RESIDUAL FUNCTION DESIGN FOR LINEAR MULTIVARIABLE SYSTEMS

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Abstract: Classical model–based fault detection schemes for linear multivariable systems require the definition of suitable residual functions. This paper shows the possibility of identifying residual generators even when the system model is unknown, by following a black–box approach. The result is obtained by using canonical input–output polynomial forms which lead to characterise in a straightforward fashion the basis of the subspace described by all possible residual generators. The performance of the proposed identification method is tested by means of a Monte Carlo simulation.

Keywords: Fault detection, linear multivariable systems, input–output polynomial models, parameter identification, Monte Carlo simulation.

1. INTRODUCTION

Most traditional fault detection methods suggested in literature (Gertler, 1998; Chen and Patton, 1999; Simani *et al.*, 2002) are based on filtering elaborations of the plant measurements. Faults are associated to residual signals which must be insensitive as much as possible to model uncertainties, disturbances and measurement noise. The origins of these methods can be found in parity space methodologies and observer–based approaches on one side and parameter estimation techniques on the other side, with many cross connections among the different approaches. Many recent investigations continue to show the advantages and disadvantages of the related residual filters. In any case almost all these approaches require the knowledge of the mathematical model of the process for which the fault detection system is designed.

This work investigates the identification problem of residual generators for linear multivariable systems with additive faults and disturbances. By following the minimal polynomial approach suggested in (Frisk, 2000; Frisk and Nyberg, 2001) and by modelling the process under investigation in terms of input–output canonical description, it is possible to compute in a straightforward fashion an analytical expression for the basis of the subspace described by all possible residual generators. In this way upper and lower bounds for the minimal order of such dynamic filters can also be obtained. These results show that the discrete–time residual generators with disturbance decoupling can be obtained without any knowledge of the mathematical model of the process under investigation, i.e. with a black–box identification approach. The design of residual generators can thus be directly realized from a finite number of input–output samples, measured in absence of faults.

It is worth noting that the setup in the paper is very similar to the well–known PCA approaches. As an example, when the problem of parity equation design and PCA is addressed, the paper by Gertler (Gertler and McAvoy, 1997) can be considered. It is discussed how PCA is related to the residual generation problem. The approach presented in this paper seems more simple and direct when compared to the technique presented by Gertler.

2. RESIDUAL GENERATOR MODEL

Let us consider a linear, time–invariant, discrete–time system described by the following input–output equation

$$P(z)\, y(t) = Q(z)\, u(t) \tag{1}$$

where z^{-1} is the unitary delay operator and $P(z)$ and $Q(z)$ are polynomial matrices with dimension $(m \times m)$ and $(m \times \ell)$ respectively, with $P(z)$ nonsingular. The terms $u(t)$ and $y(t)$ are, respectively, the ℓ–dimensional and m–dimensional input and output vectors of the considered multivariable system. Models of type (1) can be frequently found in practice by applying well–known physical laws to describe the input–output dynamical links of various systems and are a powerful tool in all fields where the knowledge of the system state does not play a direct role, such as residual generation, identification, decoupling, output controllability, etc. Algorithms to transform state–space models to equivalent input–output polynomial representations and *vice versa* are reported in (Guidorzi, 1975).

For a generic input–output model $\{P(z), Q(z)\}$, its *canonical* input–output form is the equivalent representation $\{\tilde{P}(z), \tilde{Q}(z)\}$ with $\tilde{P}(z) = M(z)\, P(z)$, $\tilde{Q}(z) = M(z)\, Q(z)$ and $M(z)$ a unimodular matrix, satisfying the following properties:

$$\deg \tilde{p}_{ii}(z) > \deg \tilde{p}_{ji}(z) \quad i \neq j \tag{2}$$

$$\deg \tilde{p}_{ii}(z) > \deg \tilde{p}_{ij}(z) \quad j > i \tag{3}$$

$$\deg \tilde{p}_{ii}(z) \geq \deg \tilde{p}_{ij}(z) \quad j < i \tag{4}$$

$$\deg \tilde{p}_{ii}(z) \geq \deg \tilde{q}_{ij}(z) \quad \forall j. \tag{5}$$

The polynomials $\tilde{p}_{ii}(z)$ are monic and, because of conditions (3) and (4), the integers $\nu_i = \deg \tilde{p}_{ii}(z)$ $(i = 1, \ldots, m)$ are equal the corresponding row–degrees of $\tilde{P}(z)$. A constructive proof of the existence and uniqueness of a canonical form for a given pair $\{P(z), Q(z)\}$ can be found in (Beghelli and Guidorzi, 1976). In the same work, an efficient and simple algorithm for transforming a generic polynomial representation to the equivalent canonical one is also described. The canonical representation $\{\tilde{P}(z), \tilde{Q}(z)\}$ leads directly to a correspondent canonical state–space realization

$$x(t+1) = \tilde{A}\, x(t) + \tilde{B}\, u(t) \tag{6}$$

$$y(t) = \tilde{C}x(t) + \tilde{D}u(t), \tag{7}$$

with order:

$$n = \sum_{i=1}^{m} \nu_i. \tag{8}$$

The integers ν_i are the ordered set of Kronecker invariants associated to the pair $\{\tilde{A}, \tilde{C}\}$ of every observable realization of $\{P(z), Q(z)\}$ (Guidorzi, 1975). In order to design residual generators of minimal order, model (1) must be firstly transformed into its canonical representation $\{\tilde{P}(z), \tilde{Q}(z)\}$, satisfying conditions (4)–(7); this step can be omitted if the minimal order constraint is relaxed. Then, matrix $\tilde{Q}(z)$ can be decomposed according to the following structure

$$\tilde{P}(z)\, y(t) = \begin{bmatrix} \tilde{Q}_c(z) & \tilde{Q}_d(z) & \tilde{Q}_f(z) \end{bmatrix} \begin{bmatrix} c(t) \\ d(t) \\ f(t) \end{bmatrix}, \tag{9}$$

where $c(t)$ is the ℓ_c–dimensional known–input vector, $d(t)$ is the ℓ_d–dimensional disturbance vector, $f(t)$ is the ℓ_f–dimensional monitored fault vector and $\ell_c + \ell_d + \ell_f = \ell$.

Equation (9) includes also the cases of additive faults on the input and output sensors. In particular, when only additive faults $f_c(t)$ on the input sensors of the system are considered, the input vector measurements can be written as

$$c(t) = c^*(t) + f_c(t) \tag{10}$$

and Eq. (9) becomes $\tilde{P}(z)y(t) = \tilde{Q}_c(z)c^*(t) + \tilde{Q}_d(z)d(t) - \tilde{Q}_c(z)f_c(t)$. Analogously, when only additive faults $f_o(t)$ on the output sensors of the system are considered the output vector measurements can be written as

$$y(t) = y^*(t) + f_o(t)\ . \tag{11}$$

In this case, it results that $\tilde{P}(z)y^*(t) = \tilde{Q}_c(z)c(t) + \tilde{Q}_d(z)d(t) + \tilde{P}(z)f_o(t)$. A general linear residual generator for the fault detection process of system (9) is a filter of type:

$$R(z)\, r(t) = S_y(z)\, y(t) + S_c(z)\, c(t)\ . \tag{12}$$

System (12) processes the known input–output data and generates the residual $r(t)$, *i.e.* a signal which is “small” (ideally zero) in the fault–free case and is “large” when a fault is acting on the system. Without loss of generality, $r(t)$ can be assumed to be a scalar signal. In such condition

$R(z)$ is a polynomial with degree greater than or equal to the row–degree of $S_c(z)$ and $S_y(z)$, in order to guarantee the physical realisability of the filter. Moreover, if $R(z)$ has all roots inside the unit circle filter (12) is asymptotically stable.

An important aspect of the design concerns the decoupling of the disturbance $d(t)$ in order to produce a correct diagnosis in all operating conditions. Equation (9) can be rewritten in the form:

$$\tilde{P}(z)\,y(t) - \tilde{Q}_c(z)\,c(t) - \tilde{Q}_f(z)\,f(t) = \tilde{Q}_d(z)\,d(t)\ . \qquad (13)$$

Premultiplying all the terms in (13) by a row polynomial vector $L(z)$ belonging to the left null–space of $\tilde{Q}_d(z)$, $\mathcal{N}_\ell(\tilde{Q}_d(z))$, we obtain:

$$\begin{aligned} L(z)\,\tilde{P}(z)\,y(t) - \\ +L(z)\,\tilde{Q}_c(z)\,c(t) - L(z)\,\tilde{Q}_f(z)\,f(t) = 0\ . \end{aligned} \qquad (14)$$

Starting from Eq. (14) with $f(t) = 0$ it is possible to obtain a residual of type (12) by setting:

$$\begin{aligned} S_y(z) &= L(z)\,\tilde{P}(z) \\ S_c(z) &= -L(z)\,\tilde{Q}_c(z) \\ R(z) &= z^{n_f}\ , \end{aligned} \qquad (15)$$

where n_f is the maximal row–degree of the pair $\left\{L(z)\,\tilde{P}(z),\, L(z)\,\tilde{Q}_c(z)\right\}$. The polynomial $R(z)$ can be arbitrarily selected, for simplicity we will consider the choice $R(z) = z^{n_f}$ which guarantees the asymptotical stability of the filter with n_f poles equal to zero. In absence of faults Equation (12) can be rewritten also in the form:

$$\begin{aligned} r(t+n_f) = z^{n_f}\,r(t) = L(z)\,\tilde{P}(z)\,y(t) - \\ +L(z)\,\tilde{Q}_c(z)\,c(t) = 0 \end{aligned} \qquad (16)$$

When a fault is acting on the system the residual generator is governed by the relation

$$r(t+n_f) = -L(z)\,\tilde{Q}_f(z)\,f(t) \qquad (17)$$

and $r(t+n_f)$ assumes values that are different from zero if $L(z)$ does not belong to $\mathcal{N}_\ell(\tilde{Q}_f(z))$. In these conditions, the design freedom in the choice of the matrix $L(z)$ can be used to optimise the sensitivity properties of $r(t)$ to the fault $f(t)$, for example by maximising the steady-state gain of the transfer function $L(z)\,\tilde{Q}_f(z)$. Another design choice regards the location of the roots of the polynomial $R(z)$ inside the unit circle, which influences the frequency response of the residual generator and, consequently, its robustness with respect to input–output measurement noises, modelling errors, parameter uncertainties, etc. It is worth noting that, in order to introduce more degrees of freedom, the polynomial $R(z)$ could be chosen as $R(z) = (z - z_1)\,(z - z_2)\ldots(z - z_{n_f})$, where its roots z_i have to be selected inside the unit circle. In other words the diagnostic features of a residual generator strongly depend on an accurate selection of the terms $L(z)$ and $R(z)$.

In order to determine the residual generators of minimal order it is necessary to compute a minimal basis of $\mathcal{N}_\ell(\tilde{Q}_d(z))$. Under the assumption that matrix $\tilde{Q}_d(z)$ is of full rank, *i.e.* $\operatorname{rank}\tilde{Q}_d(z) = \ell_d$, $\mathcal{N}_\ell(\tilde{Q}_d(z))$ has dimension $m - \ell_d$ and a minimal basis of it can be computed as suggested in (Kailath, 1980). It can be noted that in absence of disturbances $\ell_d = 0$ so that $\mathcal{N}_\ell(\tilde{Q}_d(z))$ coincides with the whole vector space. Consequently, a set of residual generators can be expressed as

$$\begin{aligned} & r_i(t+\nu_i) = z^{\nu_i}\,r(t) = \tilde{p}_i(z)\,y(t) - \tilde{q}_{c_i}(z)\,c(t) \\ & (i = 1, 2, \ldots, m) \end{aligned} \qquad (18)$$

where $\tilde{p}_i(z)$ and $\tilde{q}_{c_i}(z)$ are the i–th rows of matrices $\tilde{P}(z)$ and $\tilde{Q}_c(z)$ respectively, and ν_i is the row–degree of $\tilde{p}_i(z)$, since $\tilde{q}_{c_i}(z)$ cannot show a greater row–degree. In general, for $0 < \ell_d < m$ matrix $\tilde{Q}_d(z)$ can be partitioned in the following way

$$\tilde{Q}_d(z) = \begin{bmatrix} \tilde{Q}_{d_1}(z) \\ \tilde{Q}_{d_2}(z) \end{bmatrix}\ , \qquad (19)$$

where matrices $\tilde{Q}_{d_1}(z)$ and $\tilde{Q}_{d_2}(z)$ have dimension $\ell_d \times \ell_d$ and $(m - \ell_d) \times \ell_d$ respectively. It can be assumed, without loss of generality, that matrix $\tilde{Q}_{d_1}(z)$ is non singular. In this case it can be easily verified that a basis of $\mathcal{N}_\ell(\tilde{Q}_d(z))$ (not necessarily of minimal order) is given by the polynomial matrix:

$$B(z) = \left[\ \tilde{Q}_{d_2}(z)\,\operatorname{adj}\tilde{Q}_{d_1}(z)\ \ -\det\tilde{Q}_{d_1}(z)\,I_{m-\ell_d}\ \right] \qquad (20)$$

where $\operatorname{adj}\tilde{Q}_{d_1}(z) = 1$ if $\ell_d = 1$. By partitioning $\tilde{P}(z)$ and $\tilde{Q}_c(z)$ as $\tilde{Q}_d(z)$ in (19)

$$\tilde{P}(z) = \begin{bmatrix} \tilde{P}_1(z) \\ \tilde{P}_2(z) \end{bmatrix} \qquad \tilde{Q}_c(z) = \begin{bmatrix} \tilde{Q}_{c_1}(z) \\ \tilde{Q}_{c_2}(z) \end{bmatrix} \qquad (21)$$

a basis for the residual generators (12) of system (9) is obtained by replacing in relation (15) the row polynomial vector $L(z)$with the polynomial matrix $B(z)$, *i.e.*

$$\begin{aligned} S_y(z) &= \tilde{Q}_{d_2}(z)\,\operatorname{adj}\tilde{Q}_{d_1}(z)\,\tilde{P}_1(z) - \det\tilde{Q}_{d_1}(z)\,\tilde{P}_2(z) \\ S_c(z) &= -\tilde{Q}_{d_2}(z)\,\operatorname{adj}\tilde{Q}_{d_1}(z)\,\tilde{Q}_{c_1}(z) + \det\tilde{Q}_{d_1}(z)\,\tilde{Q}_{c_2}(z) \\ R(z) &= \operatorname{diag}\left[z^{n_{f_1}}\, z^{n_{f_2}}\, \ldots\, z^{n_{f_{m-\ell_d}}} \right]\ , \end{aligned} \qquad (22)$$

where n_{f_i} $(i = 1, \ldots, m - \ell_d)$ is the row–degree of the i–th row of matrix $S_y(z)$. It can be noted that relation (5) leads to the following inequality

$$\text{row deg}\left\{S_{y_i}(z)\right\} \geq \text{row deg}\left\{S_{c_i}(z)\right\}\ , \qquad (23)$$

where $S_{y_i}(z)$ and $S_{c_i}(z)$ denote the i–th rows of matrices $S_y(z)$ and $S_c(z)$ respectively, so that the residual generator is physically realizable.

Previous considerations can be summarised in the following theorem.

Theorem 1. The order n_f^* of a minimal order residual generator for the system (9) is constrained in the following range

$$\nu_{\min} \;\le\; n_f^* \;\le\; \min\left\{(\ell_d+1)\,\nu_{\max},\, n\right\}. \quad (24)$$

where $\nu_{\min}$ and $\nu_{\max}$ are the least and the greatest Kronecker invariant respectively and n is the order of the system.

The lower bound can be obtained in the no–disturbance case ($\ell_d = 0$) from relation (18) by selecting the rows of $\tilde{P}(z)$ associated to the minimal Kronecker invariant. The upper bound follows by considering the maximal degree of the polynomials of the matrices in (22).

A similar result, obtained with a different approach, can be found in (Frisk, 2000).

3. RESIDUAL IDENTIFICATION

In this section we will consider the problem of identifying the residual generators with minimal order n_f^*. More precisely, among the $m - l_d$ difference equations in the relation $S_y(z)\,y(t) + S_c(z)\,c(t) = 0$, we are interested in determining those with minimal order n_f^*. Note that the number of such equations is not *a priori* known. A minimal order residual generator can be expressed by a difference equation of the type

$$\sum_{i=1}^{m}\sum_{k=0}^{n_f^*}\alpha_{ik}\,y_i(t+k) + \sum_{j=1}^{\ell_c}\sum_{k=0}^{n_f^*}\beta_{jk}\,c_j(t+k) = 0, \quad (25)$$

where, in general, some coefficients α_{ik}, β_{jk} can be equal to zero.

In absence of noise in the data, the identification problem can be stated as follows.

Problem 1. Given a finite sequence of variables $y_i(t)$ $(i = 1,\dots,m)$ and $c_j(t)$ $(j = 1\dots,\ell_c)$ with $t = 1,\dots,N$ generated by a system of type (9) in absence of faults, determine the order n_f^* and the parameters α_{ik}, β_{jk} of the equations of type (25).

Define now the following vectors and matrices:

$$\begin{array}{rcl} Y_i(t) &=& \left[y_i(t)\dots y_i(t+L-1)\right]^T \\ C_j(t) &=& \left[c_j(t)\dots c_j(t+L-1)\right]^T \\ X_h(y_i) &=& \left[Y_i(1)\dots Y_i(h+1)\right] \\ X_h(c_j) &=& \left[C_j(1)\dots C_j(h+1)\right], \end{array} \quad (26)$$

for $i = 1,\dots,m$, $j = 1,\dots,\ell_c$. Define also the Hankel matrix

$$H_h = \left[\; X_h(y_1)\;\dots\; X_h(y_m)\;\; X_h(c_1)\;\dots\; X_h(c_{\ell_c})\;\right], \quad (27)$$

and compute the sample covariance matrix

$$\Sigma_h = \frac{1}{L}\,H_h^T H_h\;. \quad (28)$$

If the integer L satisfies the condition:

$$L \ge (m + l_c)\,(h+1) \quad (29)$$

the number of rows in matrix H_h is greater than or equal to the number of columns and it is easy to verify that

$$\Sigma_h > 0 \text{ for } h < n_f^* \quad (30)$$

$$\Sigma_h \ge 0 \text{ for } h \ge n_f^*. \quad (31)$$

In particular

$$\Sigma_{n_f^*}\,\Theta = 0, \quad (32)$$

where Θ is a matrix with dimension $((m{+}l_c)\,(n_f^*{+}1))\times\nu$ and the dimension ν of $\ker(\Sigma_{n_f^*})$ equals the number of the residual generators with minimal order n_f^*. The entries of Θ are the coefficients of ν relations of type (25). For simplicity, these vectors will be considered with unitary Euclidean norm. On the basis of these considerations, Problem 1 can be solved by means of the algorithm described below.

Algorithm 1

(1) Consider the sequence of symmetrical increasing dimension non negative definite matrices

$$\Sigma_1, \Sigma_2, \dots \quad (33)$$

and test the linear independence of their columns as long as a singular matrix $\Sigma_{\bar{h}}$ is encountered. Then $n_f^* = \bar{h}$ and the number of residual generators of minimal order is $\nu = (m + \ell_c)\,(\bar{h}+1) - \operatorname{rank}\Sigma_{\bar{h}}$.

(2) Compute the basis Θ of the null space of $\Sigma_{n_f^*}$.

From (26) and (29) it can be verified that the number of the available samples N must satisfy the condition $N \ge (m + l_c)\,(n_f^* + 1) + n_f^*$. In order to perform correctly the independence test in step 1 of Algorithm 1, the Hankel matrix $\left[X_h(c_1)\dots X_h(c_{\ell_c})\right]$ must be of full rank, *i.e.* the known inputs $c_1, c_2, \dots, c_{\ell_c}$ must be persistently exciting of sufficient orders (identifiability conditions). A check of this rank should thus be included in step 1.

When the input–output sequences $c(\cdot)$ and $y(\cdot)$ are corrupted by noise, the previous procedure is obviously useless since the matrices in the sequence (33) are always non singular. As a natural assumption we can state that

All the variables are affected by additive noise, *i.e.*

$$y_i^* = y_i + \tilde{y}_i \;\; i = 1,\dots,m \quad (34)$$

$$c_j^* = c_j + \tilde{c}_j \;\; j = 1,\dots,l_c \quad (35)$$

and only the noisy variables y_i^* and c_j^* are available. The processes $\tilde{y}_i$ $(i = 1,\dots,m)$ and $\tilde{c}_j$ $(j =$

$1,\ldots,l_c$) are zero–mean, ergodic and mutually uncorrelated white noise, whose variances are known up to the same scalar factor λ (unknown).

In the noisy case Problem 1 can be re–formulated as follows.

Problem 2. Given a finite sequence of noisy variables $y_i^*(t)$ $(i = 1,\ldots,m)$ and $c_j^*(t)$ $(j = 1\ldots,\ell_c)$ with $t = 1,\ldots,N$ generated by a system of type (9) in absence of faults and corrupted by noise according to Assumption 1, determine the order n_f^* and the parameters α_{ik}, β_{jk} of the equations of type (25).

Under previous assumptions, it can be easily proved that the following relation holds:

$$\Sigma_h^* = \Sigma_h + \tilde{\Sigma}_h, \tag{36}$$

where the covariance matrices are defined as

$$\Sigma_h = \lim_{L\to\infty} \frac{1}{L}\, H_h^T H_h \tag{37}$$

$$\tilde{\Sigma}_h = \lim_{L\to\infty} \frac{1}{L}\, \tilde{H}_h^T \tilde{H}_h \tag{38}$$

$$\Sigma_h^* = \lim_{L\to\infty} \frac{1}{L}\, H_h^{*T} H_h^*, \tag{39}$$

with obvious meaning of the terms. Since no correlation is assumed between the noise samples at different time lags we have:

$$\tilde{\Sigma}_h = \mathrm{diag}\Big[\tilde{\sigma}_{y_1} I_{h+1}\ \ldots\ \tilde{\sigma}_{y_m} I_{h+1}\ \ \tilde{\sigma}_{c_1} I_{h+1}\ \ldots\ \tilde{\sigma}_{c_{\ell_c}} I_{h+1}\Big] \geq 0. \tag{40}$$

Note that Assumption 1 implies the following relation

$$\tilde{\Sigma}_h = \lambda\, \tilde{\Sigma}_h^{\#} \tag{41}$$

where $\tilde{\Sigma}_h^{\#}$ is known and the scalar λ is unknown, so that equations (30) and (31) become

$$\Sigma_h = \Sigma_h^* - \lambda\, \tilde{\Sigma}_h^{\#} > 0 \quad h < n_f^* \tag{42}$$

$$\Sigma_h = \Sigma_h^* - \lambda\, \tilde{\Sigma}_h^{\#} \geq 0 \quad h \geq n_f^*. \tag{43}$$

Relation (43) leads to

$$\Sigma_h^{*-1}\, \tilde{\Sigma}_h^{\#} - \frac{1}{\lambda}\, I_h \leq 0 \quad h \geq n_f^*, \tag{44}$$

i.e. $1/\lambda$ is the maximum eigenvalue of $\Sigma_h^{*-1}\, \tilde{\Sigma}_h^{\#}$.

The solution of Problem 3 in the asymptotic case $N \to \infty$ can thus be obtained by performing the following algorithm.

Algorithm 2

(1) Consider the sequence of symmetrical increasing dimension positive definite matrices

$$\Sigma_1^*, \Sigma_2^*, \ldots \tag{45}$$

and construct the corresponding noise covariance matrices $\tilde{\Sigma}_1^{\#}, \tilde{\Sigma}_2^{\#}, \ldots$.

Compute

$$\mu_h = \max\ \mathrm{eig}\ \Sigma_h^{*-1}\, \tilde{\Sigma}_h^{\#} \tag{46}$$

and the terms

$$\frac{1}{\mu_1},\ \frac{1}{\mu_2}\ \cdots \tag{47}$$

as long as it results

$$\frac{1}{\mu_{\bar{h}+1}} = \frac{1}{\mu_{\bar{h}}}. \tag{48}$$

Then $n_f^* = \bar{h}$ and $\lambda = 1/\mu_{\bar{h}}$.

(2) Compute the matrix

$$\Sigma_{n_f^*} = \Sigma_{n_f^*}^* - \lambda\, \tilde{\Sigma}_{n_f^*}^{\#} \tag{49}$$

and determine the basis Θ of the null space of $\Sigma_{n_f^*}$.

This procedure can be used also in presence of a finite number of data, *i.e.* when only sample covariance matrices

$$\Sigma_h^* = \frac{1}{L}\, H_h^{*T} H_h^* \tag{50}$$

are available. In this case an exact value n_f^* can not be determined in step 1 because the sequence in (47) does not exhibit a stabilisation for a certain value $\bar{h}$. However, when the assumptions are only slightly violated, n_f^* can be estimated as the first value of h for which it results

$$\left|\frac{1}{\mu_{h+1}} - \frac{1}{\mu_h}\right| \Big/ \left|\frac{1}{\mu_h}\right| << \left|\frac{1}{\mu_h} - \frac{1}{\mu_{h-1}}\right| \Big/ \left|\frac{1}{\mu_{h-1}}\right|. \tag{51}$$

Note that when the system structural indexes $\{\nu_1,\ \ldots\ \nu_m\}$ are known, test (54) can be performed only for the values $\nu_{\min} \leq h \leq \min\big\{(\ell_d+1)\,\nu_{\max},\ n\big\}$.

4. NUMERICAL EXAMPLE

The method described in previous sections has been tested on a simulated system with $m = 2, \ell_c = 1, \ell_d = 1$, characterised by the following canonical representation

$$\tilde{P}(z) = \begin{bmatrix} z^2 - 0.2\,z + 0.4 & 0.2 \\ -0.2\,z - 0.1 & z + 0.4 \end{bmatrix} \tag{52}$$

$$\tilde{Q}_c(z) = \begin{bmatrix} z^2 - 0.1 \\ 0.5\,z + 0.5 \end{bmatrix} \tag{53}$$

$$\tilde{Q}_d(z) = \begin{bmatrix} z^2 - 2\,z - 0.65 \\ 0.8\,z + 1.1 \end{bmatrix}. \tag{54}$$

It can be easily verified that $\nu_1 = 2, \nu_2 = 1$ and the system admits only one residual generator with order $n_f^* = 3$. The known input $c(t)$ is a piecewise constant binary sequence while the disturbance $d(t)$ is a pseudo random binary sequence. Both sequences have zero–mean and unit variance and are corrupted by zero–mean mutually uncorrelated white noise with variances

$$\begin{bmatrix} \tilde{\sigma}_{y_1} \\ \tilde{\sigma}_{y_2} \\ \tilde{\sigma}_{c_1} \end{bmatrix} = \lambda \begin{bmatrix} 0.9574 \\ 0.2663 \\ 0.1116 \end{bmatrix}, \tag{55}$$

where λ is unknown.

The effectiveness of the method has been tested by considering different conditions of signal-to-noise ratio (SNR). For each SNR a 100 runs Monte Carlo simulation with $N = 500$ has been performed by assuming that the structural indexes of system (56)–(57) are known. In this case test (55) has been performed with $1 \leq h \leq 3$ and it has led to the correct estimation of the order n_f^* in every run. Figure 1 shows the root–mean square error (RMSE) versus the SNR, where the RMSE is defined as

$$\text{RMSE} = \sqrt{\frac{1}{100} \sum_{i=1}^{100} \|\hat{\Theta}^i - \Theta\|^2}, \tag{56}$$

and $\hat{\Theta}^i$ is the estimate, from the i–th trial, of the coefficient vector Θ. Tables 1 and 2 refer to the

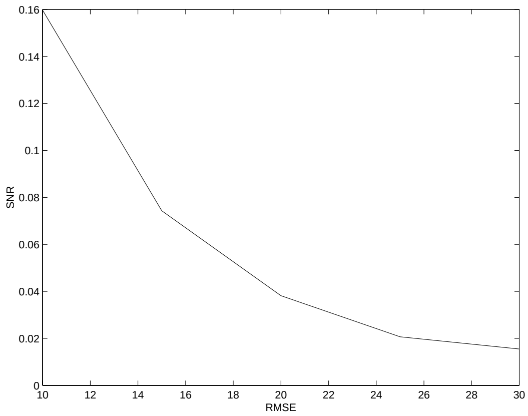

Fig. 1. RMSE versus SNR.

case SNR = 15 dB and report the true values of the coefficients of vector Θ, their mean values and the corresponding standard deviations.

Table 1. True and estimated parameters α_{ik}.

	α_{11}	α_{12}	α_{13}
true	-0.1072	0.0658	-0.1830
ident.	-0.1062 $\pm$ 0.011	0.0637 $\pm$ 0.013	-0.1832 $\pm$ 0.012
	α_{21}	α_{22}	α_{23}
true	-0.2859	-0.1372	-0.4603
ident.	-0.2850 $\pm$ 0.007	-0.1409 $\pm$ 0.032	-0.4590 $\pm$ 0.016

5. CONCLUSION

The problem of identifying residual generators for fault detection purposes in linear multivariable systems has been addressed in this work. The paper shows that the order and the parameters of these filters can be determined by following an identification approach, starting from the knowledge of a finite number of input–output samples describing the behaviour of the process in absence of faults. This result is obtained by using canonical input–output polynomial representations, which lead to a simple characterisation of the polynomial basis of the subspace described by all residual generators. The robustness of the suggested identification approach has been verified by means of a Monte Carlo simulation. The use of this procedure in real fault detection and isolation problem is currently under investigation.

Table 2. True and estimated parameters β_{jk}.

	β_{11}	β_{12}	β_{13}
true	-0.4575	0.2859	0.0615
ident.	-0.4549 $\pm$ 0.022	0.2857 $\pm$ 0.013	0.0636 $\pm$ 0.029
	β_{21}	β_{22}	β_{23}
true	0.3560	0.4575	0.0858
ident.	0.3596 $\pm$ 0.029	0.4520 $\pm$ 0.026	0.0875 $\pm$ 0.027

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