ROBUST STATE OBSERVER DESIGN BASED ON REGIONAL POLE ASSIGNMENT

Man Liu [*,**,1] **Yuan-Wei Jing** [*] **Si-Ying Zhang** [*]

[*] *School of Information Science and Engineering, Northeastern University, Shenyang, 110004, P.R. China*
[**] *Dept. of Mathematics, Dalian Nationalities University, Dalian, 116600, P.R. China.*

Abstract: The design of state observer for a linear discrete or continuous time uncertain system based on circularly regional pole assignment is studied. Taking use of algebraic Riccati equations and linear matrix inequalities, necessary and sufficient conditions for the existence of the state observer based on circularly regional pole assignment is proposed and the formulation of the gain matrix of the state observer is given. The state observer given by us has good robustness, which is illustrated by examples.

Keywords: Linear system, Regional pole assignment, State observer, Uncertainty, Robustness

1. INTRODUCTION

It is well known that the transient response of a linear system is related to the location of its poles, and therefore, pole assignment is one of the most important research problems in analysis and design of linear systems. It is difficult to assign the poles at exact location because of the parameter uncertainty caused by error in identification, aging of parts of an apparatus, linear approximation in engineering practice. On the other hand, it is enough to assign the poles in a specified region for some practical design specifications. Hence, in recent 20 years, more and more researchers focus on the regional pole assignment and much more achievements have been got, see Chilali & Gahinet (1996), Farsangi, Song & Tan(2003), Garcia & Bernussou(1995), Saeki (2001), Scherer, Gahinet & Chilali (1997), Xia & Han(2004) and the references therein.

However, few results on state observer design based on regional pole assignment are available in the literature. Li, *et al*(1997) gave a necessary and sufficient condition for the existence of the state observer based on regional pole assignment and proposed a observer design method. But only nominal continuous linear systems were considered and the results were given by general inverse of matrices, which makes calculation difficult.

In this paper, we consider the problem to design the state observer based on circularly regional pole assignment for a class of continuous and discrete linear systems with uncertainties. A necessary and sufficient condition for the existence of the state observer based on regional pole assignment is proposed and the formulation of the gain matrix of the state observer is given. Firstly, for convenience of proof, our results are expressed in terms of algebraic Riccati equations. Then, the results are transformed into linear matrix inequalities. This form of results is suitable for further study of

[1] This work was supported by the National Science Foundation of China under Grant 60274009 and Specialized Research Fund for the Doctoral Program of Higher Education under Grant 20020145007.

many other problems because many system performances can be described by linear matrix inequalities. Finally, examples are given to illustrate the methods.

Throughout this paper, the superscript T means transpose for real matrices. The expression "$(\cdot) > 0(< 0)$"means that matrix $(\cdot)$ is positive (negative) definite and I denotes the identity matrix of appropriate dimension. " $*$ " denotes the transposed elements in the symmetric position.

2. PROBLEM FORMULATION

Let us consider a continuous or discrete time system described by

$$\delta\left[x(t)\right] = (A + \triangle A)x(t) + Bu(t), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $A \in \mathscr{R}^{n\times n}$, $B \in \mathscr{R}^{n\times m}$, $C \in \mathscr{R}^{p\times n}$, $u(t) \in \mathscr{R}^{m}$ is the input, $y(t) \in \mathscr{R}^{p}$ is the output and $x(t) \in \mathscr{R}^{n}$ is the state. δ is the derivation in the continuous time case [i.e., $\delta\left[x(t)\right] = \dot{x}(t)$] and the delay operator for the discrete time one [i.e., $\delta\left[x(t)\right] = x(t+1)$]. $\triangle A$ is the uncertainty of norm bounded type written as

$$\triangle A = DFE, \tag{3}$$

where $D \in \mathscr{R}^{n\times r}$, $E \in \mathscr{R}^{l\times n}$ define the structure of the uncertainty and the parameter uncertainty F belong to the set

$$\aleph = \{F \in \mathscr{R}^{r\times l} : F^T F \le I\}. \tag{4}$$

Let the observing system be

$$\delta\left[\hat{x}(t)\right] = (A + \triangle A)\hat{x}(t) + Bu(t), \tag{5}$$

$$\hat{y}(t) = C\hat{x}(t). \tag{6}$$

Taking use of output feedback, we construct the state observer as follows:

$$\delta[\hat{x}(t)] = (A+\triangle A)\hat{x}(t) + Bu(t) + H(\hat{y}(t) - y(t)). \tag{7}$$

Substituting (6) into (7), we have that

$$\delta[\hat{x}(t)] = (A+\triangle A + HC)\hat{x}(t) + Bu(t) - Hy(t), \tag{8}$$

where $(A + \Delta A + HC)$ is the observer system matrix , H is the observer gain matrix.

Then the state estimation error system is

$$\delta\left[e(t)\right] = (A + \triangle A + HC)e(t), \tag{9}$$

where $e(t) = x(t) - \hat{x}(t)$.

The damping speed of the error between the estimated values $\hat{x}$ and the real values x depends on the poles location of the observer system matrix $(A + \Delta A + HC)$, that is, the damping speed depends on the structure of the observer gain matrix H. When constructing H, we have to consider some trade-off among damping speed, noise filtered, anti-windup and so on. When the poles location is assigned , the classical exact pole assignment method limits the damping speed, deteriorates robustness, and is hard to satisfy multi-performances in some practical applications. Thus, in order to have more degrees of freedom when designing the state observer, we consider the design of the state observer based on circularly regional pole assignment.

Specifically, the considered problem is to construct the observer gain matrix H with some degrees of freedom, such that for all $F \in \aleph$, the poles of the observer system matrix $(A + \Delta A + HC)$, which is also the system matrix of the state estimation error equation, are assigned in a specified disk $D(\alpha, r)$ with center $\alpha + j0$ and radius r, which is in the left half of the complex plane for a continuous system or in the unit disk with the center at the origin for a discrete system.

For the sake of simplicity, let us introduce following notations

$$A_\alpha = (A - \alpha I)/r, \quad C_r = C/r, \quad D_r = D/\sqrt{r},$$
$$E_r = E/\sqrt{r}, \quad A_{\alpha e} = (A + HC - \alpha I)/r,$$
$$\Delta A_{\alpha e} = \Delta A/r.$$

The following lemma is necessary.

Lemma 1 (Furuta Theorem) Let $A \in \mathscr{R}^{n\times n}$ be a given matrix. The eigenvalues of A belong to $D(\alpha, r)$ if and only if there exists a positive definite symmetric matrix $P \in \mathscr{R}^{n\times n}$ such that

$$A_\alpha^T P A_\alpha - P < 0. \tag{10}$$

Lemma 1 was given by Furuta *et al* in 1987. It is the foundation stone for studying circularly reginal pole assignment. Therefore, we refer to it as Furuta Theorem.

Now we can develop the following definition by Lemma 1.

Definition 1 The state estimation error system (9) is quadratically D stabilizable under an observer gain matrix H if and only if there exists a positive definite symmetric matrix $P \in \mathscr{R}^{n\times n}$ such that

$$(A_{\alpha e} + \Delta A_{\alpha e})^T P (A_{\alpha e} + \Delta A_{\alpha e}) - P < 0 \tag{11}$$

for all $F \in \aleph$.

3. MAIN RESULTS

We are now in a position give the main results of this paper.

Theorem 1 Let Q and R be two positive definite symmetric matrices of appropriate dimensions. The state estimation error system (9) is quadratically D stabilizable under an observer gain matrix H if and only if there exist $\varepsilon > 0$ and a positive definite symmetric matrix P satisfying the following discrete Riccati equation:

$$A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}A_\alpha^T \\ -P^{-1}+\varepsilon^{-1}D_rD_r^T+Q=0 \quad (12)$$

with

$$\varepsilon^{-1}I-E_rP^{-1}E_r^T>0. \quad (13)$$

Then the observer gain matrix H is given by

$$H=-A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}C_r^TR^{-1}. \quad (14)$$

Before proving Theorem 1, we give some lemmas.

Lemma 2 (Petersen, 1987) Let S, Y, and Z be given by $k\times k$ symmetric matrices such that $S\geq 0$, $Y<0$, and $Z\geq 0$. Furthermore, assume that $\left(\eta^TY\eta\right)^2-4\left(\eta^TS\eta\right)\left(\eta^TZ\eta\right)>0$ for all nonzero $\eta\in\mathscr{R}^k$. Then there exists a constant $\lambda>0$ such that $\lambda^2S+\lambda Y+Z<0$.

Lemma 3(Finsler Lemma) Let X be $k\times k$ symmetric matrix and $B\in\mathscr{R}^{k\times m}$ such that $\eta^TX\eta<0$ for all $\eta\neq 0$ satisfying $B^T\eta=0$. Then there exists a positive definite symmetric matrix $Y\in\mathscr{R}^{k\times k}$ such that $X-BYB^T<0$.

Lemma 4 (Petersen, 1987) Given $x\in\mathscr{R}^n, y\in\mathscr{R}^n$, then

$$Max\{(x^TDFEy)^2|F^TF\leq I\} \\ =x^TDD^Txy^TE^TEy. \quad (15)$$

Lemma 5 Let D, A, E be matrices of appropriate dimensions. Let $F\in\aleph$ and P be a positive definite symmetric matrix satisfying

$$\varepsilon^{-1}I-EP^{-1}E^T>0, \quad \varepsilon>0.$$

Then,

$$\begin{aligned} &AP^{-1}E^TF^TD^T+DFEP^{-1}A^T \\ &+DFEP^{-1}E^TF^TD^T \\ \leq &AP^{-1}E^T\left(\varepsilon^{-1}I-EP^{-1}E^T\right)^{-1}EP^{-1}A^T \\ &+\varepsilon^{-1}DD^T. \end{aligned}$$

Proof: Choosing

$$\begin{aligned} Y=&(\epsilon^{-1}I-EP^{-1}E^T)^{-\frac{1}{2}}EP^{-1}A^T \\ &-(\epsilon^{-1}I-EP^{-1}E^T)^{\frac{1}{2}}F^TD^T, \end{aligned}$$

and using $Y^TY\geq 0$, we can complete the proof.

PROOF OF THEOREM 1

*Necessity:*Suppose the system (9) is quadratically D stabilizable under an observer gain matrix H. Then there exists a positive definite symmetric matrix $P\in\mathscr{R}^{n\times n}$ such that the inequality (11) is true for all $F\in\aleph$. It is equivalent to that there exists a positive definite symmetric matrix $P\in\mathscr{R}^{n\times n}$ such that for all $\zeta\in\mathscr{R}^{2n}$ and $F\in\aleph$:

$$\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha+HC_r+D_rFE_r \\ * & -P\end{bmatrix}\zeta<0,$$

which can be written

$$\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P\end{bmatrix}\zeta+\zeta^T\begin{bmatrix}0 & HC_r \\ * & 0\end{bmatrix}\zeta \\ +\zeta^T\begin{bmatrix}0 & D_rFE_r \\ * & 0\end{bmatrix}\zeta<0.$$

And then for all $\zeta\neq 0$ such that $[0,C_r]\,\zeta=0$, we have

$$\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P\end{bmatrix}\zeta+\zeta^T\begin{bmatrix}0 & D_rFE_r \\ * & 0\end{bmatrix}\zeta<0.$$

This implies that

$$\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P\end{bmatrix}\zeta \\ <-\left\{\mathrm{Max}\zeta^T\begin{bmatrix}0 & D_rFE_r \\ * & 0\end{bmatrix}\zeta : F^TF\leq I\right\}\leq 0$$

for all $\zeta\neq 0$ such that $[0,C_r]\,\zeta=0$. And hence

$$\left(\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P_1\end{bmatrix}\zeta\right)^2 \\ >\left\{\mathrm{Max}\zeta^T\begin{bmatrix}0 & D_rFE_r \\ * & 0\end{bmatrix}\zeta : F^TF\leq I\right\}^2$$

for all $\zeta\neq 0$ such that $[0,C_r]\,\zeta=0$. By Lemma 4, we obtain

$$\left(\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P\end{bmatrix}\zeta\right)^2 \\ >4\zeta^T\begin{bmatrix}D_rD_r^T & 0 \\ 0 & 0\end{bmatrix}\zeta\zeta^T\begin{bmatrix}0 & 0 \\ 0 & E_r^TE_r\end{bmatrix}\zeta$$

for all $\zeta\neq 0$ such that $[0,C_r]\,\zeta=0$. It follows by Lemma 2 that there exists a constant $\varepsilon>0$, such that

$$\frac{1}{\varepsilon}\zeta^T\begin{bmatrix}D_rD_r^T & 0 \\ 0 & 0\end{bmatrix}\zeta+\zeta^T\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P\end{bmatrix}\zeta \\ +\varepsilon\zeta^T\begin{bmatrix}0 & 0 \\ 0 & E_r^TE_r\end{bmatrix}\zeta<0$$

for all $\zeta\neq 0$ such that $[0,C_r]\,\zeta=0$. By Lemma 3, there exists a positive definite symmetric matrix $Y=R^{-1}$, such that

$$\frac{1}{\varepsilon}\begin{bmatrix}D_rD_r^T & 0 \\ 0 & 0\end{bmatrix}+\begin{bmatrix}-P^{-1} & A_\alpha \\ * & -P\end{bmatrix} \\ +\varepsilon\begin{bmatrix}0 & 0 \\ 0 & E_r^TE_r\end{bmatrix}-\begin{bmatrix}0 \\ C_r^T\end{bmatrix}R^{-1}\begin{bmatrix}0 & C_r\end{bmatrix}<0.$$

That is

$$\begin{bmatrix}-P^{-1}+\dfrac{1}{\varepsilon}D_rD_r^T & A_\alpha \\ * & -P+\varepsilon E_r^TE_r-C_r^TR^{-1}C_r\end{bmatrix}<0.$$

By Schur Complement Lemma, the previous inequality implies

$$A_\alpha\left[P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r\right]^{-1}A_\alpha^T \\ -P^{-1}+\varepsilon^{-1}D_rD_r^T<0.$$

And there exists a positive definite symmetric matrix Q such that

$$A_\alpha\left[P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r\right]^{-1}A_\alpha^T \\ -P^{-1}+\varepsilon^{-1}D_rD_r^T+Q=0.$$

It remains to show the inequality (13). Following the same development as above, we obtain

$$\frac{1}{\varepsilon}\zeta^T\begin{bmatrix} D_rD_r^T & 0 \\ 0 & 0\end{bmatrix}\zeta+\zeta^T\begin{bmatrix} -P^{-1} & A_\alpha \\ * & -P\end{bmatrix}\zeta + \varepsilon\zeta^T\begin{bmatrix} 0 & 0 \\ 0 & E_r^TE_r\end{bmatrix}\zeta<0$$

for all $\zeta \neq 0$ such that $\left[(HC_r)^T, 0\right]\zeta = 0$. Now from Lemma 3, there exists a positive definite symmetric matrix V, such that

$$\begin{bmatrix} -P^{-1}+\frac{1}{\varepsilon}D_rD_r^T & A_\alpha \\ * & -P+\varepsilon E_r^TE_r\end{bmatrix} - \begin{bmatrix} C_r^TH^TVHC_r & 0 \\ 0 & 0\end{bmatrix}<0,$$

which, by Schur Complement Lemma, implies that

$$P-\varepsilon E_r^TE_r>0,$$

or equivalently

$$\varepsilon^{-1}I-E_rP^{-1}E_r^T>0.$$

Sufficiency: Suppose that condition of Theorem 1 is satisfied and let

$$\mathcal{L}=(A_{\alpha e}+\Delta A_{\alpha e})P^{-1}(A_{\alpha e}+\Delta A_{\alpha e})^T-P^{-1}. \tag{16}$$

Using Lemma 5, we have

$$\begin{aligned}\mathcal{L} =& A_{\alpha e}P^{-1}A_{\alpha e}^T-P^{-1}+A_{\alpha e}P^{-1}E_r^TF^TD_r^T \\ &+D_rFE_rP^{-1}A_{\alpha e})^T+D_rFE_rP^{-1}E_r^TF^TD_r^T \\ \leq& A_{\alpha e}P^{-1}A_{\alpha e}^T-P^{-1}+A_{\alpha e}P^{-1}E_r^T \\ &\cdot(\varepsilon^{-1}I-E_rP^{-1}E_r^T)^{-1}E_rP^{-1}A_{\alpha e}^T+\varepsilon^{-1}D_rD_r^T.\end{aligned}$$

Using the matrix inversion lemma, we have

$$\mathcal{L}\leq A_{\alpha e}\left(P-\varepsilon E_r^TE_r\right)^{-1}A_{\alpha e}^T-P^{-1}+\varepsilon^{-1}D_rD_r^T. \tag{17}$$

By the conditions of Theorem 1,

$$P^{-1}=A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}A_\alpha^T +\varepsilon^{-1}D_rD_r^T+Q.$$

Replacing in (17) leads to

$$\begin{aligned}\mathcal{L}\leq& A_{\alpha e}\left(P-\varepsilon E_r^TE_r\right)^{-1}A_{\alpha e}^T \\ &-A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}A_\alpha^T-Q.\end{aligned} \tag{18}$$

Now denoting $N=A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}$ and noticing that

$$\begin{aligned}&A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}A_\alpha^T \\ &=NC_r^TR^{-1}C_rN^T+N(P-\varepsilon E_r^TE_r)N^T,\end{aligned}$$

(18) is now given by

$$\begin{aligned}\mathcal{L}\leq& A_{\alpha e}\left(P-\varepsilon E_r^TE_r\right)^{-1}A_{\alpha e}^T-Q \\ &-NC_r^TR^{-1}C_rN^T-N(P-\varepsilon E_r^TE_r)N^T.\end{aligned}$$

A simple calculation shows that with H given by (14),

$$A_{ae}\left(P-\varepsilon E_r^TE_r\right)^{-1}A_{ae}^T-N(P-\varepsilon E_r^TE_r)N^T=0.$$

and then

$$\mathcal{L}\leq-[Q+HRH^T]\leq-Q<0.$$

Then, noticing that $\mathcal{L}<0$ is equivalent to (11) by Schur Complement Lemma, the system (9) is quadratically D stabilizable and the proof is completed.

For convenience of finding the gain matrix H of the state observer, we express the results in Theorem 1 in terms of linear matrix inequalities.

Theorem 2 Let R be a positive definite symmetric matrix of appropriate dimension. The state estimation error system (9) is quadratically D stabilizable under an observer gain matrix H if and only if there exist $\varepsilon>0$ and a positive definite symmetric matrix P satisfying the following linear matrix inequalities:

$$\begin{bmatrix} -P & PD_r & PA_\alpha \\ * & -\varepsilon I & 0 \\ * & * & -P-C_r^TR^{-1}C_r+\varepsilon E_r^TE_r\end{bmatrix}<0, \tag{19}$$

$$\begin{bmatrix} -\varepsilon I & \varepsilon E_r \\ * & -P\end{bmatrix}<0. \tag{20}$$

Then the observer gain matrix H is given by

$$H=-A_\alpha(P+C_r^TR^{-1}C_r-\varepsilon E_r^TE_r)^{-1}C_r^TR^{-1}.$$

Remark 1 Our result is meaningful in practice since the state observer enables to reconstruct the system state by taking use of the input and the output. So, it is necessary for us to present the result precisely although the design idea is similar to the one developed by Garcia and Bernussou (1995).

Remark 2 The results expressed in terms of linear matrix inequalities in Theorem 2 is suitable for further study of system analysis and design by synthesis because many system performances can be described by linear matrix inequalities easily.

Remark 3 We may take the positive definite symmetric matrix R for a variable in (19). The larger $\|R\|$ is, the easier the feasible solutions are to be found. Thus, the observer gain matrix H has another degree of freedom which might be available for optimizing other control performances.

If the system (1) (2) is well known(i.e., $D=0$ and $E=0$), that is

$$\delta\left[x(t)\right]=Ax(t)+Bu(t), \tag{21}$$

$$y(t)=Cx(t). \tag{22}$$

Then the state estimation error system is

$$\delta\left[e(t)\right]=(A+HC)e(t). \tag{23}$$

In that nominal system case, we have the following corollary.

Corollary 1 Let R be a positive definite symmetric matrix of appropriate dimension. The state

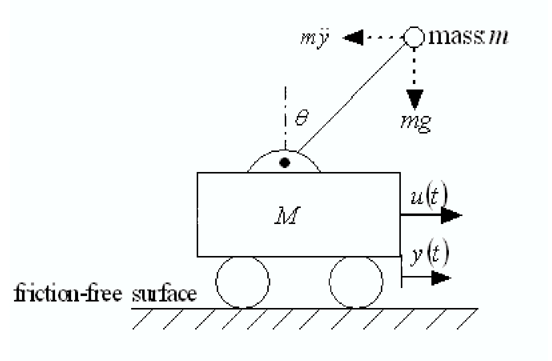

Fig. 1. Inverted Pendulum System

estimation error system (23) is D stabilizable under an observer gain matrix H (i.e., all poles of the error system (23) lie in $D(\alpha, r)$) if and only if there exists a positive definite symmetric matrix P satisfying the following linear matrix inequality:

$$\begin{bmatrix} -P & PA_\alpha \\ * & -P - C_r^T R^{-1} C_R \end{bmatrix} < 0. \tag{24}$$

Then the observer gain matrix H is given by

$$H = -A_\alpha (P + C_r^T R^{-1} C_r)^{-1} C_r^T R^{-1}.$$

Remark 4 This result in Corollary 1 is simpler than the one in Li *et al* (1997). So, judging wether the error system (23) is D stable and finding the gain matrix H by taking use of the result in Corollary 1 is easier than doing it by taking use of the result in Li *et al* (1997).

4. EXAMPLES

The examples in this section is partly borrowed from Yu (2002).

Concider a Inverted pendulum system(see Fig 1). Choose $x = \begin{bmatrix} \theta & \dot{\theta} & y & \dot{y} \end{bmatrix}^T$ be the state vector of the system, where θ is the offset angle of the pole, y is the position of the cart. u is the driving force on the cart. $p \in \mathscr{R}^2$ is the measurable output.

Continuous Case A continuous linearized model of the inverted pendulum:

$$\dot{x}(t) = (A + DFE)x(t) + Bu(t), \tag{25}$$

$$y(t) = Cx(t), \tag{26}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 10.18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.980 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -0.2 \\ 0 \\ 0.2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The uncertainty structure is described by

$$D = \begin{bmatrix} 0.1 & 1 & 0 & -0.1 \end{bmatrix}^T, \ E = \begin{bmatrix} 0.1 & -0.33 & 0.41 & 0.7 \end{bmatrix},$$

$$F = f \in \mathscr{R}, \qquad |f| \le 1.$$

We want to place the poles in a disk centered at $\alpha = -3$ and with a radius $r = 2$. The matrix R is chosen to be $R = I$. The feasible solutions of the corresponding linear matrix inequalities are

$$P = \begin{bmatrix} 1.6370 & -0.5068 & 0.0194 & -0.0111 \\ -0.5068 & 0.1592 & -0.1031 & 0.0090 \\ 0.0194 & -0.0131 & 0.1738 & -0.1061 \\ -0.0111 & 0.0090 & -0.1061 & 0.0831 \end{bmatrix},$$

$$\varepsilon = 0.0253.$$

The observer gain martix is then given by

$$H = \begin{bmatrix} -5.9454 & 0.2166 \\ -18.9650 & 0.6888 \\ 0.4081 & -3.9415 \\ 1.8638 & -3.7238 \end{bmatrix}.$$

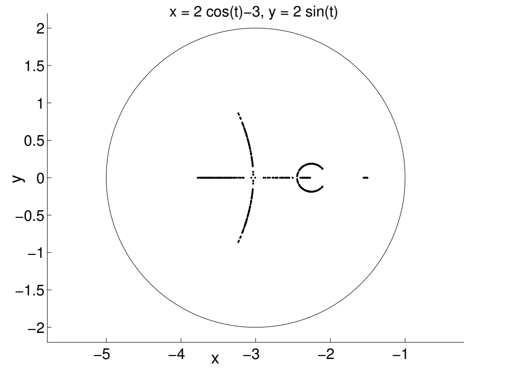

Fig. 2. Modes of Observer

Fig.2 shows the modes of the observer for 200 values of f with $-1 \le f \le 1$.

Discrete Case A discrete linearized model of the Inverted pendulum:

$$x(t+1) = (A + DFE)x(t) + Bu(t), \tag{27}$$

$$y(t) = Cx(t), \tag{28}$$

$$A = \begin{bmatrix} 1.0544 & 0.1018 & 0 & 0 \\ 1.0975 & 1.0544 & 0 & 0 \\ -0.0050 & -0.0002 & 1 & 0.1 \\ -0.0998 & -0.0049 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -0.05 \\ -1 \\ 0.05 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The uncertainty structure is described by

$$D = \begin{bmatrix} -0.08 & -0.8 & 0.08 & 0.8 \end{bmatrix}^T, \ E = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \end{bmatrix}$$

$$F = f \in \mathscr{R}, \qquad |f| \le 1.$$

We try to place the poles in the circle centered at $\alpha = 0.5$ with a radius $r = 0.5$. The matrix R is chosen to be $R = I$. The feasible solutions of the corresponding linear matrix inequalities are

$$P = 10^3 \begin{bmatrix} 20.6508 & -0.6226 & 0.1240 & -0.3219 \\ -0.6226 & 0.3576 & -0.1103 & 0.2965 \\ 0.1240 & -0.1103 & 0.0429 & -0.1018 \\ -0.3219 & 0.2965 & -0.1018 & 0.2731 \end{bmatrix},$$

$$\varepsilon = 14.1485.$$

The observing gain martix is then given by

$$H = \begin{bmatrix} -0.0229 & 0.0012 \\ -0.0916 & 0.0064 \\ 0.0156 & -0.3122 \\ 0.0643 & -0.1194 \end{bmatrix}.$$

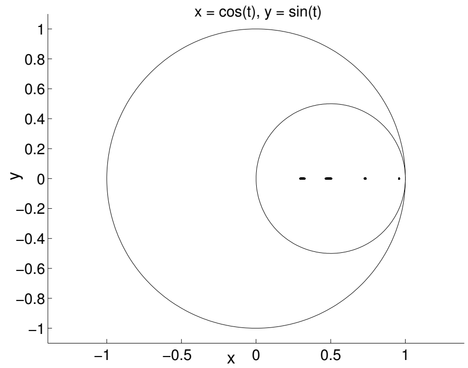

Fig. 3. Modes of Observer

Fig.3 shows the modes of the observer for 200 values of f with $-1 \leq f \leq 1$.

5. CONCLUSION

In this paper, we investigated the problem of the design of state observer for a linear discrete or continuous time uncertain system based on circularly regional pole assignment. The conditions derived are expressed by linear matrix inequalities leading to a simple procedure for the design of the observer gain matrix. And this form of results is suitable for further study for multi-objective control because many system performances can be described by linear matrix inequalities.

REFERENCES

Chilali, M., & Gahinet, P. (1996). H_∞ design with pole placement constraints: An LMI approach. *IEEE Transactions on automatic control*, 41(3): 358-367.

Farsangi, M. M., Song, Y. H., & Tan M.(2003). Multi-objective Design of damping controllers of devices via mixed H_2/H_∞ with regional pole placement. *Electrical Power & Energy Systems*, 25: 339-346.

Furuta, K. & Kim, S. B.(1987). Pole assignment in a specified disk. *IEEE Transactions on automatic control*,32(5): 423-427.

Garcia, G., & Bernussou, J.(1995). Pole assignment for uncertain systems in a specified disk by state feedback. *IEEE Transactions on Automatic Control*, 40(1), 184-190.

Li, L. W., Wang, Z. D., & Guo, Z. (1997). The algebraic parameterization of regional pole assignment for state observer. *Information and control*, 26(1): 28-32.

Petersen,R.(1987). A stabilization algorithm for a class of uncertain linear systems. *Systems and Control Letters*, 8(5): 351-357.

Saeki, M. (2001). On the pole location of a system designed by robust stability-degree assignment. *Systems & Control Letters*, 42(2): 353-361.

Scherer, C. W. Gahinet,P., & Chilali, M. (1997). Multi-objective output-feedback control via LMI optimization. *IEEE Transactions on automatic control*,42(7): 896-911.

Yu, L.(2002). *Robust Control: An LMI approach.* Beijing: Tsinghua University Press.

Xia,Y. Q. & Han, J. Q. (2004). Robut H_2 control with regional stability constraints. *Acta Automatic Sinica*, 30(3): 321-329.