifications using these sets are presented in Table 2 and Figure 9. The estimated frequency function assuming first order hold input gives a considerably better result. The estimated damping is positive and the other parameters are fairly close to their nominal values.

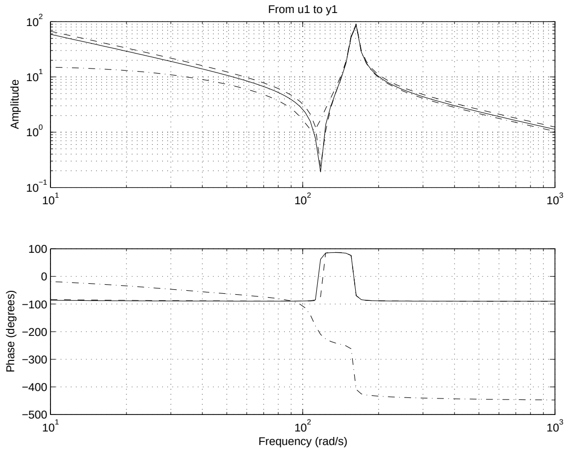

Fig. 9. Continuous time frequency functions. Solid: True system. Dashed: Estimated model assuming first order hold input. Dash-dotted: Estimated model assuming zero order hold input.

Table 2. Nominal parameter values and estimated parameter values for zero and first order hold respectively

Par.	Nom. value	ZOH	FOH
J_m	$9 \cdot 10^{-4}$	$11 \cdot 10^{-4}$	$8.2 \cdot 10^{-4}$
k	$1.5 \cdot 10^5$	$1.9 \cdot 10^5$	$1.3 \cdot 10^5$
J_a	11	16	9.3
f_m	$1 \cdot 10^{-3}$	$63 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$
d	10	-230	6.5

Figure 10 shows once more the phenomenon noted above. The estimated model, assuming zero order hold input, for the longer sampling interval tries to match the true frequency function corresponding to the shorter frequency interval within the frequency range of the input.

4. CONCLUSIONS

The consequences of using decimated data for identification of continuous time systems have been investigated. It has been illustrated that decimation of the data may lead to erroneous models and in some cases models without physical interpretation. The errors are caused by the violation of the assumption that the input is piecewise constant during the sampling interval. A possible interpretation of the behavior is that the error is caused by the difference in phase shift of the output signal for different sampling intervals. One way to improve the results is to identify the continuous time model directly and assume first order hold input of the input signal. The results are illustrated by identification of the physical parameters of a two-mass flexible mechanical system.

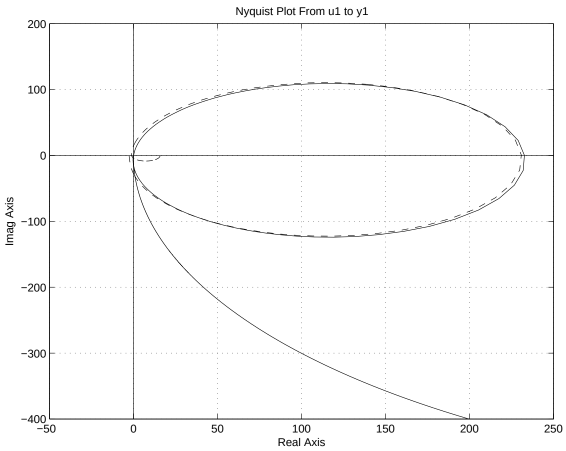

Fig. 10. Discrete time frequency functions. Solid: True system for $T = T_1$. Dashed: Estimated model using zero order hold input converted to discrete time using $T = T_2$

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