ASSESSMENT OF TUNING OF PI CONTROLLERS FOR SELF-REGULATING PROCESSES

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Abstract: A method for the performance assessment of Proportional-Integral (PI) controllers for single-input single-output (SISO) self-regulating processes is proposed. In particular, the effectiveness of the tuning of the controller with respect to its load disturbance rejection performances is determined by evaluating simultaneously different simple indexes that are calculated by considering only the manipulated variable and process output signals when an abrupt load disturbance occurs on the process. Thus, no model of the process is required. As a result, guidelines on how to improve the controller tuning are given. Simulation and experimental results demonstrate the effectiveness of the method in a process monitoring framework.

Keywords: Process control, PI control, performance evaluation, disturbance rejection, self-tuning regulators.

1. INTRODUCTION

Nowadays, process monitoring and control system performance assessment play a more and more important role in industry due to the need of increasing the quality of the products and of reducing the overall costs at the same time. In this context, there have been great efforts by the researchers to devise methodologies able to evaluate automatically if the performances of a control system are satisfactory (see (Qin, 1998) and (Harris *et al.*, 1999) and references therein contained). Though the proposed techniques can be viewed under the same framework (see (Huang and Shah, 1999) and references therein contained), they are generally divided in two categories (Qin, 1998): *stochastic performance monitoring* in which the capability of the control system to cope with stochastic disturbances is of main concern, and *deterministic performance monitoring* in which performances related to more traditional design specifications such as set-point and load rejection disturbance step response parameters are taken into account. Works that fall in the first class mainly rely on the concept of minimum variance control (Huang and Shah, 1999), whilst those that fall in the second class address different detrimental aspects for the control performances, such as presence of oscillations (Hägglund, 1995; Miao and Seborg, 1999; Hägglund and Aström, 2000), stiction in the valves (Horch, 1999), variations in the process dynamics and, obviously, bad controller tuning (Swanda and Seborg, 1999; Hägglund, 1999). In fact, it is realized that an unsatisfactory performance can be caused by different factors. Actually, as in large plants there are hundreds of control loops, it is almost impossible for operators to monitor each of them manually. Thus, it is important to have tools that are first able to automatically determine if an abnormal situation occurs and then to help the operator to understand the reason for it and possibly to suggest the way to solve the problem (for example, if a bad controller tuning is detected, then new appropriate values of controller parameters are determined). Thus, there is the need to integrate different techniques, each of them devoted to deal with a particular situation. Further, it is desirable that each technique be based as much as possible on routine operating data and that no process model is required in order to be employed in general. This paper falls in the deterministic performance monitoring category as well and it is related with the assessment of the tuning of a PI controller, which is the most adopted controller in industrial settings, with respect to its load disturbance rejection performance. In particular, the method proposed in (Hägglund, 1999), which is capable to detect sluggish control loops by calculating the so-called Idle Index, is improved by considering an additional suitable index. This is calculated very simply based on the analysis of the control signal (note that no process model is required), and it is directly related to the damping factor of the closed-loop system. By evaluating simultaneously the two indexes, the tuning of the controller is assessed. Specifically, information on the values of the proportional and integral gains (i.e. if they are too low or too high) is provided.

2. PROBLEM FORMULATION

We consider a standard unity-feedback control system where a SISO process with transfer function $P(s)$ is controlled by a PI controller described by the following transfer function:

$$C(s) = K_p \left(1 + \frac{1}{T_i s}\right) \tag{1}$$

where K_p is the proportional gain and T_i is the integral time constant. We assume that the main control specification is to guarantee good load disturbance rejection performances (note that this requirement is often of primary concern in typical industrial settings and that specifications on the set-point following performances can be pursued by employing a two degrees-of-freedom controller (Shinskey, 1994)). In this context, it is desirable that the tuning of the PI parameters is such that the integrated absolute error, defined as

$$IAE = \int |e(t)|dt \tag{2}$$

be minimized, as this ensures a low error magnitude and a stable response (i.e low settling time) at the same time (Shinskey, 1994). Thus, the aim of the proposed methodology is to verify, by evaluating an abrupt load disturbance response, if the tuning of the adopted PI controller is satisfactory (in the sense that it guarantees a good *IAE*) and to suggest the possible modifications of the controller parameters in case it results that performances can be improved.

3. THE AREA INDEX

3.1 Methodology

As already mentioned, in typical industrial processes it is in general desirable that the control law employed guarantees good load disturbance rejection performances, since this control specification is often of primary concern. The technique described hereafter is based on the analysis of the control signal when an abrupt load disturbance occurs on the process and it aims at estimating a generalized damping index of the closed-loop system. For clarity, only the underlying idea of the new technique will be presented in this subsection, whilst issues related with its implementation in practical cases are discussed in subsection 3.2.

The methodology consists of determining a suitable performance index, which will be called the *Area Index* (*AI*), based on the control signal $u(t)$ that compensates for a step load disturbance occurring on the process. Then, by evaluating the value of *AI*, it can be deduced if the control loop is too oscillatory.

Denote as $\bar{u}$ the new steady-state value achieved by the control signal after the transient load disturbance response. Denote also as t_0 the time instant in which the step load disturbance occurs (note that the value of t_0 does not need to be known) and with $t_1, \ldots, t_{n-1}$ the subsequent time instants in which it is $u(t) = \bar{u}$. Finally, denote as t_n the time instant in which the transient response ends and the manipulated variable attains its steady-state value $\bar{u}$. From a practical point of view, the value of t_n can be selected as the minimum time after that the control signal $u(t)$ remains within a one percent range of $\bar{u}$. The area delimited by the function $u(t)$ and $\bar{u}$ between two consecutive time instants t_i and t_{i+1} be defined as:

$$A_i := \int_{t_i}^{t_{i+1}} |u(t) - \bar{u}|dt. \tag{3}$$

The introduced notation is depicted in Figure 1. The Area Index *AI* is calculated as the ratio between the maximal value of the determined areas and the sum of them, excluding the area A_0, i.e. the area between the time instant in which the step load disturbance occurs and the first time instant in which it is $u(t) = \bar{u}$, from the whole analysis. In case during the overall transient response we never have $u(t) = \bar{u}$, the Area Index is simply set to one. Formally, the Area Index is therefore defined as:

$$AI := \begin{cases} 1 & \text{if} \quad n < 3 \\ \dfrac{\max\{A_1, \ldots, A_{n-2}\}}{\sum_{i=1}^{n-1} A_i} & \text{elsewhere} \end{cases}. \tag{4}$$

From formula (4) it can be trivially deduced that the value of *AI* is always in the interval $(0, 1]$. The significance of the devised index can also be evaluated by performing the following analysis. Consider the transfer function $T(s)$ from the load disturbance signal (acting at the process input) to the manipulated variable (i.e. the controller output):

$$T(s) := -\frac{C(s)P(s)}{1 + C(s)P(s)} \tag{5}$$

and assume that $T(s)$ has a pair of complex conjugate dominant poles, i.e. it can be well-approximated by the following transfer function (note that this is not always the case as it will be discussed in Section 4.2):

$$\tilde{T}(s) := -\frac{1}{T_1^2 s^2 + 2\xi T_1 s + 1}. \tag{6}$$

The Area Index *AI* has been calculated by considering the step response of $\tilde{T}(s)$ with different values of T_1 and $\xi \in (0, 1]$. It results that the value of *AI* is independent of the value of T_1 and depends only on the value of the damping factor ξ. The relation between ξ and *AI* is plotted in Figure 2. It appears that the more the value of *AI* approaches zero and the more the control loop is oscillatory, whilst the more the value of *AI* approaches one and the more the control loop is sluggish.

Remark 1. It is worth stressing that the Area Index

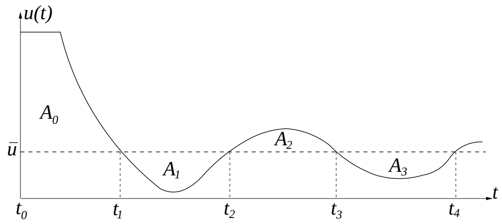

Fig. 1. Significant parameters for determining the Area Index.

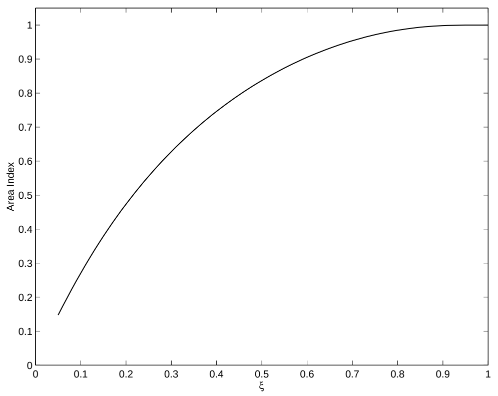

Fig. 2. Dependence of the Area Index from the damping factor ξ.

can be usefully adopted for any kind of controller. In this paper however, we focus on PI controller as the devised overall methodology can be applied to assess the values of the controller parameters.

3.2 Practical conditions

When the methodology described in subsection 3.1 has to be applied in practical cases, there are few technical problems to be solved. First, noise has to be considered. As the Area Index is determined off-line, a standard filtering procedure can be applied before calculating the different areas. Alternatively, it is sufficient to discard from the analysis those areas A_i whose value is less than a predefined threshold (because they are actually due to the noise). This threshold can be determined by considering the control signal for a sufficiently long time interval when the process is at an equilibrium point and by determining the maximum area between two consecutive crossings with respect to its steady state value (the latter can be calculated as the mean value of the control signal itself in the considered time interval). Note that this procedure is actually similar that based on the concept of noise band that has been already successfully applied in industry (Shinskey, 1994). In any case, as the overall procedure is based on the calculus of integrals, it is inherently robust to the noise.
Another aspect that has to be taken into account is that the Area Index is significant only when an abrupt load change occurs, i.e. when the load changes are fast enough with respect to the dynamics of the complementary sensitivity function (see (5)). Note that this assumption is the same that is done in (Hägglund, 1999) and also in (Petersson *et al.*, 2001) in a slightly different context. Thus, the method has to be applied only in these situations (e.g. when a sudden change in the production occurs or, obviously, when a step signal is deliberately added to the manipulated variable for this purpose), otherwise a higher value of the Area Index might result. To verify that this condition applies, the method described in (Hägglund and Aström, 2000) can be adopted.

4. TUNING ASSESSMENT

4.1 Review of the Idle Index

The method proposed by Hagglund in (Hägglund, 1999) is based on the fact that a sluggish load disturbance response is characterized by the fact that the first time derivative of the manipulated variable and of the process output signals have the same sign for a large period. Thus, it is sensible to apply to the transient response the following calculation:

$$t_{pos} = \begin{cases} t_{pos} + h & \text{if } \Delta u \Delta y > 0 \\ t_{pos} & \text{if } \Delta u \Delta y \leq 0 \end{cases}$$

$$t_{neg} = \begin{cases} t_{neg} + h & \text{if } \Delta u \Delta y < 0 \\ t_{neg} & \text{if } \Delta u \Delta y \geq 0 \end{cases}$$

where h is the sampling time and Δu and Δy are the increments of the manipulated variable and of the process output respectively. Then, the Idle Index is defined as

$$II = \frac{t_{pos} - t_{neg}}{t_{pos} + t_{neg}} \tag{7}$$

Evidently, the value of II is always in the interval $[-1, +1]$ and a positive value close to one indicates that the control loop is sluggish. The problem associated with the use of the Idle Index is that a negative value close to -1 might be obtained both from a well-tuned loop and from an oscillatory loop.

4.2 Combining the Area Index and the Idle Index

From the analysis depicted in the previous subsection, it appears, as also suggested in (Hägglund, 1999), that the Idle Index should be combined with an oscillation detection procedure. Among the different ones proposed in the literature (see e.g. (Hägglund, 1995; Miao and Seborg, 1999)), the merit of the Area Index is to provide an indication on how the controller has to be retuned, as it will be explained in the following.
Actually, it is evident that a well-tuned controller gives a low value of the Idle Index and at the same time a medium value of the Area Index, as this means that the control loop is neither sluggish nor oscillating. However, it is interesting to evaluate the values of the two indexes obtained with different controller parameters in order to adopt II and AI to give indications on how to improve the tuning. Specifically, the following analysis has been performed. Different processes with a first order plus dead time (FOPDT) transfer function have been considered, namely,

$$P(s) = \frac{1}{Ts+1} e^{-Ls} \tag{8}$$

with a normalized dead time L/T ranging from 0.1 to 1 (with a discretization step of 0.1). This choice has been motivated by the fact that this dynamics can effectively model many industrial processes and it is adopted for the large majority of PI tuning rules. Then, the methodology presented in (Silva *et al.*, 2002) has been adopted to determine the set of stabilizing PI controllers. In particular, a tight gridding has been performed on the resulting interval of values of K_p (only positive values have been considered) and for each value of the proportional gain the corresponding interval of values of T_i has been considered (and suitably discretized as well). Actually, it has to be noted that the range of the stabilizing values of the integral time constant ranges (for a given value of K_p) from a calculated lower end-point to $+\infty$ (in this latter case a simple proportional controller is adopted). Thus, the interval has been truncated when significant results have been obtained in any case. Then, for each PI controller determined in this way, a unit step load disturbance response has been simulated and the corresponding values of AI, II and IAE have been computed.

Based on the results obtained, the rules presented in Table 1 have been devised in order to assess the tuning of the PI parameters. The value of the Area Index is considered to be low if it is less than 0.35, medium if it is $0.35 < AI < 0.7$ and high if it is greater than 0.7. The value of the Idle Index is considered to be low if it is less than -0.6, medium if it is $-0.6 < II < 0$ and high if it is greater than zero.

Although these rules might appear somewhat intuitive, it is worthy to discussing two of them in some detail. First, the case when the value of AI is low and the value of II is medium/high is examined, as these seem to be two results that indicate an oscillatory loop from one side (AI) and a sluggish loop from another side (II). The situation can be evaluated by considering the following process

$$P(s) = \frac{1}{10s+1}e^{-5s} \tag{9}$$

controlled by a PI controller whose parameters are $K_p = 1.81$ and $T_i = 20$ (note that the parameters that provide the minimum IAE of 6.11 are $K_p = 1.81$ and $T_i = 10.36$ with corresponding values of $AI = 0.61$ and $II = -0.71$). The unit step load disturbance response and the corresponding control variable are plotted in Figure 3 (solid line). The resulting values of the Area Index and of the Idle Index are $AI = 0.14$ and $II = -0.21$ respectively (whilst $IAE = 11.03$). It appears that in this case the low value of AI is not associated to an oscillatory loop but to a control loop in which the dynamics of the complementary sensitivity function (see (5)) is not dominated by a pair of complex conjugate poles. Thus, although in this case the Area Index is not indicative of the damping factor of the closed-loop system (see subsection 3.1), it gives the important information that the value of the integral time constant is too high. It has to be noted that this conclusion cannot be drawn easily if a technique that reveals an oscillatory behavior of the manipulated variable (e.g. by considering the auto-correlation function) is employed only.

The second case that has to be discussed is when both values of AI and II are low. This means that the control loop is too oscillatory and this fact is motivated by a high value of the proportional gain of the controller and/or by a low value of the integral time constant. In order to provide a possible additional information on the value of T_i it is useful to calculate another simple index related to the process output signal. This fact is explained by the results shown again in Figure 3 where again the process modelled by transfer function (9) has been considered. In the first case (dashed line) the PI parameters are $K_p = 3$ and $T_i = 20$ and therefore the oscillatory response is caused by a too high value of the proportional gain. The resulting indexes are $AI = 0.19$ and $II = -0.9$ (the resulting integrated absolute error is $IAE = 9.75$). In the second case (dotted line) the PI parameters are $K_p = 2.2$ and $T_i = 6.5$ and therefore the oscillatory response is caused by both a too high value of the proportional gain and a too low value of the integral time constant. The resulting indexes are $AI = 0.23$ and $II = -0.64$ (the corresponding integrated absolute error is $IAE = 14.02$). It appears that the two considered indexes are not sufficient to distinguish the two situations. However, a look at the process output functions suggests to calculate a new index (called Output Index (OI)), namely, the ratio between the sum of the negative areas with respect to the final steady-state value and the sum of all the areas with the exception of the first one (note that a positive step load disturbance has been here assumed without loss of generality). In case the process output does never intersect its steady-state value, it has to be set simply $OI = 0$. This choice is motivated by the fact that when both K_p and T_i are high, the dynamics of the transfer function from the load disturbance to the process output is not dominated by a pair of complex conjugate poles only. The resulting values of OI are 0.26 and 0.56 for the first and second case respectively.

Summarizing, when both the values of the Area Index and of the Idle Index are low it is convenient to evaluate the devised Output Index. In case $OI < 0.35$ it can be concluded that both the proportional gain and integral time constant values are too high. Otherwise, the oscillatory response is caused by a too high value of K_p and/or a too low value of T_i. Note that in this latter case experience suggests to decrease the value of the proportional gain anyway.

Remark 2. It is well-known that an oscillatory response can be caused either by unsuitable controller parameters or by the excessive presence of stiction in the actuators. Thus, before applying the devised methodology it is sound to determine if valves require maintenance. This can be done by applying one (or more) of the different algorithm proposed in the literature for this purpose (see for example (Horch, 1999)).

Remark 3. It is worth noting that there are situations where the PI parameters that minimize the IAE value might yield to a too oscillatory control variable (with respect to the application) (Skogestad, 2003). These cases can be easily handled by the devised methodology, as the range of the medium values of the Area Index can be suitably modified to address the operator specifications.

Table 1. Rules for the assessment of the PI tuning. (*): an additional test is useful.

Value of AI	Value of II	Tuning assessment
high	high	K_p too low, T_i too high
high	low	K_p too low
medium/high	medium	K_p too low, T_i too low
medium	low	K_p ok, T_i ok
low	medium/high	T_i too high
low	low	K_p too high and/or T_i too low (*)

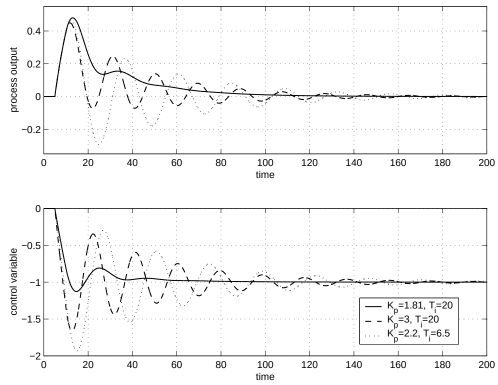

Fig. 3. Example of a load disturbance response for a too high value of T_i (solid line), for too high values of K_p and T_i (dashed line) and for too high value of K_p and a too low value of T_i (dotted line).

5. SIMULATION EXAMPLES

As a first example, the following process has been considered:

$$P_1(s) = \frac{1}{10s+1}e^{-2s} \qquad (10)$$

The PI parameters that minimize the IAE index have been found by applying a genetic algorithm. It results $K_p = 4.61$ and $T_i = 6.06$ and the corresponding indexes are $IAE = 1.42$, $AI = 0.61$ and $II = -0.81$. Thus, according to Table 1, the proposed method suggests correctly that the tuning is good. The unit step load disturbance response, together with the corresponding manipulated variable signal is plotted in Figure 4 (solid line). Then, it has been fixed $K_p = 2.5$ (maintaining the same value as before of the integral time constant). The performance obtained is shown again in Figure 4 (dashed line) and the calculated indexes are $AI = 1$ and $II = -0.61$ ($IAE = 2.42$). Thus, the too low value of the proportional gain is recognized by the devised technique.

As a second example, the following fourth-order process has been considered:

$$P_2(s) = \frac{1}{(s+1)^4} \qquad (11)$$

Note that (11) can be approximated by a FOPDT transfer function with a time constant $T = 2.1$ and a dead time $L = 1.9$. Again, the optimal tuning has been found by a genetic algorithm. It results $K_p = 1.65$ and $T_i = 4.15$, which yields to a minimum IAE of 2.79 and $AI = 0.36$ and $II = -0.80$. The control system response to a unit step load disturbance is plotted in Figure 5 (solid line). By decreasing both values of the parameters to $K_p = 1.2$ and $T_i = 2$ the results shown in Figure 5 (dashed line) are obtained. The corresponding considered indexes are $IAE = 4.07$, $AI = 0.40$ and

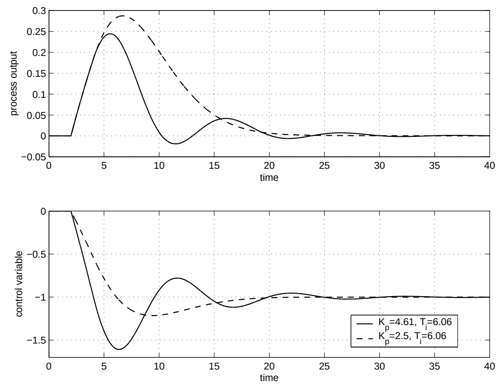

Fig. 4. Load disturbance responses for $P_1(s)$ with $K_p = 4.61$ and $T_i = 6.06$ (solid line) and with $K_p = 2.5$ and $T_i = 6.06$ (dashed line).

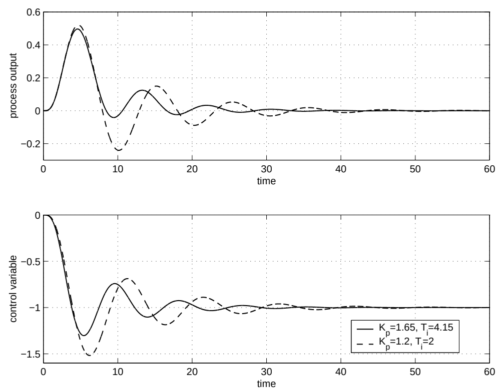

Fig. 5. Load disturbance response for $P_2(s)$ with $K_p = 1.65$ and $T_i = 4.15$ (solid line) and with $K_p = 1.2$ and $T_i = 2$ (dashed line).

$II = -0.55$. Thus, the effectiveness of the proposed method is confirmed, since from Table 1 it results that both the PI parameters are too low. It should be noted that whereas the technique proposed confirms that the tuning is good the achieved integrated absolute error value is not far from the optimum.

6. EXPERIMENTAL RESULTS

In order to prove the effectiveness of the devised technique in practical applications, a laboratory experimental setup (made by KentRidge Instruments) has been employed. Specifically, the apparatus consists of a small perspex tower-type tank (whose area is 40 cm^2) in which a level control is implemented by means of a PC-based controller. The tank is filled with water by means of a pump whose speed is set by a DC voltage (the manipulated variable), in the range 0-5 V, through a PWM circuit. The tank is fitted with an outlet at the base in order for the water to return to a reservoir. The measure of the level of the water is given by a capacitive-type probe that provides an output signal between 0 (empty tank) and 5 V (full tank). A second inflow (driven by a second pump) is adopted as a disturbance input. Specifically, when the system is at the steady-state with the process output sensor at 3 V, the second pump is activated by applying a step signal from 0 to 1.8 V. Since the apparent dead time of the system is very small with respect to its

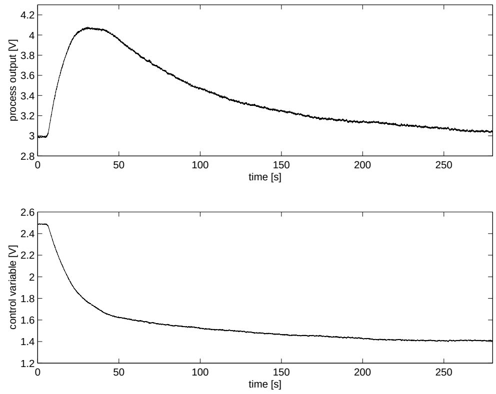

Fig. 6. Experimental load disturbance response with $K_p = 0.5$ and $T_i = 50$.

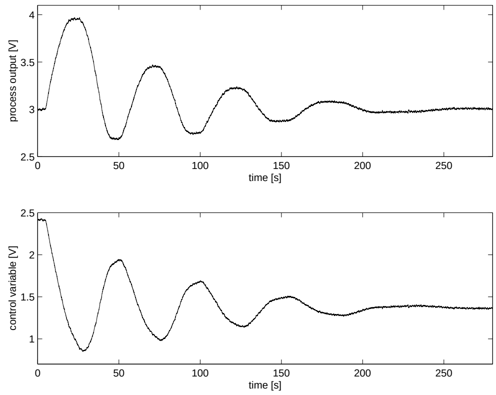

Fig. 7. Experimental load disturbance response with $K_p = 1$ and $T_i = 25$.

dominant time constant, a time delay of 10 s has been added via software to the plant input. It has also to be noted that the system is nonlinear, as the output flow rate of the tank depends on the square root of the level value.

Three experiments are presented in the following. In the first, the PI parameters have been set to $K_p = 0.5$ and $T_i = 50$. The load response is plotted in Figure 6. The calculated indexes are $AI = 1$ and $II = 0.22$ (the integrated absolute error is 106.5) and therefore Table 1 suggests to increase the proportional gain value and to decrease the integral time constant. With the values of the PI parameters modified to $K_p = 1$ and $T_i = 25$ the response shown in Figure 7 has been obtained. In this case it results $AI = 0.25$ and $II = -0.73$ (and $IAE = 49.17$). The oscillatory response is detected by the low value of the Area Index and according to Table 1 the value of the proportional gain has to be decreased. This fact is confirmed by the third experiment, where $K_p = 0.8$ and $T_i = 25$ have been selected. The corresponding response is plotted in Figure 8 (note the different range of the time axis with respect to the previous cases). It is $AI = 0.40$ and $II = -0.69$, indicating that the PI controller is well tuned, as it is ascertained also by the obtained value of $IAE = 34.83$.

7. CONCLUSIONS

In this paper a novel technique for the assessment of the tuning of a PI controller for self-regulating processes has been presented. Its main feature is that an

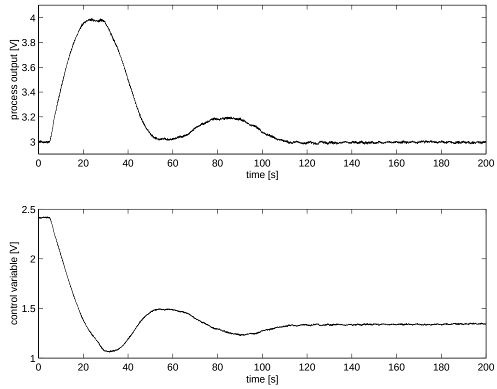

Fig. 8. Experimental load disturbance response with $K_p = 0.8$ and $T_i = 25$.

indication on how the controller parameters have to be modified to achieve better performances is provided. A key role in the overall methodology is played by the newly devised Area Index, whose significance, with respect to other methods for oscillations detection, has been highlighted. Simulation as well as experimental results show that, together with other functionalities devoted to the purpose of process monitoring, the method appears to be suitable to implement in the industrial context as a useful tool for operators.

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