SIMULTANEOUS RECONSTRUCTION OF STATES, SENSOR FAULTS AND UNCERTAINTIES IN LINEAR SYSTEMS

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Abstract: In this paper the problem of simultaneous reconstruction of the state, the sensor fault and the uncertainty in linear systems is considered. Inspired by the idea in chaotic masking, one scheme for chaotic secure communication, this paper proposes a new scheme to resolve the above problem. Together with iterative learning strategy, the generalized state observer proposed in chaotic masking can be used to simultaneously reconstruct the state, the sensor fault and the uncertainty. Theoretical analysis and numerical simulations verify the effectiveness of this scheme.

Keywords: sensor fault, fault detection and isolation, state observer, iterative learning, chaos synchronization, chaotic masking

1. INTRODUCTION

In the past couple of decades, much attention has been paid to the problem of robust sensor fault detection and isolation (FDI). A number of interesting robust sensor FDI problem formulations have been proposed (Frank, 1990; Gertler, 1988; Nikoukhah, 1994; Piercy, 1992). In addition, an augmentation approach was used for representing any sensor faults in the form of actuator faults for an augmented system, thereby permitting the use of actuator FDI methods to accomplish sensor FDI under some conditions (Saif and Guan, 1993; Park et al, 1994). It is well known that sensor FDI has some special properties that are different from those for actuator FDI. The response of sensor faults can not be restricted to a line direction in general by using a detection filter, but only to a plane. Xiong and Saif (2000) illustrated some inherent difference between sensor FDI and actuator FDI although the FDI system design approach can be almost the same. It showed that the augmentation didn't not improve the isolability of sensor faults, that is to say, the isolation condition for the augmented system was often stricter than that for the original system.

Recently, chaotic systems attracted lots of attention in the nonlinear field. Chaotic systems are simple deterministic nonlinear systems behaving complex, noise-like and unpredictable behavior. Among many studies on chaotic systems, chaos synchronization is one of the main topics. Chaos synchronization is defined as the synchronization between two identical or different chaotic systems. As reported in the literature (Morgul, 1999; Morgul et al, 2003; Liao and Huang, 1999), chaos synchronization suggests possibility for communication using chaotic waveforms as carriers. Due to the fact that chaotic signals are noise-like and unpredictable in nature, such signals can be used to establish a potentially secure means of communication. One approach employed to achieving secure communication uses a chaotic signal to mask the message signal (Morgul, 1999; Morgul et al, 2003; Liao and Huang, 1999), which can be realized via an observer based chaos synchronization schemes. The message signal hidden in transmitter can be recovered by a suitable receiver. In some chaotic masking schemes (Boutayeb et al, 2002; Liao and Huang, 1999), the hidden message signal could be regarded as the sensor fault, which is recovered or reconstructed via an observer.

As far as we known, many robust sensor FDI schemes only consider the reconstruction of the state and the sensor fault. They can be estimated if the uncertainty in dynamical system is bounded by a known bounding function or decoupled from the uncertainty. Scarce papers consider the problem of simultaneous reconstruction of the state, the sensor fault and the uncertainty in dynamical systems. Borrowing the idea from chaotic masking (Boutayeb et al, 2002), one scheme for chaotic secure communication, this paper proposes a new scheme to resolve the above problem. Together with the iterative learning strategy (Chen and Saif, 2001, 2002, 2003), the generalized state observer proposed in chaotic masking (Boutayeb et al, 2002) can be used to simultaneously reconstruct the state, the sensor fault and the uncertainty in linear systems under some mild conditions. The special structure of the generalized state observer ensures that the scheme for FDI has no any restrictions on sensor faults.

2. PROBLEM FORMULATION

Let us consider uncertain linear systems given by

$$\begin{aligned} \dot{x} &= Ax + Bu + \eta(t) \\ y &= Cx + Df(t) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector of the system, $u \in R^m$ is the control input vector, $y \in R^r$ is the output vector, $f \in R^p (p \leq r)$ is the sensor fault vector, A, B, C and D are matrices with appropriate dimensions. Further, the uncertainty $\eta(t) \in R^n$ stands for actuator faults or component faults in system (1).

In this paper we consider the problem of simultaneous reconstruction of the state x, the sensor fault f and the uncertainty η in system (1). This means that the uncertainty η can be estimated approximately while the state x and the sensor fault f are reconstructed without residual generation. In order to do, system (1) must satisfy the following assumptions A1 and A2.

A1: In system (1) matrix D is full column rank. Moreover, the uncertainty vector $\eta(t)$ can be norm bounded by a known positive constant δ, namely $||\eta(t)|| < \delta$.

A2: For arbitrary complex number μ with $\mathrm{Re}(\mu) \geq 0$, the following condition holds:

$$\begin{aligned} \mathrm{rank} \begin{pmatrix} \mu I_n - A & 0 \\ C & D \end{pmatrix} &= n + \mathrm{rank} \begin{pmatrix} 0 \\ D \end{pmatrix} \\ &= n + p \end{aligned} \tag{2}$$

3. MAIN RESULT

In this section we will consider the problem of simultaneous reconstruction of the state, the sensor fault and the uncertainty in system (1). The combination of the generalized state observer proposed in chaotic masking (Boutayeb et al, 2002) and the iterative learning strategy (Chen and Saif, 2001, 2002, 2003) can effectively resolve this problem.

3.1 *The Augmentation Strategy*

Among many works, one way to deal with the sensor faults f is the augmentation strategy (Saif and Guan, 1993; Park et al, 1994). Any sensor fault can be regarded as an actuator fault in an augmented system, thereby permitting the use of actuator FDI methods to accomplish sensor FDI. The augmentation strategy can be explained as the following proposition:

Proposition 1 (Saif and Guan, 1993; Park et al, 1994): For any piecewise continuous vector function $f \in R^p$, and a stable $p \times p$ matrix A_f, there will always exist an input $\zeta \in R^p$ such that

$$\dot{f} = A_f f + \zeta \tag{3}$$

Augmenting system (1) with system (3) results in the following $n + p$ dimensional system

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{f} \end{pmatrix} &= \begin{pmatrix} A & 0 \\ 0 & A_f \end{pmatrix} \begin{pmatrix} x \\ f \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \\ & \quad \begin{pmatrix} I_n \\ 0 \end{pmatrix} \eta + \begin{pmatrix} 0 \\ I_p \end{pmatrix} \zeta \\ y &= \begin{pmatrix} C & D \end{pmatrix} \begin{pmatrix} x \\ f \end{pmatrix} \end{aligned} \tag{4}$$

Note that the uncertainty in the augmented system (4) includes two parts: one is the original uncertainty η; another is the additional uncertainty ζ via the augmentation. Further, the additional uncertainty ζ is totally unknown.

Based on the augmented system (4), the problem of reconstructing the state and the sensor fault can be resolved by the UIO method (Xiong and Saif, 2000; Saif and Guan, 1993; Park et al, 1994), the learning system approach (Polycarpou and Helmicki, 1995) and the iterative learning estimator (Chen and Saif, 2001, 2002, 2003). However, the information on the original uncertainty η in system (1) could not be derived because the uncertainty in system (4) includes additional uncertainty ζ owing to the augmentation strategy. Further, Xiong and Saif showed that the above augmentation didn't improve the isolability of faults (Xiong and Saif, 2000). That is to say, the isolation condition for the augmented system is often stricter than that for the original system.

3.2 *The Generalized State Observer*

In chaotic masking literature, Boutayeb et al (2002) proposed a generalized state observer to estimate the state and the hidden message signal when there exists no uncertainty in the transmitter. In this paper, together with the iterative learning strategy, this kind of observer can be applied to resolve the problem of simultaneous reconstruction of the state, the sensor fault and the uncertainty. The following symbols are similar to those in Boutayeb et al (2002). Let

$$E = [I_n \ \ 0], \quad H = [C \ \ D] \quad \text{and} \quad M = [A \ \ 0] \tag{5}$$

Because matrix D is full column rank, matrices P and Q can be constructed as follows

$$[P \ \ Q] = \left[\begin{pmatrix} E \\ H \end{pmatrix}^T \begin{pmatrix} E \\ H \end{pmatrix} \right]^{-1} \begin{pmatrix} E \\ H \end{pmatrix}^T \tag{6}$$

Hence we have the following relationship:

$$PE + QH = I_{n+p} \tag{7}$$

Let the augmented states be $\xi = col(x, \ f)$. For system (1), a generalized state observer is constructed as

$$\begin{aligned} \dot{z} &= Nz + Ly + PBu + Pv \\ \hat{\xi} &= z + Qy \end{aligned} \tag{8}$$

where matrices N and L are to be determined. Further, the iterative learning estimator $v(t)$ is selected as

$$v(t) = \begin{cases} K_1 v(t-\tau) + K_2(y(t-\tau) \\ \qquad\qquad -H\hat{\xi}(t-\tau)), & t > \tau \\ 0, & t \le \tau \end{cases} \tag{9}$$

where $\tau > 0$ is a time delay constant, $K_1 \in R^{n\times n}$ and $K_2 \in R^{n\times r}$ are to be determined, $y(t-\tau)$ and $\hat{\xi}(t-\tau)$ denote the outputs of previous time.

Let the error signal be $e = \hat{\xi} - \xi$. Hence we have

$$\begin{aligned} e &= \hat{\xi} - \xi = z + Qy - \xi = z + QH\xi - \xi \\ &= z + (QH - I_{n+p})\xi = z - PE\xi \end{aligned}$$

Therefore the error dynamics is

$$\begin{aligned} \dot{e} &= Nz + LH\xi + PBu + Pv - PE \\ &\quad \times \left[\begin{pmatrix} M \\ * \end{pmatrix} \xi + \begin{pmatrix} B \\ * \end{pmatrix} u + \begin{pmatrix} I_n \\ * \end{pmatrix} \eta \right] \\ &= N(e + PE\xi) + LH\xi - PM\xi + Pv - P\eta \\ &= Ne + (N + FH - PM)\xi + Pv - P\eta \end{aligned}$$

where $F = L - NQ$, and the signal " $*$ " stands for the unknown term with respect to the sensor fault f. Choosing matrix $N = PM - FH$, then we have

$$\dot{e} = Ne + Pv - P\eta \tag{10}$$

Remark 1. From above analysis, the generalized state observer based FDI method only requires the full column rank of matrix D, which ensures the existence of matrices P and Q. Compared with the augmentation strategy (Xiong and Saif, 2000; Saif and Guan, 1993; Park et al, 1994), this method has no any restrictions on the sensor fault f because of the special structure of observer (8), namely the special relationship (7).

Before we give the result, two lemmas should be given as follows:

Lemma 1 (Boutayeb et al, 2002). Matrix N is stable if and only if Assumption A2 holds, i.e., the system (A, B_1, C, D) is of minimum phase where B_1=0.

Lemma 2 (Chen and Saif, 2001, 2002, 2003). If the iterative learning estimator is chosen as (9), the following inequality holds:

$$\begin{aligned} v^T(t)v(t) &\le 2v^T(t-\tau)K_1^T K_1 v(t-\tau) \\ &\quad + 2e^T(t-\tau)(K_2H)^T(K_2H)e(t-\tau) \end{aligned} \tag{11}$$

Now we give our main result.

Theorem 1. For system (1) and its observer (8) with the iterative learning estimator (9), suppose that Assumptions A1 and A2 hold. The error $e(t)$ of system (10) can be bounded if the following conditions are satisfied:
(i) Matrix F is chosen to make $N = PM - FH$ be Hurwitz stable, and $L = F + NQ$;
(ii) Matrices K_1 and K_2 satisfy

$$4(K_2H)^T(K_2H) \le R_1 \quad \text{and} \quad 4K_1^T K_1 \le I_n \tag{12}$$

where $R_1 \in R^{(n+p)\times(n+p)}$ is a symmetric positive definite matrix.
(iii)For a given symmetric positive definite matrix $Q_1 = Q_1^T \in R^{(n+p)\times(n+p)}$, the following algebraic equation has one unique symmetric positive definite matrix solution $P_1 \in R^{(n+p)\times(n+p)}$:

$$N^T P_1 + P_1 N + P_1 P P^T P_1 + R_1 = -Q_1 \tag{13}$$

Proof: Under condition (i), it can be deduced from Lemma 1 that matrix N can be Hurwitz stable.

For the error dynamics (10), let a Lyapunov function be

$$V(t) = e^T(t)P_1e(t) + \int_{t-\tau}^{t} e^T(\theta)R_1e(\theta)\mathrm{d}\theta + \int_{t-\tau}^{t} v^T(\alpha)v(\alpha)\mathrm{d}\alpha$$

So its derivative with respect to time is

$$\begin{aligned}\dot{V} &= \dot{e}^TP_1e(t) + e^T(t)P_1\dot{e}(t) + e^T(t)R_1e(t) \\ &\quad -e^T(t-\tau)R_1e(t-\tau) + v^T(t)v(t) \\ &\quad -v^T(t-\tau)v(t-\tau) \\ &= e^T(t)[N^TP_1 + P_1N + R_1]e(t) \\ &\quad +2e^T(t)P_1Pv(t) - 2e^T(t)P_1P\eta \\ &\quad -e^T(t-\tau)R_1e(t-\tau) + v^T(t)v(t) \\ &\quad -v^T(t-\tau)v(t-\tau)\end{aligned}$$

Since

$$\begin{aligned}2e^T(t)P_1Pv(t) &\le 2||e^T(t)P_1P|| \cdot ||v(t)|| \\ &\le e^T(t)P_1PP^TP_1e(t) + v^T(t)v(t)\end{aligned}$$

we have

$$\begin{aligned}\dot{V} \le\ & e^T(t)[N^TP_1 + P_1N + P_1PP^TP_1 + R_1]e(t) \\ & +2v^T(t)v(t) - e^T(t-\tau)R_1e(t-\tau) \\ & -v^T(t-\tau)v(t-\tau) + 2\delta||P|| \cdot ||P_1|| \cdot ||e(t)||\end{aligned}$$

From Lemma 2, we get the following inequality

$$\begin{aligned}\dot{V} \le\ & e^T(t)[N^TP_1 + P_1N + P_1PP^TP_1 + R_1]e(t) \\ & +2\delta||P|| \cdot ||P_1|| \cdot ||e(t)|| \\ & +v^T(t-\tau)(4K_1^TK_1 - I_n)v(t-\tau) \\ & +e^T(t-\tau)(4(K_2H)^T(K_2H) - R_1)e(t-\tau)\end{aligned}$$

From conditions (ii) and (iii), the above inequality becomes

$$\begin{aligned}\dot{V} &\le -e^T(t)Q_1e(t) + 2\delta||P|| \cdot ||P_1|| \cdot ||e(t)|| \\ &\le -\lambda_{\min}(Q_1)e^T(t)e(t) + 2\delta\lambda_{\max}(P_1)||P|| \cdot ||e(t)||\end{aligned}$$

in which $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ stand for the minimum and maximum eigenvalues of a matrix, respectively. Therefore if

$$||e(t)|| \ge \frac{2\delta\lambda_{\max}(P_1)||P||}{\lambda_{\min}(Q_1)} \tag{14}$$

we have $\dot{V} < 0$.

Summing up the above analysis, the error $e(t)$ can be bounded by the constant $\frac{2\delta\lambda_{\max}(P_1)||P||}{\lambda_{\min}(Q_1)}$ as the time tends to infinity. □

3.3 *Estimation of Uncertainty* η

From the proof of the above theorem, the error $e(t)$ and its derivative $\dot{e}(t)$ are bounded, which results in the boundedness of $Pv - P\eta$, thereby the term Pv can approximately estimate the uncertain term $P\eta$. Moreover, error dynamics (10) between system (1) and its observer (8) only includes the information on the original uncertainty η, which is important to reconstruct this uncertainty. Now we conclude that the original uncertainty η can be estimated by the iterative learning estimator (9) in the sense of least mean square. From the proof of Lemma 1 given by Boutayeb et al (2002), matrix P satisfies

$$P = \Psi\begin{pmatrix} I_n \\ 0 \end{pmatrix}, \text{where} \Psi = \begin{pmatrix} I_n + C^TC & C^TD \\ D^TC & D^TD \end{pmatrix}^{-1} \tag{15}$$

Thus, we have

$$\Psi^{-1}P = \begin{pmatrix} I_n \\ 0 \end{pmatrix} \tag{16}$$

From the above two equations, we obtain

$$\mathrm{Rank}(P) = \mathrm{Rank}\begin{pmatrix} I_n \\ 0 \end{pmatrix} \tag{17}$$

It means that matrix P is of full column rank. According to Theorem 1, error system (10) is practically stabilized. Denote $O(e) = Pv - P\eta$ where $O(e)$ is due to the bounded errors $e(t)$ and $\dot{e}(t)$. This implies that

$$v - \eta = (P^TP)^{-1}P^TO(e)$$

In other words, the original uncertainty η can be reconstructed by the iterative learning estimator v in the sense of least mean square, that is to say

$$\hat{\eta} \approx v \tag{18}$$

where $\hat{\eta}$ is the estimated uncertainty.

4. NUMERICAL SIMULATION

In this section, one simple two-dimensional linear system is illustrated to show the effectiveness of the proposed method. Its dynamics is described by

$$\begin{aligned}\dot{x} &= Ax + Bu + \eta \\ y &= Cx + Df\end{aligned} \tag{19}$$

where

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \text{ and } \eta = \begin{bmatrix} 0.02\sin(t) \\ 0.005\sin(t) \end{bmatrix}$$

In system (19), the sensor has an abrupt fault given by

$$f = \begin{cases} 0, & t \le 20\,\mathrm{s} \\ 0.01, & \text{otherwise} \end{cases} \tag{20}$$

Clearly, system (19) satisfies Assumption A2. Therefore, matrices P, Q, N and L in observer (8) can be selected as follows:

$$P = \begin{bmatrix} 0.3333 & 0 \\ 0 & 1.0000 \\ 0 & -0.5000 \end{bmatrix}, Q = \begin{bmatrix} 0.3333 & 0.3333 \\ 0 & 0 \\ -0.2500 & 0.2500 \end{bmatrix}$$

$$N = \begin{bmatrix} -14.1768 & 0.1089 & -0.4489 \\ 33.3408 & -5.9411 & -3.8822 \\ -17.1354 & -4.9410 & -13.8821 \end{bmatrix}$$

$$L = \begin{bmatrix} 2.0295 & 2.0295 \\ -6.0568 & -6.0568 \\ 3.1059 & 3.1059 \end{bmatrix}$$

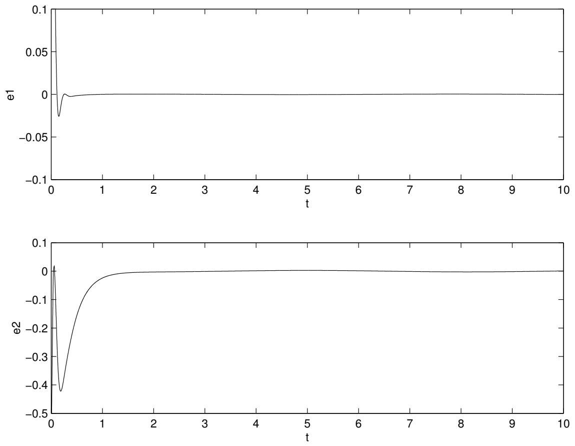

Fig. 1. The estimate errors on states

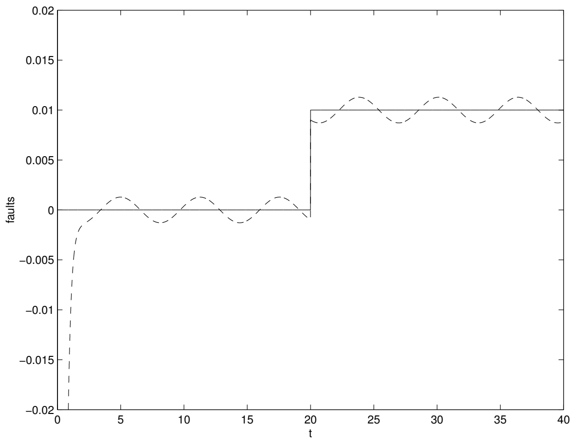

Fig. 2. The real (solid line) and reconstructed (dashed line) faults

The iterative learning estimator $v(t)$ is selected as (9), in which the time delay constant $\tau_1 = 0.02$,

$$K_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \quad \text{and} \quad K_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}$$

In this simulation the initial conditions for system (19) and its observer (8) are chosen as $(0.5\ -0.5)^T$ and $(-0.5\ 0.2\ -0.4)^T$, respectively. Simulation results are shown in figures 1 to 3. In figure 1, the estimation errors of states $x_i (i = 1, 2)$ approach the origin. Figure 2 shows the results of the real and reconstructed abrupt fault (20). The solid and dashed lines represent the real and reconstructed faults, respectively, which means that this abrupt fault can be reconstructed approximately. In figure 3, the real uncertainty η and the iterative learning control (9) are plotted. From these figures, we conclude that the combination of the generalized state observer and the iterative learning strategy can simultaneously reconstruct the state, the sensor fault and the uncertainty.

5. CONCLUSION

This paper considers the problem of simultaneous reconstruction of the state, the sensor fault and the uncertainty in linear systems. Borrowing from the idea in chaotic masking, this paper proposes a new scheme to resolve the above problem. The combination of the generalized state observer proposed in chaotic masking and the iterative learning strategy can simultaneously reconstruct the state, the sensor fault and the uncertainty. Theoretical analysis and numerical simulations verify the effectiveness of this scheme.

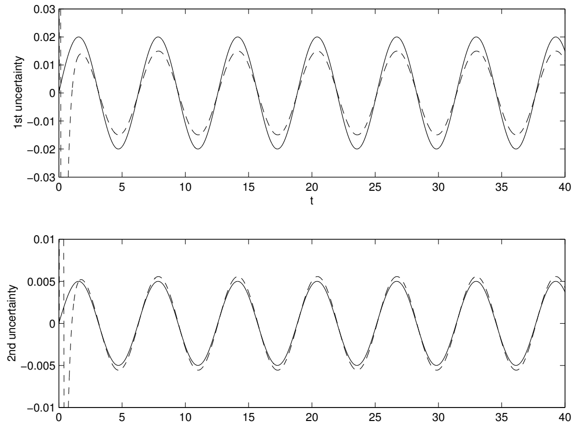

Fig. 3. The estimate errors on states

6. ACKNOWLEDGEMENT

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