$\mathcal{H}_2/\mathcal{H}_\infty$ CONTROL WITHOUT LYAPUNOV MATRIX OR FREQUENCY WEIGHTS

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Abstract: The multiobjective control problem is difficult and remains mostly open. Although the Linear Matrix Inequalities (LMI) provide a powerful analysis technique, a trade-off must be done in the relaxation to restore LMI's properties in control synthesis. The Youla parameterization is one of the skill to restore LMI's properties while generating all possible compensators. In this paper, it is shown that Youla parameterization can lead to $\mathcal{H}_2$ optimisation without introducing additional decision variables, and allows to perform frequency shaping (such as flexible modes attenuation) without having to choose frequency weights.

Keywords: multiobjective control, LMI, $\mathcal{H}_2$ norm, Youla parameterization.

1. INTRODUCTION

The aim of multiobjective control problems is to mix different criteria - which are mathematical translations of the manufacturer specifications - into the compensator synthesis. One common framework for multiobjective control is $\mathcal{H}_2/\mathcal{H}_\infty$ synthesis, however the problem formulation must be well chosen to obtain an efficient solution.

The Linear Matrix Inequalities (LMI) provide such an attractive formulation due to several reasons:

- the problem feasibility can be checked
- the optimal solution can be reached, because they lead to convex optimisation problems
- powerful numerical optimisation techniques can be used to solve these problems.

Although most of the analysis criteria can be expressed using LMI formulations, the convexity property is often lost when the synthesis problem of designing a controller is considered. Different works have been done to restore this property, as e.g. (Gahinet and Apkarian, 1994) for $\mathcal{H}_\infty$ control using the elimination lemma. In this paper, another approach is considered for plants with one control input. Thanks to the Youla parameterization, which defines a convex set describing all stabilizing controllers, the synthesis problem becomes convex. It will be shown that this parameterization leads to a new LMI formulation for $\mathcal{H}_2$ optimisation which avoids the introduction of additional variables, as involved in the standard formulation of (Boyd *et al.*, 1994).

As another interesting point, $\mathcal{H}_\infty$ constraints on specified frequency ranges can be handled without roll-off filters or frequency-dependent weights, like in standard $\mathcal{H}_\infty$ approaches. It allows to consider attenuation specifications of bending modes without acting on the control bandwidth. The proposed approach uses the μ-analysis formulation instead of using a finite frequency Kalman-Yakubovich-Popov (KYP) lemma as proposed in (Iwasaki *et al.*, 2000).

The paper is organized as follows: section 2 contains a brief presentation of Youla parameterization; the main results appear in sections 3, where a new LMI formulation of $\mathcal{H}_2$ synthesis is proposed, and 4, which explains how $\mathcal{H}_\infty$ constraints can be taken into account. An illustrative example is finally presented in section 5.

2. YOULA PARAMETERIZATION

Since the work of (Raggazini and Franklin, 1958), the Youla parameterization has often been used in multiobjective control problems (Hindi *et al.*, 1998; Scherer, 1999). Consider a continuous or discrete-time plant G with state space realization:

$$G: \begin{array}{c} \\ \\ z \\ y \end{array} \begin{array}{c} \begin{array}{cc} \quad\quad w & \quad u \end{array} \\ \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right) \end{array} \tag{1}$$

where z is the output to be controlled despite disturbance w, using control input u and measurement y. All stabilizing controllers are described by the Redheffer product $K = J * Q$ (see the interconnection structure of Figure 1), where the Youla parameter Q is any stable transfer function. System J depends both on G_{22} (the transfer between u and y) and an initial compensator K_0.

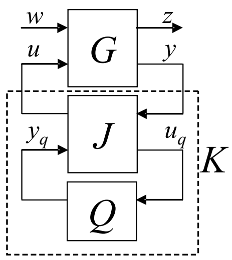

Fig. 1. Closed-loop structure using Youla parameterization

An initial compensator must therefore be known to use the Youla parameterization. In general, it is a static or low order controller. In the literature two kinds of representation for J have been proposed. The first one results from coprime factorizations of G_{22} and K_0 (Walker and Ridgely, 1995):

$$J = \begin{pmatrix} K_0 & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1}N \end{pmatrix} \tag{2}$$

with $K_0 = \tilde{V}_0^{-1}\tilde{U}_0 = U_0 V_0^{-1}$ and $G_{22} = NM^{-1}$.

The second representation uses the LQG form (3)- see the top of the next page - if K and G are of same order: K_c and K_f are respectively a state space and an observer gain. If K has an order greater than G, an initial Youla parameter Q_0 - with state space matrices A_{q_0}, B_{q_0}, C_{q_0}, D_{q_0} - has to be added (4). This form can not be used if the compensator has an order less than the plant.

The interconnection $G * J$ exhibits a transfer function identically equal to 0 between u_q and y_q (Figure 1). As an interesting result, the closed loop transfer matrix G_{zw} (between input w and output z) is affine in Q:

$$G_{zw} = H - UQV \tag{5}$$

where H, U and V are stable transfer functions, resulting from the interconnection $G * J$.

Using the most common formulations of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ constraints (Boyd *et al.*, 1994), such an affine dependence of the plant results in matrix inequalities being bi-affine in the decision variables. In the literature two kinds of methods are proposed to restore LMI constraints. By using a change of variables on the Lyapunov matrix (Scherer, 1999), LMI constraints with respect to the output matrices of G_{zw} can be obtained in most cases. However this technique can not be applied automatically, because the choice of the change of variables is not obvious. A second approach (Hindi *et al.*, 1998) consists in increasing the representation of G_{zw} using the Kronecker product: again LMI constraints with respect to the output matrices of G_{zw} are obtained in most cases. The inconvenient is a large number of state space variables, which can yield a numerically infeasible problem if the number of inputs and/or outputs is high.

For the case where the plant has only one control input, and if it is possible to consider only one controlled output (in the case of more than one output, a weighted combination of them can be considered), U is scalar and G_{zw} can be rewritten:

$$G_{zw} = H - QUV \tag{6}$$

From state space realizations of H, U, V and Q, a non minimal realization of G_{zw} is as follows:

$$G_{zw} = \left(\begin{array}{c|c} A_{zw} & B_{zw} \\ \hline C_{zw} & D_{zw} \end{array}\right) = \tag{7}$$
$$\left(\begin{array}{cccc|c} A_h & 0 & 0 & 0 & B_h \\ 0 & A_q & B_q C_v & B_q D_v C_u & B_q D_v D_u \\ 0 & 0 & A_v & B_v C_u & B_v D_u \\ 0 & 0 & 0 & A_u & B_u \\ \hline C_h & -C_q & -D_q C_v & -D_q D_v C_u & D_h - D_q D_v D_u \end{array}\right)$$

According to (7), matrices C_q, D_q enter linearly in C_{zw}, D_{zw} and not in A_{zw}, B_{zw}. Suppose A_q

$$J = \left(\begin{array}{c|cc} A - B_2K_c - K_fC_2 + K_fD_{22}K_c & K_f & B_2 - K_fD_{22} \\ \hline -K_c & 0 & I \\ -C_2 + D_{22}Kc & I & -D_{22} \end{array}\right) \tag{3}$$

$$J = \left(\begin{array}{cc|cc} A - B_2K_c - K_fC_2 - B_2D_{q_0}C_2 & B_2C_{q_0} & K_f + B_2D_{q_0} & B_2 \\ -B_{q_0}C_2 & A_{q_0} & B_{q_0} & 0 \\ \hline -K_c - D_{q_0}C_2 & C_{q_0} & D_{q_0} & I \\ -C_2 & 0 & I & 0 \end{array}\right) \tag{4}$$

and B_q are chosen; for instance in discrete time one can use an FIR structure or a real Kautz filter (Paarero, 2003) for the Youla parameter; similarly the one proposed by (Akcay and Ninness, 1999) can be used for continuous time.

Note that the order of the Youla parameter is a priori chosen. In that case, different LMI formulations with respect to C_q, D_q can be obtained: they are derived in sections 3 and 4.

3. $\mathcal{H}_2$ OPTIMIZATION

Only the discrete-time case is considered in this section, but similar results are easily obtained in continuous time. The commonly used LMI formulation of discrete-time $\mathcal{H}_2$ optimization is:

$$\begin{aligned} &\min_{x,X} \xi \text{ such that:} \\ &\begin{pmatrix} A_{zw}^TXA_{zw} - X & * \\ B_{zw}^TXA_{zw} & B_{zw}^TXB_{zw} - I \end{pmatrix} < 0, \\ &\begin{pmatrix} X & * & * \\ 0 & I & * \\ C_{zw}(x) & D_{zw}(x) & S \end{pmatrix} > 0 \\ &Trace(S) - \xi < 0 \end{aligned} \tag{8}$$

where vector x contains the decision variables in C_q ,D_q and ξ is an upper bound of $\|G_{zw}\|_2^2$.

It can be noticed that the constraints in problem (8) are LMI's in x and X, where the additional decision variable X is a symmetric matrix with the same size as A_{zw}. Therefore such a problem can be solved using convex optimization techniques.

Using the time-domain definition of the $\mathcal{H}_2$ norm, a more simple formulation can be obtained, where the introduction of this additional variable is avoided. It is derived in the following.

The $\mathcal{H}_2$ norm measures the energy of output z for a unitary white noise w. Suppose that only one controlled output is considered (in the case of multi-outputs, a $\mathcal{H}_2$ constraint is associated to each output). In such case, the trace in (8) can be omitted and the $\mathcal{H}_2$ norm can be written:

$$\|G_{zw}\|_2^2 = \lim_{n\to\infty} \mathrm{E}\left\{y(nT)y(nT)^T\right\} \tag{9}$$

with $y(nT) = C_{zw}(x)\left(\sum_{k=1}^{n} A_{zw}^{k-1}B_{zw}w(n-k)\right)+ D_{zw}(x)w(n)$.

The white noise w having unity variance, one obtains equivalently the relation:

$$\begin{aligned} &\|G_{zw}\|_2^2 = \\ &\qquad C_{zw}(x)W_cC_{zw}(x)^T + D_{zw}(x)D_{zw}(x)^T \end{aligned} \tag{10}$$

where $W_c = \sum_{k=0}^{\infty} A_{zw}^kB_{zw}B_{zw}^T\left(A_{zw}^k\right)^T$ represents the controllability grammian, which is commonly computed as the solution of the Lyapunov equation:

$$A_{zw}W_cA_{zw}^T - W_c + B_{zw}B_{zw}^T = 0$$

Therefore (8) can be reformulated as:

$$\begin{aligned} &\min_{x} \xi \text{ such that:} \\ &C_{zw}(x)W_cC_{zw}(x)^T + D_{zw}(x)D_{zw}(x)^T - \xi < 0 \end{aligned} \tag{11}$$

Writing $W_c = W_c^{1/2}(W_c^{1/2})^T$ and using the Schur lemma, one finally obtains the following problem:

$$\begin{aligned} &\min_{x} \xi \text{ such that:} \\ &\begin{pmatrix} \xi & * \\ \begin{pmatrix} (W_c^{1/2})^T & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} C_{zw}(x)^T \\ D_{zw}(x)^T \end{pmatrix} & I \end{pmatrix} > 0 \end{aligned} \tag{12}$$

Having no additional variable, this last LMI problem is particularly interesting for plants with large state space dimensions, which is often the case when using the Youla parameterization.

4. FREQUENCY CONSTRAINTS

One of the biggest problems encountered in robust synthesis is the conception of weighting filters, particularly when a trade-off must be done between high and low frequency behavior (see e.g. the mixed sensitivity problem), or when there is only specifications on particular intervals of frequencies and no constraint in the rest of the frequency domain: this is for instance the case when attenuation of bending modes is required.

In this section, an LMI problem is proposed for frequency shaping with specification on particular intervals. It is based on a result introduced in (Friang *et al.*, 1998) to avoid frequency griding in μ-analysis, which allows to consider the frequency as an uncertain real parameter. Using this formulation, an LMI constraint will be derived for control synthesis using Youla parameterization.

As in (Friang *et al.*, 1998), continuous-time systems will be considered in the following. Equivalent results are obtained for the discrete-time case using a Tustin transform with sample time T. This transformation does not affect the result of the $\mathcal{H}_\infty$ problem, if the frequency values are computed by considering the warped frequency axis. It results in state-space matrices given below (index c indicates the matrices of the equivalent continuous-time system):

$$\begin{array}{l} A_{zw_c} = -\frac{2}{T}(I - A_{zw})R, \quad B_{zw_c} = \frac{4}{T}RB_{zw} \\ C_{zw_c}(x) = C_{zw}(x)R \\ D_{zw_c}(x) = D_{zw}(x) - C_{zw}(x)RB_{zw} \end{array} \tag{13}$$

with $R = (I + A_{zw})^{-1}$

As seen in (13), this transformation preserves the affine dependence in x of the output matrices C_{zw_c} and D_{zw_c}.

The transfer G_{zw_c} can be expressed as the upper LFT $G_{zw_c}(s) = \mathcal{F}_u(\mathcal{M}, s^{-1}I)$ (Figure 2) with:

$$\mathcal{M} = \begin{pmatrix} A_{zw_c} & B_{zw_c} \\ C_{zw_c}(x) & D_{zw_c}(x) \end{pmatrix} \tag{14}$$

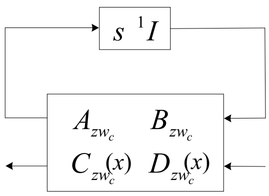

Fig. 2. G_{zw_c} expressed in LFT form

Considering that $s = j\omega$ and assuming ω belongs to $[\omega_1, \omega_2]$ (with ω_1 possibly 0 and ω_2 possibly $+\infty$), one can note that:

$$\frac{1}{jw}I = \mathcal{F}_u(\mathcal{N}(\omega_1, \omega_2), \delta I) \qquad \delta \in [-1, +1] \tag{15}$$

with $\mathcal{N}(\omega_1, \omega_2) = \begin{pmatrix} -aI & bI \\ jcI & -jdI \end{pmatrix}$ and:

$$a = \frac{\omega_2 - \omega_1}{\omega_1 + \omega_2 + 2\omega_1\omega_2}, \; d = \frac{\omega_2 + \omega_1 + 2}{\omega_1 + \omega_2 + 2\omega_1\omega_2}$$

$$b = c = \frac{\sqrt{2(\omega_2 - \omega_1)(1 + \omega_1)(1 + \omega_2)}}{\omega_1 + \omega_2 + 2\omega_1\omega_2}$$

Using these results, the frequency response $G_{zw_c}(j\omega)$ for $\omega \in [\omega_1, \omega_2]$ is described as the interconnection $\tilde{\mathcal{M}} = \mathcal{N}(\omega_1, \omega_2) * \mathcal{M}$, with $\delta \in [-1, +1]$ (Figure 3). $\tilde{\mathcal{M}}$ is written using complex valued matrices as:

$$\tilde{\mathcal{M}} = \begin{pmatrix} \tilde{A}_{zw_c} & \tilde{B}_{zw_c} \\ \tilde{C}_{zw_c}(x) & \tilde{D}_{zw_c}(x) \end{pmatrix} \tag{16}$$

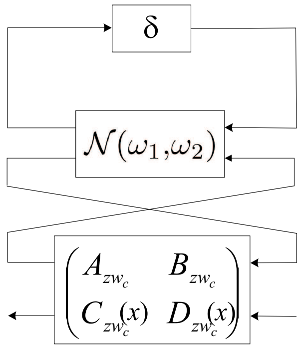

Fig. 3. G_{zw_c} with frequency as real uncertain parameter

Again the vector of decision variables x enter linearly in $\tilde{C}_{zw_c}$ and $\tilde{D}_{zw_c}$ only.

Considering δ as an uncertain real parameter and using the upper bound of the structured singular value in (Fan *et al.*, 1991), the frequency response $G_{zw_c}(j\omega)$ is guaranteed to have a magnitude less than γ if the matrix inequality (17) holds, with $L = L^H > 0$ and $P = P^H$.

Using the Schur lemma, one obtains finally the inequality (18), which is an LMI in x, L and P.

Remark 1. Matrices L and P are complex valued. To save time calculation and memory space in the case of bending modes attenuation, this LMI can be used with the rigid model of the plant (i.e. the plant without bending modes) by choosing adequately the value of γ (see the example in the next section).

5. NUMERICAL EXAMPLE

The digital control of a hard-disk read/write head is considered. It is taken from a Matlab demo (Grace *et al.*, 1995). The head-disk assembly (HDA) and actuators are modeled by a SISO system where the input is the current i_c driving the voice coil motor and the output is the position error signal $\epsilon_\theta = \theta_{ref} - \theta$. The order of the state space representation is 10 including two rigid-body modes and the first four resonances. The model also includes a small delay $T_r = 10^{-5}$ sec.

Only the rigid modes are considered for the compensator design, although the bending modes are to be attenuated while rejecting an input disturbance with zero static error. The rigid model is first discretized using a zero-order hold with sample time $T = 7.10^{-5}$ sec. To handle disturbance rejection, a $\mathcal{H}_2$ performance ξ_1 is considered between input disturbance and error ϵ_θ. To reduce the control effort, another $\mathcal{H}_2$ performance ξ_2 is defined between input disturbance and control input i_c. The following functional will be min-

$$\tilde{\mathcal{M}}^H \begin{pmatrix} L & 0 \\ 0 & I \end{pmatrix} \tilde{\mathcal{M}} + j\left[\begin{pmatrix} P & 0 \\ 0 & 0 \end{pmatrix}\tilde{\mathcal{M}} - \tilde{\mathcal{M}}^H \begin{pmatrix} P & 0 \\ 0 & 0 \end{pmatrix}\right] < \begin{pmatrix} L & 0 \\ 0 & \gamma^2 I \end{pmatrix} \tag{17}$$

$$\begin{pmatrix} \tilde{A}_{zw_c}^H L \tilde{A}_{zw_c} + jP\tilde{A}_{zw_c} - j\tilde{A}_{zw_c}^H P - L & * & * \\ \tilde{B}_{zw_c}^H L \tilde{A}_{zw_c} - j\tilde{B}_{zw_c}^H P & \tilde{B}_{zw_c}^H L \tilde{B}_{zw_c} - \gamma^2 I & * \\ \tilde{C}_{zw_c}(x) & \tilde{D}_{zw_c}(x) & -I \end{pmatrix} < 0 \tag{18}$$

imized to determine the trade-off between both objectives:

$$\xi = \lambda_1\xi_1 + \lambda_2\xi_2 \tag{19}$$

where λ_1 and λ_2 are real positive parameters weighting each $\mathcal{H}_2$ performance.

The design follows two steps: the first one uses objective (19) without considering any attenuation constraint, and compares formulations (8) and (12). The Youla parameter has an FIR form.

The initial compensator is taken from the Matlab demo:

$$K_{init}(z) = \frac{46.29z^2 - 89.32z + 43.09}{z^2 - 0.2801z - 0.7199} \tag{20}$$

The maximum value of the resulting error for a unit impulse disturbance is $\epsilon_\theta = 0.0281$ (Figure 4). The initial compensator having the same order as the rigid model, system J is represented using (3).

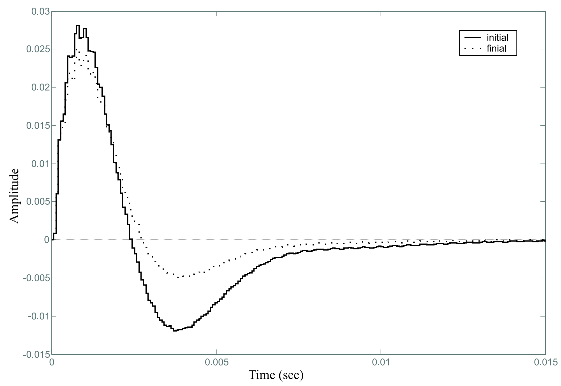

Fig. 4. Impulse response

For comparing both $\mathcal{H}_2$ formulations (8) and (12), the values $\lambda_1 = \lambda_2 = 0.5$ are considered. Table 1 outlines this comparison for different orders of the Youla parameter, using a Pentium4 2.53 GHz processor.

n_q	LMI (12)		LMI (8)	
	time (s)	result	time (s)	result
1	0.16	0.553	26.07	0.557
2	0.18	0.553	26.74	0.555
3	0.22	0.553	47.73	0.554
4	0.27	0.553	59.99	0.553
5	0.29	0.553	107.63	0.553

Table 1. Comparison of $\mathcal{H}_2$ formulations

Although the value of objective (19) in both LMI fomulations is nearly the same, the computation time is much higher when using (8). This gap increases when the order of the Youla parameter rises. As shown by the table, the value of the objective does not really decrease when the order of the Youla parameter rises. For this reason, the order of the Youla parameter n_q is kept equal to 1 in the following.

For choosing λ_1 and λ_2 (taking $\lambda_1+\lambda_2=1$), different values are considered. The control input i_c is not affected by λ_1 and λ_2, but it must be taken into account for getting a reasonable bandwidth. Table 2 shows the maximal value of error ϵ_θ with respect to a unit impulse disturbance when the resulting compensator is applied on the complete hard-disk model.

λ_1	λ_2	$(\epsilon_\theta)_{\max}$
1	0	unstable
0.9	0.1	unstable
0.8	0.2	0.0279
0.7	0.3	0.0298
0.6	0.4	0.0318
0.5	0.5	0.0341

Table 2. Choice of λ_1,λ_2

The value of ϵ_θ decreases when λ_1 increases, but for $\lambda_1 = 1$ and $\lambda_1 = 0,9$ the closed loop system is unstable (although it is stable when only the rigid model is considered). The frequency response between the input disturbance and the output of the compensator is helpful to analyze the reason of this instability (Figure 5).

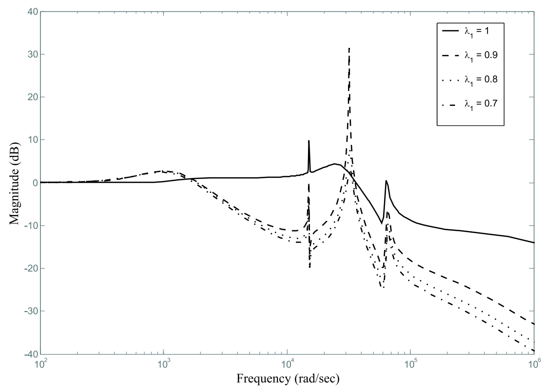

Fig. 5. Frequency response according to λ_1 and λ_2

$\lambda_1 = 1$ results in a large bandwith which implies that flexible modes cannot be neglected. Concerning $\lambda_1 = 0.9$, the instability is due to the resonance peak. This last point is interesting, because instability can thus be avoided by minimizing γ

under LMI constraint (18). The compensator obtained previously for $\lambda_1 = 0.9$ and $\lambda_2 = 0.1$ is used as an initial compensator with J under the form (4).

$\mathcal{H}_2$ constraints are simultaneously taken into account with the fixed values $\xi_1 = 0.0109$ and $\xi_2 = 1.1169$ obtained in the first step, in order to preserve the time-domain performance. The range of frequencies where γ is minimized is $[2.9.10^4, 3.5.10^4]$. Figure 6 outlines the benefit of using (18), for different values of n_q. It can be noticed that the LMI constraint (18) acts efficiently in the chosen frequency domain. The value $n_q = 1$ is not sufficient to reduce significantly the magnitude. For $n_q = 2$ the peak resonance is significantly reduced and stability is restored, without acting on the bandwidth. Furthermore the disturbance rejection is also improved, with maximal error $\epsilon_\theta = 0.0264$ (Figure 4).

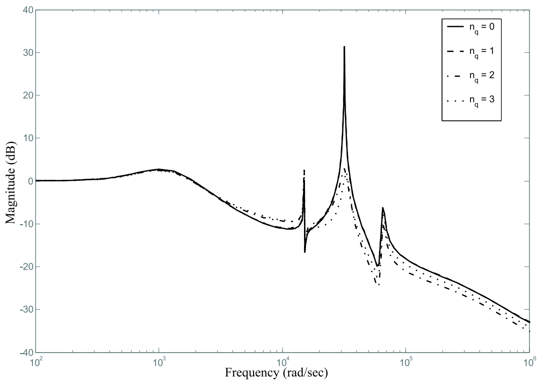

Fig. 6. Reduction of the peak magnitude

6. CONCLUSION

Designing a multiobjective controller, by considering $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance (for time responses and bending modes attenuation respectively), can be done through the Youla parameterization without introducing the Lyapunov matrix for $\mathcal{H}_2$ performance and without weighting filters for $\mathcal{H}_\infty$ attenuation.

The numerical efficiency of the proposed developments using LMI constraints has been showed by considering an example where a compensator has been gradually designed in a very simple way according to engineering specifications. Contrary to most design methods, the proposed approach does not require to tune finely the design parameters. The FIR base can be replaced by another more efficient one, but it should be notice that FIR orthonormal functions provide a good initialization to determine the parameters (i.e. the poles) of the suitable base. Another way to determine the poles of the Youla parameter is the one proposed in (Henrion *et al.*, 2004).

The same developments can be proposed to consider gain and delay margins, which will be the subject of a forthcoming publication.

7. ACKNOWLEDGEMENT

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