STABILIZATION OF INVARIANT SETS OF SWITCHED SYSTEMS BY OUTPUT FEEDBACK [1]

Yu-Ping Tian [*,2] **Xiang-Ze Lin** [*]

[*] *Department of Automatic Control, Southeast University, Nanjing, 210096, P.R.China*

Abstract: In this paper, the notion of the input/output-to-state stability (IOSS) is extended to the case of the input/output-to-$V(x)$ stability (IOVS), which implies detectability of the zero-value set of a storage function. Based on this notion and the notion of passivity of nonlinear systems, we propose and prove sufficient conditions under which an invariant state set of a class of switched nonlinear systems can be stabilized by output feedback. Then we show that a nonlinear system is IOVS if $V(x)$ is an IOSS-Lyapunov function of the system.

Keywords: Nonlinear systems, Switches, Output feedback, Stabilization

1. INTRODUCTION

In control literature there are many papers devoted to stabilization problem for the following nonlinear affine system

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \tag{1}$$

where $x \in X = R^n$ is the state, $u \in U = R^m$ is the control input which is measurable and locally essentially bounded, $y \in Y = R^m$ is the measurable output, $f : R^n \times R^m \to R^n$ is smooth mapping, $h : R^n \to R^m$ is a continuous function.

Under the hypothesis that the unforced system of (1), $\dot{x} = f(x)$, is linear (i.e., $f(x) = Ax$) and Lypunove stable, Jurdjevic and Quinn proposed a sufficient condition under which the equilibrium point of the system can be asymptotically stabilized by a smooth state feedback (Jurdjevic and Quinn, 1978). Since then, various Jurdjevic-Quinn type sufficient conditions have been developed for system (1) (e.g., see, Kalouptsidis and Tsinias (1984); Lee and Arapostathis (1988); Lin (1995) and references therein).

The stabilization problem for system (1) has been also extensively studied under the hypothesis that the system is passive. The complete answer to the equilibrium point stabilization problem was given by Byrnes *et al.* in Byrnes, Isidori and Willems (1991): the zero-state equilibrium of the unforced nonlinear passive system with a proper positive-definite storage function is stabilizable if the system is zero-state detectable. Recently, this result was extended by Shiriaev to the case of the invariant set stabilization via introducing a notion of V-detectability (Shiriaev, 2000).

In this paper we study the invariant set stabilization problem for the following switched system

$$\begin{cases} \dot{x} = f_q(x, u) \\ y = h(x) \end{cases} \tag{2}$$

[1] This work was supported by the National Natural Science Foundation of China for Distinguished Young Scholars (under the grant 60425308) and the Doctoral Research Foundation of the Educational Ministry of China (under the grant 20040286004).

[2] Corresponding author.(yptian@seu.edu.cn)

where $q \in Q = \{1, 2, \cdots, N\}$ is determined by some switching signal $\sigma(t) : [0, \infty) \rightarrow Q$. For each $q \in Q$, the q-th subsystem of (2) satisfies all the assumptions given for the system (1), and is passive with a nonnegative storage function. The question we raise in this paper is: *Can the passification approach introduced by Byrnes, Isidori and Willems (1991) and then extended by Shiriaev (2000) be applied to the switched nonlinear system or not?*

To answer this question, we extend the notion of the input/output-to-state stability (IOSS) introduced by Sontag and Wang (1997), to the case of the input/output-to-$V(x)$ stability (IOVS). We show that IOVS includes the V-detectability introduced by Shiriaev as a special case in which the control input is set to be zero. Based on this new notion and the notion of passivity of nonlinear systems, we propose and prove sufficient conditions under which an invariant state-set of the switched system (2) can be stabilized by output feedback. A multiple-storage-function like theorem is obtained for solving the problem. The relationship between IOVS and IOSS is also studied in this paper based on the concept of IOSS-Lyapunov function (Sontag and Wang, 1997). A checkable criterion for the IOVS property is provided: the nonlinear system (1) is IOVS if $V(x)$ is an IOSS-Lyapunov function of the system.

2. PASSIVITY, INPUT/OUTPUT-TO-V STABILITY AND V-DETECTABILITY

In this section we focus on some notions of the nonlinear affine control system (1). Throughout the paper we denote by $\|z\|_J$ the supremum norm of a signal z on the interval $J \subset [0, \infty)$.

We first recall the notion of passivity of system (1).

Definition 2.1 (Willems, 1972). The system (1) is passive if there exists a C^0 nonnegative storage function $V(x) : X \rightarrow R, V(0) = 0$, such that for all piece-wise continuous control input $u(s)$ along the solution $x(t) = x(t, x_0)$ defined on the maximal interval $[0, t_*)$ the inequality

$$V(x) - V(x_0) \leq \int_0^t y^T(s)u(s)\mathrm{d}s, \; t \in [0, t_*), \quad (3)$$

holds.

A function V is said to be *proper* if the set $\{x \in R^n : V(x) \leq c\}$ is compact for all $c \geq 0$.

Note that if $V(x)$ is a C^r $(r \geq 1)$ function, the passivity condition, inequality (3), is equivalent to

$$\dot{V} \leq y^T u, \; \forall u \in U. \quad (4)$$

In Sontag and Wang (1997) the input/output-to-state stability for system (1) was defined as follows.

Definition 2.2. The system (1) is input/output-to-state stable (IOSS) if there exist some $\beta \in \mathcal{KL}$ and $\gamma_1, \gamma_2 \in \mathcal{K}$ such that

$$|x(t, x_0, u)| \leq \beta(|x_0|, t) + \gamma_1(\|y\|_{[0,t]}) + \gamma_2(\|u\|_{[0,t]}) \quad (5)$$

for every initial state $x(0) = x_0 \in X$ and control input $u \in U$.

This definition describes the property that "small input and small output mean (eventually) small state trajectory". If we consider the behavior of the nonnegative storage function $V(x)$ instead of the state trajectory, it is natural to extend the above notion to the case of input/output-to-V stability.

Definition 2.3. The system (1) is input/output-to-V stable (IOVS) if there exist some $\beta \in \mathcal{KL}$ and $\gamma_1, \gamma_2 \in \mathcal{K}$ such that

$$V(x) \leq \beta(V(x_0), t) + \gamma_1(\|y\|_{[0,t]}) + \gamma_2(\|u\|_{[0,t]}) \quad (6)$$

for every initial state $x(0) = x_0 \in X$ and control input $u \in U$. If Eq.(6) holds only for $x_0 \in N = \{x \in X : V(x) < c\}$, where $c > 0$ is a constant, then system is locally input/output-to-V stable.

Obviously, when $V(x) = |x|$, IOVS is identical to IOSS.

The notion of IOVS can also be considered as a generalization of the V-detectability of system (1) introduced by Shiriaev which is given below.

Definition 2.4 (Shiriaev, 2000). The system (1) is locally V-detectable if there exists a constant $c > 0$ such that for every $x_0 \in N = \{x \in X : V(x) < c\}$ and $t > 0$ the following implication holds

$$h(x(t, x_0, 0)) = 0 \;\Rightarrow\; \lim_{t \to \infty} V(x(t, x_0, 0)) = 0. \quad (7)$$

If $N = X$, then system is said to be V-detectable.

Indeed, if $u(t) \equiv 0$ for any $t \geq 0$, then from (6) it immediately follows the implication (7).

3. STABILIZATION OF INVARIANT SETS OF SWITCHED SYSTEMS

In this section we consider the set stabilization problem for the switched system (2), in which $q \in Q = \{1, 2, \cdots, N\}$, is determined by the switching signal $\sigma(t) : [0, \infty) \to Q$. $\sigma(t)$ can be considered as a right-continuous, piece-wise, constant-value function. Let $t_i, i = 0, 1, 2, \cdots$, be the sequence of time at which $\sigma(t)$ is discontinuous, i.e., the field of system (2) is switched from one to another at t_i. Assume that the dwell time at each subsystem is no less than a positive constant τ, i.e.,

$$t_{i+1} - t_i \geq \tau > 0, \ i = 0, 1, 2, \cdots. \tag{8}$$

Note that the sequence $t_0, t_1, t_2, \cdots, t_M, \cdots$, may be finite or infinite. However, the finite case can always be regarded as a special case of the infinite case. Indeed, in the finite case, we can artificially introduce infinitely many "switching points" $t_{M+1}, \cdots, t_{M+j} \cdots$, after the last real switch at t_M with $\sigma(t_{M+j})$ having the same value for all j. Then, all the further results hold. So in the rest of the paper we will consider only the infinite case.

Theorem 3.1. *Suppose that the switched system (2) satisfies the following assumptions*

(1) For each $q \in Q$, the q-th subsystem of (2) is passive with a nonnegative C^r-smooth storage function $V_q(x), r \geq 1$.
(2) For each $q \in Q$, $V_q^0 = \{x \in X : V_q(x) = 0\}$ is a compact set.
(3) For each $q \in Q$, the q-th subsystem of (2) is input/output-to-V_q stable in a neighborhood of V_q^0 defined as

$$V_q^{c_q} = \{x \in X : V_q(x) \leq c_q\}.$$

(4) At each switching point $t_i, i = 1, 2, \cdots$, the following inequality holds

$$V_q(x(t_i)) \geq V_p(x(t_i)), \tag{9}$$

where $q = \sigma(t), \forall t \in [t_{i-1}, t_i)$, and $p = \sigma(t_i)$.

Let $\phi_q : Y \to U$ be any continuous function satisfying $\phi_q(0) = 0$, $y^T\phi_q(y) > 0$ for $y \neq 0$. Then under the feedback control

$$u_q = -\phi_q(y), \tag{10}$$

the set $V_0 = \bigcup_{q=1}^{N} V_q^0$ is invariant and asymptotically stable for the switched system (2). Moreover, if the function V_q is proper and, for each $q \in Q$, the q-th subsystem of (2) is input/output-to-V_q stable, then, under (10), V_0 is a globally asymptotically stable invariant set of the switched system (2).

Proof. First of all, we note that assumption 4 implies any trajectory starting from V_q^0 will enter V_p^0 when the field is switched from subsystem q to subsystem p. Therefore, V_0 is indeed an invariant set for the switched system. The rest of this proof consists of two parts: the first part is devoted to the local asymptotic stability of V_0 and the second part to the global asymptotic stability.

(1) Since the set $V_q^0, \forall q \in Q$, is compact and $V_q, \forall q \in Q$, is a smooth function, there exists a small positive constant ε_q such that the set $V_{\varepsilon_q} = \{x \in R^n : V_q(x) \leq \varepsilon_q\}$ is also compact. Let $\varepsilon = \min\{\min_{q\in Q} \varepsilon_q, \min_{q\in Q} c_q\}$, and $V_\varepsilon = \bigcup_{q=1}^{N} V_q^\varepsilon$, where $V_q^\varepsilon = \{x \in R^n : V_q(x) \leq \varepsilon\}$. Then V_ε is compact and for any $x_0 \in V_q^\varepsilon$ subsystem q is IOVS by assumption 3.

By assumption 1 of the theorem, subsystem q is passive with a nonnegative smooth storage function $V_q(x)$. So from the passivity condition (4) and the feedback control law (10) it follows that

$$\dot{V}_q(x) \leq y^T u_q = y^T(-\phi_q(y)) = -y^T\phi_q(y) \leq 0. \tag{11}$$

Inequality (11) plus the assumption 4 of the theorem guarantees the set V_ε is also invariant for the switched system.

The invariance of the set V_ε implies for any time interval $[t_i, t_{i+1})$ with $\sigma(t) = q, \forall t \in [t_i, t_{i+1})$, we have $x(t_i) \in V_q^\varepsilon$. So from the IOVS property of subsystem q in V_q^ε we get

$$\begin{aligned} V_q(x(t)) \leq \beta(V_q(t_i), t) + \gamma_1(\|y\|_{[t_i,t]}) \\ +\gamma_2(\|u\|_{[t_i,t]}) \end{aligned} \tag{12}$$

for $t \in [t_i, t_{i+1})$, where $\beta \in \mathcal{KL}$, $\gamma_1, \gamma_2 \in \mathcal{K}$. For any pair of consecutive time intervals $[t_m, t_{m+1}), [t_n, t_{n+1})$ on which $\sigma = q$ we have

$$V_q(x(t_n)) \leq V_q(x(t_m)).$$

So from (12) we further get

$$\begin{aligned} V_q(x(t)) \leq \beta(V_q(t_{i_0^q}), t) + \gamma_1(\|y\|_{[t_i,t]}) \\ +\gamma_2(\|u\|_{[t_i,t]}), \end{aligned} \tag{13}$$

where $t_{i_0^q}$ denotes the time at which subsystem q becomes active for the first time.

In the infinite sequence $t_i, i = 0, 1, 2, \cdots$, we can always find an infinite subsequence $t_{i_j^q}, j = 0, 1, 2, \cdots$, such that $\sigma(t_{i_j^q}) = q$, $q \in Q$. Inequality (11) and the assumption 4 tell that $V_q(x(t_{i_j^q})), j = 0, 1, 2, \cdots$ is non-increasing in j. On the other hand, $V_q(x(t_{i_j^q})) \geq 0, \forall j \geq 0$. Therefore, there exists a constant $\eta_q \geq 0$ such that

$$\lim_{j\to\infty} V_q(x(t_{i_j^q})) = \eta_q.$$

Next we will show that $\eta_q = 0$. Since $\lim_{j\to\infty} V_q(x(t_{i_j^q}))$ exists, for any small $\varepsilon_1 \in (0, \varepsilon)$ there must exist an integer K such that for any $j > K$ the following inequality holds.

$$|V_q(x(t_{i_{j+1}^q})) - V_q(x(t_{i_j^q}))| < \varepsilon_1.$$

Since we have assumed that the dwell time at each subsystem is no less than a positive constant τ, there exist $t_a, t_b \in [t_{i_j^q}, t_{i_j^q+1}), t_a < t_b$, such that $\sigma(t) = q$ for all $t \in [t_a, t_b]$. Then we have

$$\begin{aligned}|V_q(x(t_b)) - V_q(x(t_a))| &\le |V_q(x(t_{i_j^q+1})) - V_q(x(t_{i_j^q}))| \\ &\le |V_q(x(t_{i_{j+1}^q})) - V_q(x(t_{i_j^q}))| \\ &< \varepsilon_1.\end{aligned}$$

From the above equation and the passivity of subsystem q we get

$$\begin{aligned}-\varepsilon_1 &< V_q(x(t_b)) - V_q(x(t_a)) \\ &\le -\int_{t_a}^{t_b} y^T(s)\phi_q(y(s))\mathrm{d}s \le 0.\end{aligned}$$

So we conclude that $y \to 0$ and hence $u_q \to 0$ according the property of the function $\phi_q(y)$ when $t \to \infty$. With this conclusion let us be back to Eq. (13). Since γ_1, γ_2 are $\mathcal{K}$ functions and β is a $\mathcal{KL}$ function, we get from (13)

$$V_q(x(t)) \to 0$$

when $t \to \infty$. This proves the asymptotic stability of the invariant set V_0 according to the definition of asymptotic stability of a set (see, e.g., Bacciotti, Mazzi and Sabatini (1996)).

(2) The proof of the global asymptotic stability of V_0 immediately follows from the proof of part (1). Indeed, by assumption, the set V_ε is compact for any $\varepsilon > 0$ and all the subsystems of (2) are IOVS and hence all the arguments in part (1) hold with any initial state $x_0 \in X$. Q.E.D.

Remark 3.2. If $V_q(x)$'s are all positively definite, V_0 contains the only state $x = 0$. In that case, Theorem 3.1 gives a result for the equilibrium stabilization problem of the switched system.

4. RELATIONSHIP BETWEEN IOVS AND IOSS

Sontag and Wang introduced the following notion to characterize the IOSS property of system (1).

Definition 4.1 (Sontag and Wang, 1997). An IOSS-Lypunov function for system (1) is any smooth function V with the following properties:

(1) There exist $\mathcal{K}_\infty$-functions α_1 and α_2 such that

$$\alpha_1(|x|) \le V(x) \le \alpha_2(|x|), \ \forall x \in X. \quad (14)$$

(2) There exist $\mathcal{K}_\infty$-functions α_3, σ_1 and σ_2 such that

$$\begin{aligned}&\frac{\partial V(x)}{\partial x}[f(x) + g(x)u(t)] \le \\ &\quad -\alpha_3(|x|) + \sigma_1(|u|) + \sigma_2(|y|), \ \forall x \in (15)\end{aligned}$$

By Theorem 2.4 of Krichman, Sontag, and Wang (2001), system (1) is IOSS if and only if it admits an IOSS-Lyapunov function. Using the notion of IOSS-Lyapunov function we can establish the following theorem which gives a sufficient condition of IOVS and reveals the relationship between IOVS and IOSS.

Theorem 4.2. *(1) If $V(x)$ is an IOSS-Lyapunov function of system (1), then system (1) is input/output-to-V stable; (2) If system (1) is input/output-to-V stable, and there exist $\mathcal{K}_\infty$ functions α_1, α_2 such that $\alpha_1(|x|) \le V(x) \le \alpha_2(|x|)$, $\forall x \in X$, then the system is inpit/output-to-state stable.*

Proof. (1) If $V(x)$ is an IOSS-Lyapunov function of system (1), then, by Theorem 2.4 in Krichman, Sontag, and Wang (2001), system (1) is IOSS, i.e., there exist $\beta \in \mathcal{KL}$ and $\gamma_1, \gamma_2 \in \mathcal{K}$ such that

$$\begin{aligned}|x(t, x_0, u)| &\le \beta(|x_0|, t) + \gamma_1(\|y\|_{[0,t]}) \\ &\quad + \gamma_2(\|u\|_{[0,t]}), \ \forall x_0 \in X, \ \forall u \in U.\end{aligned}$$

By the definition of the IOSS-Lyapunov function we have

$$\begin{aligned}V(x) &\le \alpha_2(|x|) \\ &\le \alpha_2(\beta(|x_0|, t) + \gamma_1(\|y\|_{[0,t]}) + \gamma_2(\|u\|_{[0,t]})) \\ &\le \alpha_2(3\max\{\beta(|x_0|, t), \gamma_1(\|y\|_{[0,t]}), \\ &\qquad \gamma_2(\|u\|_{[0,t]})\}) \\ &\le \alpha_2(3\beta(|x_0|, t)) + \alpha_2(3\gamma_1(\|y\|_{[0,t]})) \\ &\quad + \alpha_2(3\gamma_2(\|u\|_{[0,t]})).\end{aligned}$$

Now, from Proposition 7 of Sontag (1998) we know that there exist $\mathcal{K}_\infty$ functions θ_1 and θ_2 such that

$$\begin{aligned}V(x) &\le \alpha_2(3\theta_1(\theta_2(|x_0|)e^{-t})) \\ &\quad + \alpha_2(3\gamma_1(\|y\|_{[0,t]})) + \alpha_2(3\gamma_2(\|u\|_{[0,t]})) \\ &\le \alpha_2(3\theta_1(\theta_2(\alpha_1^{-1}(V(x_0)))e^{-t}))\end{aligned}$$

$$+\alpha_2(3\gamma_1(\|y\|_{[0,t]}))+\alpha_2(3\gamma_2(\|u\|_{[0,t]}))$$
$$=\tilde{\beta}_1(V(x_0),t)+\tilde{\gamma}_1(\|y\|_{[0,t]})+\tilde{\gamma}_2(\|u\|_{[0,t]})$$

where

$$\tilde{\beta}_1(V(x_0),t)=\alpha_2(3\theta_1(\theta_2(\alpha_1^{-1}(V(x_0)))e^{-t})),$$
$$\tilde{\gamma}_1(\|y\|_{[0,t]})=\alpha_2(3\gamma_1(\|y\|_{[0,t]})),$$
$$\tilde{\gamma}_2(\|u\|_{[0,t]})=\alpha_2(3\gamma_2(\|u\|_{[0,t]})).$$

It is easy to see that $\tilde{\beta}_1(\cdot,\cdot)\in\mathcal{KL}$ and $\tilde{\gamma}_1,\tilde{\gamma}_2\in\mathcal{K}$. Then, by Definition 2.3, we can conclude that system (1) is input/output-to-V stable.

(2) Suppose that system (1) is input/output-to-V stable and there exist $\mathcal{K}_\infty$ functions α_1,α_2 such that $\alpha_1(|x|)\le V(x)\le\alpha_2(|x|)$, $\forall x\in X$. Then, by Proposition 7 of Sontag (1998) we know that there exist $\theta_1,\theta_2\in\mathcal{K}_\infty$ such that

$$\begin{aligned}\alpha_1(|x|)&\le V(x)\\&\le\beta(V(x_0),t)+\gamma_1(\|y\|_{[0,t]})+\gamma_2(\|u\|_{[0,t]})\\&\le\theta_1(\theta_2(V(x_0))e^{-t})+\gamma_1(\|y\|_{[0,t]})\\&\quad+\gamma_2(\|u\|_{[0,t]}),\quad\forall x_0\in X,\ \forall t>0.\end{aligned}$$

Therefore,

$$\begin{aligned}|x|&\le\alpha_1^{-1}(\theta_1(\theta_2(V(x_0))e^{-t})+\gamma_1(\|y\|_{[0,t]})\\&\quad+\gamma_2(\|u\|_{[0,t]}))\\&\le\alpha_1^{-1}(3\max\{\theta_1(\theta_2(V(x_0))e^{-t}),\\&\quad\gamma_1(\|y\|_{[0,t]}),\gamma_2(\|u\|_{[0,t]})\})\\&\le\alpha_1^{-1}(3\theta_1(\theta_2(V(x_0))e^{-t}))+\alpha_1^{-1}(3\gamma_1(\|y\|_{[0,t]}))\\&\quad+\alpha_1^{-1}(3\gamma_2(\|u\|_{[0,t]}))\\&\le\alpha_1^{-1}(3\theta_1(\theta_2(\alpha_2(x_0))e^{-t}))+\alpha_1^{-1}(3\gamma_1(\|y\|_{[0,t]}))\\&\quad+\alpha_1^{-1}(3\gamma_2(\|u\|_{[0,t]}))\\&=\tilde{\beta}_2(|x_0|,t)+\tilde{\psi}_1(\|y\|_{[0,t]})+\tilde{\psi}_2(\|u\|_{[0,t]})\end{aligned}$$

where

$$\tilde{\beta}_2(|x_0|,t)=\alpha_1^{-1}(3\theta_1(\theta_2(\alpha_2(x_0))e^{-t})),$$
$$\tilde{\psi}_1(\|y\|_{[0,t]})=\alpha_1^{-1}(3\gamma_1(\|y\|_{[0,t]})),$$
$$\tilde{\psi}_2(\|u\|_{[0,t]})=\alpha_1^{-1}(3\gamma_2(\|u\|_{[0,t]})).$$

It is easy to see that $\tilde{\beta}_2(\cdot,\cdot)\in\mathcal{KL}$ and $\tilde{\psi}_1,\tilde{\psi}_2\in\mathcal{K}$. Then we can conclude that system (1) is input/output-to-state stable. Q.E.D.

5. CONCLUSION

In this paper, we have proposed and solved the problem of the global (local) stabilization of invariant sets of a class of switched systems, every subsystem of which is a passive and nonlinear system. The main idea of our approach is to extend the passification scheme for equilibrium point stabilization introduced by Byrnes, Isidori and Willems (1991), to the problem of invariant set stabilization for switched nonlinear systems. To this end, we have extended the notion of the input/output-to-state stability introduced by Sontag and Wang (1997), to the case of the input/output-to-$V(x)$ stability, which implies detectability of the zero-value set of a storage function. Based on this notion and the notion of passivity of nonlinear systems, we have proposed and proved sufficient conditions under which an invariant state set of the switched nonlinear system can be stabilized by output feedback. We have also shown that a nonlinear system is IOVS if $V(x)$ is an IOSS-Lyapunov function of the system. The relationship between IOVS and IOSS is also characterized by the concept of IOSS-Lyapunov function.

REFERENCES

Bacciotti, A., L. Mazzi and M. Sabatini (1996). Generation of stable limit cycles in controllable linear systems. *Systems & Control Letters*, **28**, 43-48.

Byrnes, C. I., A. Isidori and J. C. Willems (1991). Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE Trans. Automatic Control*, **36**, 1228-1240.

Jurdjevic, V. and J. P. Quinn (1978). Controllability and stability. *J. Differential Equations*, **28**, 381-389.

Krichman, M., E. D. Sontag and Y. Wang (2001). Input-output-to-state stability. *SIAM J. Control Optim.*, **39**, 1874-1928.

Kalouptsidis, N. and J. Tsinias (1984). Stability improvement of nonlinear systems by feedback. *IEEE Trans. Automatic Control*, **29**, 364-367.

Lee, K. K. and A. Arapostathis (1988). Remarks on smooth feedback stabilization of nonlinear system. *Systems & Control Letters*, **10**, 41-44.

Lin, W. (1995). Feedback stabilization of general nonlinear control systems: A passive system approach. *Systems & Control Letters*, **25**, 41-52.

Shiriaev, A. S. (2000). The notion of V-detectability and stabilization of invariant sets of nonlinear systems. *Systems & Control Letters*, **39**, 327-338.

Sontag, E. D. (1998). Comments on integral variants of ISS. *Systems & Control Letters*, **34**, 93-100.

Sontag, E. D. and Y. Wang (1997). Output-to-state stability and detectability of nonlinear systems. *Systems & Control Letters*, **29**, 279-290.

Willems, J. C. (1972). Dissipative dynamics systems. Part 1, general theory. *Arch. Rat. Mech. Anal.*, **45**, 321-351.