DECENTRALIZED ADAPTIVE ROBUST TRACKING AND MODEL FOLLOWING FOR UNCERTAIN LARGE SCALE SYSTEMS

Hansheng Wu *

* *Department of Information Science*
Hiroshima Prefectural University
Shobara–shi, Hiroshima 727–0023, Japan
Email: hansheng@bus.hiroshima–pu.ac.jp

Abstract: The problem of decentralized robust tracking and model following is considered for a class of uncertain large scale systems. In this paper, it is assumed that the upper bounds of the uncertainties and external disturbances are unknown. A modified adaptation law with σ–modification is introduced to estimate such unknown bounds, and on the basis of the updated values of these unknown bounds, a class of decentralized local state feedback controllers is constructed for robust tracking of dynamical signals. It is shown that the proposed decentralized adaptive robust tracking controllers can guarantee that the tracking errors between each subsystem and the corresponding local reference model decrease uniformly asymptotically to zero.

Keywords: Large scale systems, decentralized control, robust tracking control, model following, adaptive control, reference model.

1. INTRODUCTION

The robust tracking and model following problem for composite dynamical systems with significant uncertainties has been widely investigated over the last decades. Some approaches to tracking dynamical signals in such uncertain dynamical systems have been developed (see, e.g., (Hopp and Schmitendorf, 1990), (Wu, 2000*b*), (Shyu and Chen, 1995), (Oucheriah, 1999), and the references therein). In recent years, there also are some works in which the problem of decentralized robust tracking and model following is considered for uncertain large scale systems. In (Shigemaru and Wu, 2001), for example, the problem of decentralized robust tracking and model following for large scale interconnected systems with uncertainties is considered, and a class of continuous (nonlinear) decentralized state feedback controllers is proposed. It is also shown in (Shigemaru and Wu, 2001) that the proposed decentralized robust tracking controllers can guarantee that the tracking errors between each subsystem and the corresponding local reference model decrease asymptotically to zero.

It is well known that in the control literature, for dynamical systems with uncertainties and external disturbances, the upper bounds of uncertainties and external disturbances are generally supposed to be known, and such bounds are employed to construct some types of stabilizing feedback controllers. However, in a number of practical control problems, such bounds may be unknown, or be partially known. Specially, in the problem of robust tracking and model following, it is also difficult to evaluate the upper bounds of uncertainties and external disturbances. Therefore, adaptive schemes should be introduced to update these unknown bounds. For such uncertain systems, several types of adaptive robust state feedback controller have been proposed (see, e.g., (Brogliato and Neto, 1995), (Choi and Kim, 1993), (Wu, 1999), (Wu, 2000*a*)). In particular, in a recent paper (Wu, 2004), the problem of adaptive robust tracking and model following is considered for uncertain time–delay dynamical systems. However, few efforts are made to consider the problem of decentralized robust tracking and model following for uncertain large scale systems with the unknown bounds of uncertainties and external disturbances.

In this paper, similar to (Shigemaru and Wu, 2001), the problem of decentralized robust tracking and model following is also considered for a class of large scale interconnected systems with time–varying uncertain parameters and external disturbance. It is assumed that the upper bounds of the uncertainties and external disturbances are unknown. For such a class of uncertain large scale systems, some decentralized local state feedback controllers will be developed for robust tracking of dynamical signals. For this purpose, similar (Wu, 2004), a class of modified adaptation laws with σ–modification is first introduced to estimate the unknown bounds of the uncertainties and external disturbances. Then, by making use of the updated values of these unknown bounds a class of decentralized adaptive robust tracking controllers is constructed. It will be shown that by employing the proposed decentralized adaptive robust tracking controllers, one can guarantee that the tracking errors between each subsystem and the corresponding local reference model decrease uniformly asymptotically to zero. That is, it is possible for each subsystem to tracks exactly the given local reference system.

2. PROBLEM FORMULATION

Consider an uncertain large scale system S composed of N interconnected subsystems $S_i, i = 1, 2, \cdots, N$, described by

$$\frac{dx_i(t)}{dt} = [\, A_i + \Delta A_i(\upsilon_i, t)\,]x_i(t) + [\, B_i + \Delta B_i(\nu_i, t)\,]u_i(t) + \sum_{j=1}^{N} A_{ij}(\zeta_i, t)x_j(t) + w_i(q_i, t) \quad \text{(1a)}$$

$$y_i(t) = C_i x_i(t) \quad \text{(1b)}$$

where $x_i(t) \in R^{n_i}$ is the state vector, $u_i(t) \in R^{m_i}$ is the control (or input) vector, $y_i(t) \in R^{l_i}$ is the output vector, and A_i, B_i, C_i are known constant matrices of appropriate dimensions. In particular, $A_{ij}(\cdot)$ stands for the extent of interconnection between S_i and S_j, and are assumed to be continuous in all their arguments. $\Delta A_i(\cdot)$, $\Delta B_i(\cdot)$ represent the uncertainties of the systems, and are also assumed to be continuous in all their arguments. Moreover, the uncertain parameters $(\upsilon_i,\ \nu_i,\ \zeta_i,\ q_i) \in \Psi_i \subset R^{L_i}$ are Lebesgue measurable and take values in a known compact bounding set Ω_i. In this paper, $x(t) \in R^n$ denotes $\left[x_1^\top(t) \ \cdots \ x_N^\top(t)\right]^\top$, where $n = n_1 + \cdots + n_N$.

On the other hand, for each $i \in \{1, 2, \cdots, N\}$, the reference sign $\hat{y}_i(t)$, which should be followed by the output $y_i(t)$ of each subsystem S_i, is assumed to be the output of a reference model $\hat{S}_i$ described by the differential equation of the form:

$$\frac{d\hat{x}_i(t)}{dt} = \hat{A}_i\hat{x}_i(t), \qquad \hat{y}_i(t) = \hat{C}_i\hat{x}_i(t) \quad (2)$$

where $\hat{x}_i(t) \in R^{\hat{n}_i}$ is the state vector of the reference model, $\hat{y}_i(t) \in R^{\hat{l}_i}$ is the output vector of the reference model, and $\hat{A}_i$, $\hat{C}_i$ are known constant matrices of appropriate dimensions. Here, $\hat{y}_i(t)$ has the same dimension as $y_i(t)$, i.e. $\hat{l}_i = l_i$. Furthermore, it is required that the model state must be bounded, i.e. for each reference model $\hat{S}_i$, $i \in \{1, 2, \ldots, N\}$, there exists a finite positive constant M_i such that for all $t \geq t_0$, $\|\hat{x}_i(t)\| \leq M_i$, $i \in \{1, 2, \cdots, N\}$.

As pointed out in (Hopp and Schmitendorf, 1990), not all models of the form given in (2) can be tracked by a corresponding subsystem given in (1) with a feedback controller. Similar to (Hopp and Schmitendorf, 1990), in this paper, the requirement for the developed decentralized local controller to track the model described by (2) is the existence of the matrices $G_i \in R^{n_i \times \hat{n}_i}$, $H_i \in R^{m_i \times \hat{n}_i}$, such that for each $i \in \{1, 2, \ldots, N\}$, the following matrix algebraic equation holds.

$$\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} \begin{bmatrix} G_i \\ H_i \end{bmatrix} = \begin{bmatrix} G_i\hat{A}_i \\ \hat{C}_i \end{bmatrix} \quad (3)$$

For each $i \in \{1, 2, \ldots, N\}$, if a solution cannot be found to satisfy this algebraic matrix equation, a different model or output matrix C_i must be chosen. In particular, the approach to finding the solution to (3) is also discussed in detail in (Hopp and Schmitendorf, 1990) and (Shyu and Chen, 1995).

Provided that all states are available, the decentralized local state feedback controller for each subsystem can be represented by a function:

$$u_i(t) = p_i(x_i(t), t), \quad i \in \{1, 2, \ldots, N\} \quad (4)$$

Now, the question is how to synthesize a decentralized local state feedback controller $u_i(t)$ such that the output $y_i(t)$ of each subsystem follows the output $\hat{y}_i(t)$ of the corresponding local reference model.

In this paper, for (1) the following standard assumptions are introduced.

Assumption 2.1. For any $i \in \{1, 2, \cdots, N\}$, the pair (A_i, B_i) is completely controllable.

Assumption 2.2. For all $(\upsilon_i,\ \nu_i,\ \zeta_i,\ q_i) \in \Psi_i,\ i \in \{1,2,\dots,N\}$, there exist some continuous and bounded matrix functions $N_i(\cdot)$, $E_i(\cdot)$, $D_{ij}(\cdot)$, $\tilde{w}_i(\cdot)$ of appropriate dimensions such that for each $i \in \{1,2,\dots,N\}$,

$$\begin{aligned}\Delta A_i(\upsilon_i,t) &= B_i N_i(\upsilon_i,t)\\ \Delta B_i(\nu_i,t) &= B_i E_i(\nu_i,t)\\ A_{ij}(\zeta_i,t) &= B_i D_{ij}(\zeta_i,t), \quad j=1,2,\dots,N\\ w_i(q_i,t) &= B_i \tilde{w}_i(q_i,t)\end{aligned}$$

Remark 2.1. It is obvious that Assumption 2.2 defines the matching condition about the uncertainties and external disturbance, and is a rather standard assumption for robust control problem (see, e.g., (Hopp and Schmitendorf, 1990), (Wu, 2000*b*), (Oucheriah, 1999), (Choi and Kim, 1993), and the references therein). It is well known that these matching conditions restrict the structure of each subsystem by stipulating that all uncertainties and interconnections should fall into the range space of the control vector B_i. However, this fact is true for a large class of systems, particularly mechanical systems.

For convenience, the following notations are introduced which represent the bounds of the uncertainties and external disturbances.

$$\begin{aligned}\rho_i(t) &:= \max_{\upsilon_i} \|N_i(\upsilon_i,t)\|\\ \rho_{ij}(t) &:= \max_{\zeta_i} \|D_{ij}(\zeta_i,t)\|, \quad j=1,2,\dots,N\\ \kappa_i(t) &:= \max_{q_i} \|\tilde{w}_i(q_i,t)\|\\ \mu_i(t) &:= \min_{\nu_i} \left[\frac{1}{2}\lambda_{\min}\left(E_i(\nu_i,t)+E_i^\top(\nu_i,t)\right)\right]\end{aligned}$$

Here, the functions $\rho_i(t)$, $\rho_{ij}(t)$, $\kappa_i(t)$, $\mu_i(t)$ are assumed to be unknown. Moreover, the uncertain $\rho_i(t)$, $\rho_{ij}(t)$, $\kappa_i(t)$, $\mu_i(t)$ are also assumed, without loss of generality, to be uniformly continuous and bounded for any $t \in R^+$.

By employing the notations given above, for (1) the following standard assumption is introduced.

Assumption 2.3. For every $t \geq t_0$, $\mu_i(t) > -1,\ i \in \{1,2,\dots,N\}$.

Remark 2.2. It is well known that in (Shigemaru and Wu, 2001), the upper bounds of the uncertainties and external disturbances are assumed to be known, and such bounds are employed to construct their decentralized local robust tracking controllers. That is, $\rho_i(t)$, $\rho_{ij}(t)$, $\kappa_i(t)$, $\mu_i(t)$, are assumed to be the known continuous and bounded functions, and the proposed decentralized robust tracking control schemes include such bounds $\rho_i(t)$, $\rho_{ij}(t)$, $\kappa_i(t)$, $\mu_i(t)$. In this paper, it has been assumed for the bounds $\rho_i(t)$, $\rho_{ij}(t)$, $\kappa_i(t)$, $\mu_i(t)$ to be unknown. For such a problem of decentralized robust tracking and model following, a class of decentralized local adaptive robust tracking controllers will be proposed. It will be also show that the proposed decentralized tracking controller can guarantee that tracking errors between each subsystem and the local reference model decrease to zero asymptotically.

On the other hand, it follows from Assumption 2.1 that for any given positive constant η_i and positive definite matrix $Q_i \in R^{n_i \times n_i}$, the algebraic Riccati equation of the form

$$A_i^\top P_i + P_i A_i - \eta_i P_i B_i B_i^\top P_i \;=\; -Q_i \qquad (5)$$

has a solution $P_i \in R^{n_i \times n_i}$, which is also a positive definite matrix.

3. MAIN RESULTS

In this section, a class of decentralized local adaptive robust state feedback controllers is proposed, which can guarantee that (i) the output $y_i(t)$ of each subsystem follows the output $\hat{y}_i(t)$ of the corresponding local reference model and (ii) the tracking error decreases asymptotically to zero. For this, let the tracking error between each subsystem and the local reference model be defined as

$$e_i(t) \;=\; y_i(t) - \hat{y}_i(t), \quad i \in \{1,2,\dots,N\} \quad (6)$$

then the decentralized local state feedback tracking control laws can be constructed as

$$u_i(t) \;=\; H_i \hat{x}_i(t) + \tilde{p}_i(t), \quad i \in \{1,2,\dots,N\} \ (7)$$

where $H_i \in R^{m_i \times \hat{n}_i}$ is assumed to be satisfy the matrix algebraic equation described by (3), and $\tilde{p}_i(t)$ is auxiliary control function which will be given later.

Here, for each subsystem, a new state vector $z_i(t),\ i \in \{1,2,\dots,N\}$, is first defined as follows:

$$z_i(t) \;:=\; x_i(t) - G_i \hat{x}_i(t) \qquad (8)$$

where $G_i \in R^{n_i \times \hat{n}_i}$ is still assumed to be satisfy the algebraic equation described by (3).

It can be obtained from (3) and (8) that the relationship between the tracking error $e_i(t)$ and the auxiliary state vector $z_i(t)$ is as follows.

$$e_i(t) \;=\; C_i z_i(t), \quad i \in \{1,2,\dots,N\} \qquad (9)$$

For each subsystem, applying (7) to (1) yields an auxiliary subsystem $\hat{S}_i,\ i \in \{1,2,\dots,N\}$, of the form:

$$\begin{aligned}\frac{dz_i(t)}{dt} \;&=\; [\,A_i + \Delta A_i(\upsilon_i,t)\,]\,z_i(t)\\ &+ [\,B_i + \Delta B_i(\nu_i,t)\,]\,\tilde{p}_i(t) + \sum_{j=1}^{N} A_{ij}(\zeta_i,t) z_j(t)\\ &+ g_i(\upsilon_i,\nu_i,\zeta_i,q_i,\hat{x}_i,t) \end{aligned} \qquad (10)$$

where

$$g_i(\upsilon_i, \nu_i, \zeta_i, q_i, \hat{x}_i, t)$$
$$:= [\, \Delta A_i(\upsilon_i, t)G_i + \Delta B_i(\nu_i, t)H_i \,]\, \hat{x}_i(t)$$
$$+ \sum_{j=1}^{N} A_{ij}(\zeta_i, t)G_j \hat{x}_j(t) + w_i(q_i, t) \qquad (11)$$

Then, by making use of the matching condition (see Assumption 2.2), (11) can be reduced to

$$g_i(\cdot) \;=\; B_i F_i(\cdot) \qquad (12)$$

where

$$F_i(\upsilon_i, \nu_i, \zeta_i, q_i, \hat{x}_i, t)$$
$$:= [\, N_i(\upsilon_i, t)G_i + E_i(\nu_i, t)H_i \,]\, \hat{x}_i(t)$$
$$+ \sum_{j=1}^{N} D_{ij}(\zeta_i, t)G_j \hat{x}_j(t) + \tilde{w}_i(q_i, t) \qquad (13)$$

Furthermore, for (13) the following notation is introduced.

$$\beta_i(t) \;:=\; \max\Big\{ \big\| F_i(\upsilon_i, \nu_i, \zeta_i, q_i, \hat{x}_i, t) \big\| \;: \;(\upsilon_i, \nu_i, \zeta_i, q_i) \in \Psi_i,\ \|\hat{x}_i(t)\| \le M_i,\ t \in R^+ \Big\}$$

Here, the uncertain $\beta_i(t)$ is still assumed to be uniformly continuous and bounded.

In this paper, since the bounds $\rho_i(t)$, $\rho_{ij}(t)$, $\mu_i(t)$, $\beta_i(t)$ have been assumed to be continuous and bounded for any $t \in R^+$, it can be supposed that there exist some positive constants ρ_i^*, ρ_{ij}^*, μ_i^*, β_i^*, which are defined by

$$\rho_i^* := \max\left\{ \rho_i(t) :\ t \in R^+ \right\}$$
$$\rho_{ij}^* := \max\left\{ \rho_{ij}(t) :\ t \in R^+ \right\}$$
$$\mu_i^* := \min\left\{ \mu_i(t) :\ t \in R^+ \right\} \;>\; -1$$
$$\beta_i^* := \max\left\{ \beta_i(t) :\ t \in R^+ \right\}$$

Here, it is worth pointing out that the constants ρ_i^*, ρ_{ij}^*, μ_i^*, β_i^*, are still unknown. Therefore, such unknown bounds can not be directly employed to construct the decentralized robust tracking controllers.

Without loss of generality, the following definition is also introduced:

$$\psi_i^* := \frac{1}{1+\mu_i^*}\left(1 + (\rho_i^*)^2 + \sum_{j=1}^{N} \left(\rho_{ij}^*\right)^2 \right) \qquad (14a)$$

$$\phi_i^* := \frac{\beta_i^*}{1+\mu_i^*} \qquad (14b)$$

where for any $i \in \{1, 2, \ldots, N\}$, ψ^* and ϕ^* are obviously unknown positive constants.

Now, the auxiliary control function $\tilde{p}_i(t)$ is given as follows. That is, for any $i \in \{1, 2, \ldots, N\}$,

$$\tilde{p}_i(t) \;=\; p_{i1}(z_i(t), t) + p_{i2}(z_i(t), t) \qquad (15a)$$

where $p_{i1}(\cdot)$ and $p_{i2}(\cdot)$ are given by

$$p_{i1}(z_i(t), t) = -\frac{1}{2}\, \eta_i \hat{\psi}_i(t) B_i^\top P_i z_i(t) \qquad (15b)$$

$$p_{i2}(z_i(t), t) = -\frac{\hat{\phi}_i^2(t) B_i^\top P_i z_i(t)}{\big\| B_i^\top P_i z_i(t) \big\|\, \hat{\phi}_i(t) + \sigma_i(t)} \qquad (15c)$$

and where $\sigma_i(t) \in R^+$ is any positive uniform continuous and bounded function which satisfies

$$\lim_{t\to\infty} \int_{t_0}^{t} \sigma_i(\tau) d\tau \;\le\; \bar{\sigma}_i \;<\; \infty \qquad (15d)$$

where $\bar{\sigma}_i$ is any positive constant. Here for any $i \in \{1, 2, \ldots, N\}$, $P_i \in R^{n_i \times n_i}$ is the solution of the Riccati equation described by (5), η_i is a positive constant which is chosen such that

$$Q_i - (1+N)\eta^{-1} I_i \;>\; 0 \qquad (15e)$$

where $\eta := \min\{\eta_i,\ i = 1, 2, \ldots, N\}$.

In particular, for any $i \in \{1, 2, \ldots, N\}$, $\hat{\psi}_i(\cdot)$ and $\hat{\phi}_i(\cdot)$ in (15) are, respectively, the estimates of the unknown ψ_i^* and ϕ_i^*, which are, respectively, updated by the following adaptive laws:

$$\frac{d\hat{\psi}_i(t)}{dt} = -\gamma_i \sigma_i(t) \hat{\psi}_i(t) + \eta_i \gamma_i \big\| B_i^\top P_i z_i(t) \big\|^2 \qquad (16a)$$

$$\frac{d\hat{\phi}_i(t)}{dt} = -m_i \sigma_i(t) \hat{\phi}_i(t) + m_i \big\| B_i^\top P_i z_i(t) \big\| \qquad (16b)$$

where γ_i, m_i are any positive constants, and $\hat{\psi}_i(t_0)$, $\hat{\phi}_i(t_0)$ are finite.

Moreover, let $\hat{\psi}(t) \in R^N$ and $\hat{\phi}(t) \in R^N$ denote, respectively, $\hat{\psi}(t) := \left[\hat{\psi}_1(t) \ \cdots \ \hat{\psi}_N(t) \right]^\top$ and $\hat{\phi}(t) := \left[\hat{\phi}_1(t) \ \cdots \ \hat{\phi}_N(t) \right]^\top$.

For each auxiliary subsystem, applying the auxiliary control function given in (15) to (10) yields the following closed–loop auxiliary subsystem:

$$\frac{dz_i(t)}{dt} \;=\; \left[A_i - \frac{1}{2}\eta_i \hat{\psi}_i(t) B_i B_i^\top P_i \right] z_i(t)$$
$$+ \left[\Delta A_i(\cdot) - \frac{1}{2}\eta_i \hat{\psi}_i(t) \Delta B_i(\cdot) B_i^\top P_i \right] z_i(t)$$
$$+ \Big[B_i + \Delta B_i(\nu_i, t) \Big] p_{i2}(z_i(t), t)$$
$$+ \sum_{j=1}^{N} A_{ij}(\zeta_i, t) z_j(t) + g_i(\upsilon_i, \nu_i, \zeta_i, q_i, \hat{x}_i, t) \qquad (17)$$

On the other hand, letting $\tilde{\psi}_i(t) = \hat{\psi}_i(t) - \psi_i^*$ and $\tilde{\phi}_i(t) = \hat{\phi}_i(t) - \phi_i^*$, the adaptive laws given in (16) can rewritten as the following error system:

$$\frac{d\tilde{\psi}_i(t)}{dt} = -\gamma_i \sigma_i(t) \tilde{\psi}_i(t) + \eta_i \gamma_i \big\| B_i^\top P_i z_i(t) \big\|^2$$
$$-\gamma_i \sigma_i(t) \psi^* \qquad (18a)$$

$$\frac{d\tilde{\phi}_i(t)}{dt} = -m_i\sigma_i(t)\tilde{\phi}_i(t)+m_i \left\|B_i^\top P_i z_i(t)\right\| \\ -m_i\sigma_i(t)\phi^* \quad (18b)$$

Here, $\tilde{\psi}(t) \in R^N$ and $\tilde{\phi}(t) \in R^N$ denote, respectively, $\tilde{\psi}(t) := \left[\tilde{\psi}_1(t) \ \cdots \ \tilde{\psi}_N(t)\right]^\top$ and $\tilde{\phi}(t) := \left[\tilde{\phi}_1(t) \ \cdots \ \tilde{\phi}_N(t)\right]^\top$.

In the following, $(z, \tilde{\psi}, \tilde{\phi})(t)$ denotes a solution of the closed–loop auxiliary system and the error system. Then, the following theorem can be obtained.

Theorem 3.1. Consider the adaptive closed–loop auxiliary system described by (17) and (18), which satisfies *Assumptions 2.1* to *2.3*. Then, the solutions $(z, \tilde{\psi}, \tilde{\phi})$ $(t;\ t_0, z(t_0), \tilde{\psi}(t_0), \tilde{\phi}(t_0))$ of the closed-loop auxiliary system described by (17) and the error system described by (18) are uniform bounded and

$$\lim_{t\to\infty} z(t;\ t_0, z(t_0)) \ = \ 0 \quad (19)$$

Proof : For the adaptive closed–loop auxiliary system described by (17) and (18), a Lyapunov function candidate is fist defined as follows.

$$V(z, \tilde{\psi}, \tilde{\phi}) = \sum_{i=1}^{N} z_i^\top(t) P_i z_i(t) \\ +\frac{1}{2}\tilde{\psi}^\top(t)\big(I+\mu^*\big)\Gamma^{-1}\tilde{\psi}(t) \\ +\tilde{\phi}^\top(t)\big(I+\mu^*\big)M^{-1}\tilde{\phi}(t) \quad (20)$$

where for each $i \in \{1, 2, \ldots, N\}$, P_i is the solution to (5), and $(I+\mu^*) \in R^{N\times N}$, $\Gamma^{-1} \in R^{N\times N}$, $M^{-1} \in R^{N\times N}$ are positive definite matrices which are defined by

$$\big(I+\mu^*\big) := \mathrm{diag}\left\{\big(1+\mu_1^*\big),\ \ldots,\ \big(1+\mu_N^*\big)\right\} \\ \Gamma^{-1} := \mathrm{diag}\left\{\gamma_1^{-1},\ \ldots,\ \gamma_N^{-1}\right\} \\ M^{-1} := \mathrm{diag}\left\{m_1^{-1},\ \ldots,\ m_N^{-1}\right\}$$

Let $(z(t),\ \tilde{\psi}(t),\ \tilde{\phi}(t))$ be the solutions to (17) and (18) for $t \geq t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (17) and (18), and by making use of some trivial manipulations, it is obtained that

$$\frac{dV(z, \tilde{\psi}, \tilde{\phi})}{dt} \ \leq \ \sum_{i=1}^{N}\Bigg\{-z_i^\top(t) Q_i z_i(t) \\ +\eta_i \left\|B_i^\top P_i z_i(t)\right\|^2 + 2\rho_i^* \left\|B_i^\top P_i z_i(t)\right\| \|z_i(t)\| \\ -\eta_i\hat{\psi}_i(t)\big(1+\mu_i^*\big) \left\|B_i^\top P_i z_i(t)\right\|^2 \\ -\frac{2\hat{\phi}_i^2(t)\big(1+\mu_i^*\big)\left\|B_i^\top P_i z_i(t)\right\|^2}{\left\|B_i^\top P_i z_i(t)\right\| \hat{\phi}_i(t) \ + \ \sigma_i(t)} \\ +\sum_{j=1}^{N} 2\rho_{ij}^* \left\|B_i^\top P_i z_i\right\| \|z_j\| + 2\beta_i^* \left\|B_i^\top P_i z_i\right\|\Bigg\} \\ +\tilde{\psi}^\top(t)\big(I+\mu^*\big)\Gamma^{-1}\frac{d\tilde{\psi}(t)}{dt} \\ +2\tilde{\phi}^\top(t)\big(I+\mu^*\big)M^{-1}\frac{d\tilde{\phi}(t)}{dt} \quad (21)$$

Notice the fact that for any positive constant $c > 0$,

$$2ab \ \leq \ \frac{1}{c}a^2 + cb^2, \quad \forall\, a,\ b\ >\ 0$$

Then, it can be further obtained from (21) that for any $t \geq t_0$,

$$\frac{dV(z, \tilde{\psi}, \tilde{\phi})}{dt} \ \leq \ \sum_{i=1}^{N}\Bigg\{-z_i^\top(t)\tilde{Q}_i z_i(t) \\ +\big(1+\mu_i^*\big)\Bigg[-\eta_i\hat{\psi}_i(t)\left\|B_i^\top P_i z_i\right\|^2 + \eta_i\psi_i^*\left\|B_i^\top P_i z_i\right\|^2 \\ -\frac{2\hat{\phi}_i^2(t)\left\|B_i^\top P_i z_i\right\|^2}{\left\|B_i^\top P_i z_i\right\|\hat{\phi}_i(t)+\sigma_i(t)} + 2\phi_i^*\left\|B_i^\top P_i z_i\right\|\Bigg] \\ +\big(1+\mu_i^*\big)\gamma_i^{-1}\tilde{\psi}_i(t)\frac{d\tilde{\psi}_i(t)}{dt} \\ +2\big(1+\mu_i^*\big)m_i^{-1}\tilde{\phi}_i(t)\frac{d\tilde{\phi}_i(t)}{dt}\Bigg\} \quad (22)$$

where $\tilde{Q}_i \ := \ Q_i - \eta^{-1}(1+N)I_i \ > \ 0.$

Notice that the facts that

$$\hat{\psi}_i(t) = \tilde{\psi}_i(t) + \psi_i^*, \quad \hat{\phi}_i(t) = \tilde{\phi}_i(t) + \phi_i^*$$

it follows from (18) and (22) that

$$\frac{dV(\tilde{z}(t))}{dt} \ \leq \ -\bar{\eta}_{\min}\|z(t)\|^2 + \sum_{i=1}^{N}\bar{\varepsilon}_i\sigma_i(t) \quad (23)$$

where

$$\tilde{z}(t) := \left[\ z^\top(t) \quad \tilde{\psi}^\top(t) \quad \tilde{\phi}^\top(t)\ \right]^\top \\ \bar{\varepsilon}_i := \frac{1}{4}\big(1+\mu_i^*\big)\left[8 + |\psi_i^*|^2 + 2\,|\phi_i^*|^2\right] \\ \bar{\eta}_{\min} := \min\Big\{\ \lambda_{\min}\left(\tilde{Q}_i\right),\ i = 1, \ldots, N\ \Big\}$$

On the other hand, in the light of the definition, given in (20), of Lyapunov function, there always exist two positive constants $\delta_{\min}$ and $\delta_{\max}$ such that for any $t \geq t_0$,

$$\tilde{\gamma}_1(\|\tilde{z}(t)\|) \ \leq \ V(\tilde{z}(t)) \ \leq \ \tilde{\gamma}_2(\|\tilde{z}(t)\|) \quad (24)$$

where

$$\tilde{\gamma}_1(\|\tilde{z}(t)\|) := \delta_{\min}\|\tilde{z}(t)\|^2 \\ \tilde{\gamma}_2(\|\tilde{z}(t)\|) := \delta_{\max}\|\tilde{z}(t)\|^2$$

Then, in the light of (23) and (24), by employing the known method (see, e.g. (Wu, 2004)), it can be easily shown that the solutions $\tilde{z}(t)$ of the adaptive closed–loop auxiliary system are uniformly bounded, and that the auxiliary state $z(t)$ converges asymptotically to zero. Thus, the proof of this theorem can be completed. $\nabla\nabla\nabla$

Thus, from Theorem 3.1, the following theorem can be obtained, which shows that by employing the decentralized local controllers described in (7) with (15) and (16), one can guarantee the zero–error tracking between each subsystem and the local reference model.

Theorem 3.2. Consider the model following problem of uncertain large scale system (1) satisfying *Assumptions 2.1* to *2.3*. Then, by using the decentralized local state feedback controllers $u_i(t)$ described in (7) with (15) and (16), one can guarantee that the tracking error $e_i(t)$, $i \in \{1, 2, \ldots, N\}$, between each subsystem and local reference model, decreases uniformly asymptotically to zero.

Proof: It has been shown in *Theorem 3.1* that each adaptive closed–loop auxiliary subsystem described by (17) and (18) is uniformly bounded and their auxiliary states can converge uniformly asymptotically to zero. That is, for the auxiliary state $z_i(t)$, it can be obtained that

$$\lim_{t\to\infty} \|z_i(t)\| = 0, \quad i \in \{1, 2, \ldots, N\}$$

Then, it can easily be obtained from the relationship between $e_i(t)$ and $z_i(t)$, i.e. $e_i(t) = C_i z_i(t)$, that each local tracking error $e_i(t)$, $i \in \{1, 2, \ldots, N\}$, also decreases uniformly asymptotically to zero. $\nabla\nabla\nabla$

Remark 3.1. In a recent paper (Wu, 2004), the adaptation laws with σ–modification have been improved to guarantee an asymptotic stability result. In this paper, the improved adaptation laws, described by (16), are extended to the problem of decentralized robust tracking model following for uncertain large scale interconnected systems to develop a class of decentralized local adaptive robust tracking controllers.

Remark 3.2. In order to illustrate the validity of the results obtained in the paper, a numerical example is also given, and the simulation is carried out. It is known from the results of the simulation that the proposed decentralized adaptive robust tracking controllers can indeed guarantee that the tracking errors between each subsystem and the corresponding local reference model decrease uniformly asymptotically to zero. (The details of the illustrative numerical example and the figures of the simulation will be displayed in the presentation.)

4. CONCLUDING REMARKS

The problem of decentralized robust tracking and model following for a class of large scale interconnected systems with time–varying uncertain parameters and external disturbance has been considered. Here, the upper bounds of the uncertainties and external disturbances are assumed to be unknown. For such a class of uncertain large scale systems, a class of decentralized adaptive robust state feedback controllers has been proposed for robust tracking of dynamical signals. It has been shown that by employing the proposed decentralized adaptive robust tracking controllers, one can guarantee that the tracking errors between each subsystem and the corresponding local reference model decrease uniformly asymptotically to zero. Therefore, our results may be expected to have some applications to practical robust tracking and model following problems of uncertain large scale systems.

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