CALCULATING DYNAMIC DISTURBANCE REJECTION MEASURES

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Abstract: A number of indicators for the determination of the disturbance rejection capability of a system have been proposed in the literature in the last two decades. These tools are useful for evaluating early stage process designs and screening regulatory control structures, since they are based on minimal modelling requirements. In previous publications (Hovd and Braatz, 2000; Hovd *et al.*, 2003; Kookos and Perkins, 2003), it has been shown how to evaluate at steady state the disturbance rejection measures proposed by Skogestad and Wolff (1992). If acceptable values for these disturbance rejection measures are not achieved, no controller can achieve satisfactory control. In this paper, it is shown how to obtain upper and lower bounds for these disturbance rejection measures also as a function of frequency. The upper and lower bounds can be made arbitrarily accurate at the expense of increasing the size of the optimization problems involved.

Keywords: Controllability measures, disturbances,dynamics

1. INTRODUCTION

The importance of designing processes which can be acceptably controlled is widely recognized, and has been studied by many researchers (Fisher *et al.*, 1988; Hovd and Skogestad, 1992; Straub and Grossmann, 1993; Braatz *et al.*, 1996; Heath *et al.*, 2000; Ma *et al.*, 2002). A significant consideration is whether it is possible to reduce the effect of disturbances to an acceptable level using the available manipulated variables. Three relevant questions in this context are:

(1) What is the minimum output error that is obtainable for the worst possible combination of disturbances with the optimal use of the manipulated variables?
(2) What is the minimum required magnitude for the manipulated variables to obtain an acceptable output error for the worst possible combination of disturbances?
(3) What is the smallest disturbance for which the minimum output error is of the maximum acceptable magnitude, provided the manipulated variables are used optimally?

While the mathematical formulation of each of these questions in terms of optimization problems has been provided (Wolff, 1994; Skogestad and Wolff, 1992), no explanation was given on how to solve the resulting optimization problems. The calculation of these measures is non-trivial even for linear plants, and only recently have meth-

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ods for calculating these measures been proposed. Hovd and Braatz (2000) present a non-convex optimization formulation. Although global optimization software is required, the resulting optimization problem can be solved in acceptable time for problems of moderate size (i.e. up to about 12×12). An alternative approach is presented by Kookos and Perkins (2003), who present mixed-integer linear programming (MILP) formulations for the same problems.

Pervious publications have shown how to calculate the disturbance rejection measures at steady state only. This paper presents how to obtain upper and lower bounds for the disturbance rejection measures, where the bounds can be evaluated also at non-zero frequencies. In order to increase the accuracy of the bounds, the size of the corresponding optimization problems has to increase. The approach chosen here is therefore an extension to the approach of Kookos and Perkins (Kookos and Perkins, 2003), since the required computational time with their approach is expected to scale more favorably with problem size. The proposed formulation is presented in detail for the minimum output error problem, and the modifications needed for the two other problems are explained briefly.

We assume that manipulated variables, disturbances and outputs are related through a linear transfer function model

$$y(s) = G(s)u(s) + G_d(s)d(s) \tag{1}$$

and assume that the model has been scaled such that manipulated variables and disturbances are less than one in magnitude at all frequencies ω (where $s = j\omega$), whereas controlled variables are scaled such that the maximum acceptable control error is of magnitude one.

2. THE MINIMUM OUTPUT ERROR PROBLEM

Question 1 in the Introduction is essentially a statement of the minimum output error problem. Stated in mathematical terms, this becomes

$$\max_{\|d\|_\infty \leq 1} \min_{\|u\|_\infty \leq 1} \|y\|_\infty \tag{2}$$

subject to fulfilling the model equation (1). The main problem which has prevented the use of previously published solution approaches from at non-zero frequencies, is the fact that the infinity norm of a vector depends non-linearly on the real and imaginary components of a vector element.

2.1 Linear approximations to the infinity norm

The infinity norm of a vector is simply the magnitude of the largest vector element. The infinity norm is therefore convenient for expressing constraints on disturbances, manipulated variables, and controlled outputs. It is well known that the magnitude of a vector element w_k is

$$|w_k| = \sqrt{w_k^* w_k} = \sqrt{(w_k^R)^2 + (w_k^I)^2}$$

where superscripts R and I denote the real and complex components, respectively, of vector element w_k. Thus, the magnitude, and hence also the infinity norm, depends non-linearly on the magnitude of the real and complex components of the vector elements. In the following, two approximation to constraints in the infinity norm will be used, which are formulated as linear constraints on the real and imaginary components of the vector elements. One approximation will allow vectors of slightly too large infinity norm than the exact constraint, whereas the other approximation will disallow some vectors of acceptable infinity norm. For a vector w to have $\|w\|_\infty \leq r$, the real and imaginary components of all vector elements must lie on a disc of radius r in the complex plane. This disc of radius r is approximated using n linear constraints. For simplicity, we assume $n = 2^k$, $k \geq 2$, and select n uniformly distributed points along the perimeter of the disk. Thus, the coordinates for point i will be

$$(a_i, b_i) = (r \cos v_i, r \sin v_i) \tag{3}$$

where $v_i = \frac{2\pi i}{n}$. For convenience, the point $(r, 0)$ may be given either index 0 or index n in the sequel. The disc of radius r may be approximated by a polygon described by the tangent lines to the disc at all points (a_i, b_i). A vector which has one or more elements on polygon edges will clearly have an infinity norm no smaller than r, i.e., this polygon allows some vectors of infinity norm larger than r.

Alternatively, a polygon may be constructed from straight lines passing through adjacent points on the disc. A vector which has one or more elements on the edges of this polygon will have an infinity norm no larger than r, and thus this polygon will exclude some vectors of acceptable infinity norm. This idea is illustrated in Fig. 1 for the case when $n = 8$. The polygon described by the solid lines allows some vectors of infinity norm larger than r, whereas the polygon resulting from the dashed lines excludes some vectors of acceptable infinity norm. The dashed line passing through points i and $i-1$ on the edge of the disc in Fig. 1 is described by

$$\begin{aligned}(a_i - a_{i-1})w_i^I &= \\ (b_i - b_{i-1})w_i^R &+ (a_i b_{i-1} - b_i a_{i-1})\end{aligned} \tag{4}$$

The polygon is described in terms of inequalities simply by substituting $\leq$ for the equality sign in (4) for $0 < i \leq 2^{k-1}$, and substituting $\geq$ for

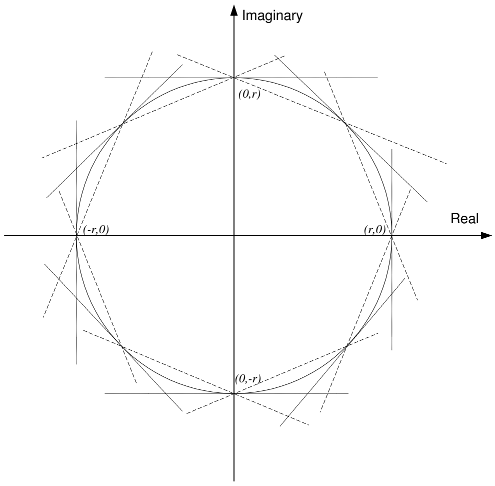

Fig. 1. Approximating the disc of radius r by polygons.

$2^{k-1} < i \leq 2^k$. Thus, for each vector element we get n constraints relating the real and imaginary components of each element. The coefficients describing these constraints may be collected in column vectors C^I, C^R, and C^0, whose dimension and element values will depend on the number of points n along the circle of radius r. Normalizing both sides of Eq. (4) by dividing with r all terms become independent of r except term $(a_i b_{i-1} - b_i a_{i-1})/r$ which depends linearly on r. For a m-dimensional column vector w, split into a real part w^R and imaginary part w^I, the constraint that $\|w\|_\infty \leq r$ is therefore conservatively be approximated by the set of constraints

$$I_m \otimes C^I w^I + I_m \otimes C^R w^R - \underline{1}_m \otimes C^0 r \leq 0 \quad (5)$$

where $\otimes$ denotes the Kronecker product, I_m is the m by m identity matrix, and $\underline{1}_m$ is an m-dimensional column vector of ones. Note that the linear dependency on r has explicitly been factored out of the vector C^0, since the value of r will typically be one of the free variables in our optimization problems.

A solid line in Fig. 1 tangent to the disc of radius r at point i is described by

$$\sin v_i w_i^I + \cos v_i w_i^R - r = 0 \quad (6)$$

A non-conservative approximation to the constraint $\|w\|_\infty \leq r$ can therefore be expressed as

$$I_m \otimes N^I w^I + I_m \otimes N^R w^R - \underline{1}_m \otimes N^0 r \leq 0 \quad (7)$$

where the elements of N^I, N^R and N^0 follow from Eq. (6). The accuracy of the non-conservative and conservative approximations to the constraint in the infinity norm can be obtained by studying the areas covered by the corresponding polygons compared to the circle that would represent the exact infinity norm constraint. This is shown in Table 1. It can be seen that even with eight points, giving eight linear constraints per vector element, the approximations can be useful in many applications.

Table 1. Areas covered by the approximations to a disc of unit radius.

Number of points	8	16	32	64
Conservative	2.828	3.061	3.107	3.137
Exact	π	π	π	π
Non-conservative	3.314	3.183	3.152	3.144

Clearly, using the conservative approximation for $\|u\|_\infty$ and $\|y\|_\infty$, and the non-conservative approximation for $\|d\|_\infty$ in Eq. 2, will give an upper bound on the true solution. Conversely, using the non-conservative approximation for $\|u\|_\infty$ and $\|y\|_\infty$, and the conservative approximation for $\|d\|_\infty$ will give a lower bound. This paper will describe the calculation the upper bound, calculation of the lower bound follows by trivial modifications.

2.2 The inner minimization problem

This subsection will describe the re-formulation of the inner minimization problem in Eq.2, using the conservative approximation to both $\|u\|_\infty$ and $\|y\|_\infty$. The notation $\tilde{C}^I_{n_y} = I_{n_y} \otimes C^I$, $\tilde{C}^I_{n_u} = I_{n_u} \otimes C^I$, etc., is used, where n_y and n_u are the dimensions of the output and manipulated variable vectors, respectively. The vectors u, y, and d are decomposed into real and imaginary components, denoted by superscripts R and I, and the transfer functions G and G_d are similarly decomposed. We thus get

$$\min_{\|u\|_\infty} \|y\|_\infty \;\text{ s.t. }\; y = Gu + G_d d \quad (8)$$

$$\approx \min_{u^R, u^I, r_y} r_y \quad (9)$$

$$\text{s.t. } \tilde{C}^I_{n_y} \left\{ \left[G^I \; G^R\right] \begin{bmatrix} u^R \\ u^I \end{bmatrix} + \left[G^I_d \; G^R_d\right] \begin{bmatrix} d^R \\ d^I \end{bmatrix} \right\}$$
$$+ \tilde{C}^R_{n_y} \left\{ \left[G^R \; -G^I\right] \begin{bmatrix} u^R \\ u^I \end{bmatrix} \right.$$
$$\left. + \left[G^R_d \; -G^I_d\right] \begin{bmatrix} d^R \\ d^I \end{bmatrix} \right\} - \tilde{C}^0_{n_y} r_y \leq 0$$
$$\left[\tilde{C}^R_{n_u} \; \tilde{C}^I_{n_u}\right] \begin{bmatrix} u^R \\ u^I \end{bmatrix} - \tilde{C}^0_{n_u} \cdot 1 \leq 0$$

In order to achieve a more compact notation, we define

$$\tilde{C}^{Ru}_{n_y} = \tilde{C}^R_{n_y} \left[G^R \; -G^I\right] \quad (10)$$
$$\tilde{C}^{Rd}_{n_y} = \tilde{C}^R_{n_y} \left[G^R_d \; -G^I_d\right] \quad (11)$$
$$\tilde{C}^{Iu}_{n_y} = \tilde{C}^I_{n_y} \left[G^I \; G^R\right] \quad (12)$$
$$\tilde{C}^{Id}_{n_y} = \tilde{C}^I_{n_y} \left[G^I_d \; G^R_d\right] \quad (13)$$
$$\tilde{u} = \begin{bmatrix} u^R \\ u^I \end{bmatrix}; \quad \tilde{d} = \begin{bmatrix} d^R \\ d^I \end{bmatrix} \quad (14)$$

The Lagrangian function of the approximation to the inner minimization problem in Eq. (8) is then

$$
\begin{aligned}
L = r_y + \lambda^T \Big\{ & \left(\tilde{C}^{Ru}_{n_y} + \tilde{C}^{Iu}_{n_y}\right)\tilde{u} \\
& + \left(\tilde{C}^{Rd}_{n_y} + \tilde{C}^{Id}_{n_y}\right)\tilde{d} - \tilde{C}^{0}_{n_y} r_y \Big\} \\
& + \nu^T \left\{ \left[\tilde{C}^{R}_{n_u}\ \tilde{C}^{I}_{n_u}\right]\tilde{u} - \tilde{C}^{0}_{n_u}\cdot 1 \right\}
\end{aligned} \tag{15}
$$

where λ and ν are non-negative Lagrange multipliers. The KKT optimality conditions for Eq. (9) are then (Luenberger, 1984)

$$
\frac{\partial L}{\partial r_y} = 1 - \left(\tilde{C}^{0}_{n_y}\right)^T \lambda = 0 \tag{16}
$$

$$
\frac{\partial L}{\partial \tilde{u}} = \left(\tilde{C}^{Ru}_{n_y} + \tilde{C}^{Iu}_{n_y}\right)^T \lambda + \left[\tilde{C}^{Ru}_{n_u}\ \tilde{C}^{Iu}_{n_u}\right]^T \nu = 0 \tag{17}
$$

$$
\begin{aligned}
\lambda \times \Big\{ & \left(\tilde{C}^{Ru}_{n_y} + \tilde{C}^{Iu}_{n_y}\right)\tilde{u} \\
& + \left(\tilde{C}^{Rd}_{n_y} + \tilde{C}^{Id}_{n_y}\right)\tilde{d} - \tilde{C}^{0}_{n_y} r_y \Big\} = 0
\end{aligned} \tag{18}
$$

$$
\nu \times \left\{ \left[\tilde{C}^{R}_{n_u}\ \tilde{C}^{I}_{n_u}\right]\tilde{u} - \tilde{C}^{0}_{n_u}\cdot 1 \right\} = 0 \tag{19}
$$

where the symbol $\times$ denotes element-by element multiplication. The complementarity conditions, Eqs. (18,19) complicate the solution of the problem since they destroy the linearity and convexity of the problem. Following (Kookos and Perkins, 2003), we will use the method proposed by (Fortuny-Amat and B., 1981) to transform the KKT conditions to a mixed-integer linear problem. For illustration, consider the condition

$$
\gamma x = 0 \tag{20}
$$

Introduce the integer variable Γ, defined such that $\Gamma = 1$ if $x = 0$, and $\Gamma = 0$ otherwise. Then the condition in Eq. (20) is replaced by

$$
\begin{aligned}
-(1-\Gamma)B_m \le x \le (1-\Gamma)B_m \\
\gamma \le \Gamma B_m
\end{aligned} \tag{21}
$$

where B_m is a sufficiently large number. When $\Gamma = 1$ then $x = 0$, while when $\Gamma = 0$ then $\gamma = 0$, and as a result the condition in Eq. (20) is always satisfied.

It has been found that when the same points along the unit disc are used to define the linear constraints approximating the infinity-norm constraints for outputs and manipulated variables, the inner minimization problem becomes non-unique, leading to numerical problems in the solution. Such problems are removed by using different points along the unit disc used to define the linear constraints for outputs and manipulated variables.

2.3 An approximate solution to the minimum output error problem

Transforming the KKT conditions in Eqs. (16 - 19) for the inner minimization problem using binary variables, we can formulate the approximate solution for the overall problem. In the following, capital Greek letters denote binary variables. In the description for the inner minimization problem above, we have used the conservative approximation to the infinity norm for u and y. A non-conservative approximation for $\|d\|_\infty$ is therefore used, in order to calculate an upper bound to the minimum output error. Clearly, using a non-conservative approximation for $\|u\|_\infty$ and $\|y\|_\infty$, and a conservative approximation for $\|d\|_\infty$, would result in a lower bound on the minimum output error.

$$
\begin{aligned}
& \max_{\tilde{d}} r_y \\
& \text{s.t. } \left[\tilde{N}^{R}_{n_d}\ \tilde{N}^{I}_{n_d}\right]\tilde{d} - \tilde{N}^{0}_{n_d} \le 0 \\
& \left(\tilde{C}^{0}_{n_y}\right)^T \lambda = 1 \\
& \left(\tilde{C}^{Ru}_{n_y} + \tilde{C}^{Iu}_{n_y}\right)^T \lambda + \left[\tilde{C}^{Ru}_{n_u}\ \tilde{C}^{Iu}_{n_u}\right]^T \nu = 0 \\
& \left(\tilde{C}^{Ru}_{n_y} + \tilde{C}^{Iu}_{n_y}\right)\tilde{u} + \left(\tilde{C}^{Rd}_{n_y} + \tilde{C}^{Id}_{n_y}\right)\tilde{d} - \tilde{C}^{0}_{n_y} r_y \le 0 \\
& -\left(\tilde{C}^{Ru}_{n_y} + \tilde{C}^{Iu}_{n_y}\right)\tilde{u} - \left(\tilde{C}^{Rd}_{n_y} + \tilde{C}^{Id}_{n_y}\right)\tilde{d} \\
& \qquad + \tilde{C}^{0}_{n_y} r_y \le (\underline{1} - \Theta)B_m \\
& \lambda \le \Theta B_m \\
& \left[\tilde{C}^{R}_{n_u}\ \tilde{C}^{I}_{n_u}\right]\tilde{u} - \tilde{C}^{0}_{n_u}\cdot 1 \le 0 \\
& -\left[\tilde{C}^{R}_{n_u}\ \tilde{C}^{I}_{n_u}\right]\tilde{u} + \tilde{C}^{0}_{n_u}\cdot 1 \le (\underline{1} - \Phi)B_m \\
& \nu \le \Phi B_m;\ \lambda \ge 0;\ \nu \ge 0 \\
& \Theta \in \{0,1\}^{n_y \cdot n};\ \ \Phi \in \{0,1\}^{n_u \cdot n}
\end{aligned} \tag{22}
$$

3. OTHER DISTURBANCE REJECTION MEASURES

In this section, we will briefly explain how to formulate optimization problems which allow obtaining quantitative answers to Questions (2) and (3) in the Introduction.

3.1 The required input magnitude

The required input magnitude problem is stated in Question (2) in the Introduction, and is formulated mathematically as

$$
\max_{\|d\|_\infty \le 1}\ \min_{\|y\|_\infty \le 1} \|u\|_\infty \tag{23}
$$

It is straight forward to reformulate this using linear constraints to approximate the infinity norms, in the same way as for the minimum output error

Table 2. Variables in the FCC example

y	Measurements
y_1	Riser outlet temperature
y_2	Regenerator cyclone temperature
y_3	Regenerator dense bed temperature
d	Disturbances
d_1	Feed temperature
d_2	Air temperature
d_3	Feed flowrate
u	Manipulated variables
u_1	Air flowrate
u_2	Catalyst circulation rate
u_3	Feed composition

problem above. Note however that if $G(j\omega)$ does not have full row rank, the inner minimization problem (minimizing $\|u\|_\infty$ for a fixed d) need not have any feasible solution. The reader is referred to the discussion in Hovd *et al.* (2003) for a more detailed discussion.

3.2 Acceptable disturbance magnitude

The acceptable disturbance magnitude problem corresponds to Question (3) in the Introduction. This is stated mathematically as

$$\max \|d\|_\infty \text{ s.}t.\ \epsilon(d) = 0 \tag{24}$$

where $\epsilon(d)$ is defined as

$$\begin{aligned} \epsilon(d) &= \max_d \min_u \mu \\ \|y\|_\infty &\leq 1 + \mu \\ \|u\|_\infty &\leq 1 + \mu \end{aligned} \tag{25}$$

This problem can be reformulated using the KKT optimality conditions, see (Kookos and Perkins, 2003) for more details. Approximating the infinity norms with linear constraints then results in a problem similar to the minimum output error problem. Alternatively, the acceptable disturbance magnitude problem may be solved by iteratively scaling the disturbance magnitude in the minimum output error problem until a minimum output error of 1 is obtained.

4. EXAMPLE

Here a Fluid Catalytic Cracker (FCC) described in (Wolff *et al.*, 1992) is considered. The FCC has three controlled variables, three manipulated variables, and three disturbances. The variables considered are described in Table 2 [2]. The scaled linearized model used for this example can be found in (Wolff, 1994). The minimum output error at steady state has previously been found to be zero (Hovd and Braatz, 2000; Kookos and Perkins,

Table 3. Bounds for the minimum output error for the example at $\omega = 0.1$ rad/s.

Number of points	16	24	32
Conservative approx.	0.3033	0.2237	0.1963
CPU time (s)	57	502	2028
Non-conservative approx.	0.1611	0.1613	0.1614
CPU time (s)	166	1180	1015

2003). Calculating upper and lower bounds for the frequency-dependent minimum output error we get the results in 2. These results are obtained using 16 linear constraints to approximate the infinity-norm constraints for disturbances, outputs and manipulated variables. This case study

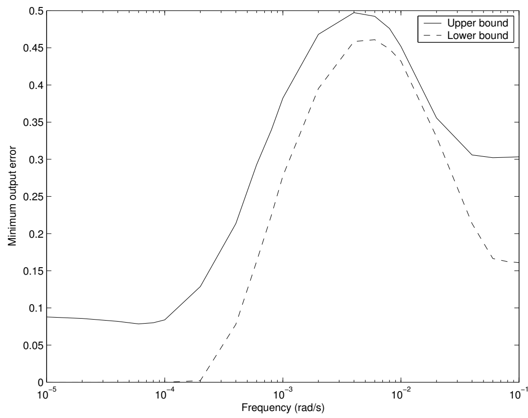

Fig. 2. The minimum output error for the FCC example.

was solved on an Intel P4 2.66 GHz/512Mb computer using the GAMS interface to CPLEX MILP solver (Brooke *et al.*, 1998).From 2 it is clear that the steady state minimum output error is too optimistic, since the peak value over all frequencies for the lower bound of the minimum output error is above 0.45 (compared to 0.0 at steady state). Nevertheless, the results are consistent with the previous steady state result, since the lower bound for the minimum output error is found to be 0.0 at low frequencies.

Table 3 illustrates how the accuracy of the approximations depend on the number of points along the radius-r disc used to approximate the infinity-norm constraints $\|\cdot\|_\infty \leq r$. It is clear, as expected, that the difference between the upper and lower bound is reduced by using more points (i.e., more linear constraints) in the approximation. For this particular example, it turns out that it is the upper bound that is more sensitive to the number of points used.

5. DISCUSSION AND CONCLUSIONS

This paper shows how to calculate upper and lower bounds for the disturbance rejection measures proposed by Skogestad and Wolff (1992) for

[2] Actually, u_3 is the rate constant for coke formation for the feed, which is a direct function of feed composition.

non-zero frequencies. The example illustrates that using the steady-state values only for the disturbance rejection measures may be misleading.

Upper and lower bounds for the disturbance rejection measures are obtained, instead of their exact values. This is because linear constraints are used to approximate the original infinity-norm constraints for disturbances, outputs and manipulated variables. Increasing the number of linear constraints used in these approximations gives more accurate bounds, at the cost of having to solve a larger and more complex optimization problem. The resulting optimization problems are of the MILP type, and the increase in computational complexity with increased number of linear constraints results from an increase in the number of integer-valued variables.

In principle, the problem sizes that can be solved using the methods described in this paper is only limited by available computational power. However, the GAMS implementation of the algorithm proposed in this paper, when applied to the FCC case study, was used excessive computational time (more than an hour) when more than 32 points were used in approximating the infinity norm. A huge number of alternative integer solutions need to be explored in this case (more than $2^{192} \approx 6.3 \cdot 10^{57}$) and a refinement of the proposed formulation is needed in order to extend its applicability to large scale problems. Nevertheless, the exact value of the objective function is seldom important since the upper and lower bounds are enough to help the control engineer to judge the closed loop sensitivity to disturbances. Using 8 or 16 points is often enough to achieve this goal and at the same time generating a MILP problem that can be solved in a few minutes even for large problems. Future work will aim at reducing the computational load, although many problems of industrial relevance can be studied with the present implementation.

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