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SUPER MECHANO-SYSTEMS: FUSION OF CONTROL AND MECHANISM

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Abstract: Super Mechano-System is the name of the research project at Tokyo Institute of Technology sponsored by the Japanese Ministry of Education, Culture, Sports, Science and Technology. The aim is creating a New Mechanical Systems with self-organizing capabilities of its structure and functions adapting to the environment by the fusion of the control and mechanism. The system may have hyper redundant components with autonomous intelligence or several different functions, some of which integrate to have the most appropriate system for the objective in the varying environment by the fusion of control and mechanisms. This paper presents an aspect of the project relating to the control for the integration and its application to the control of the pendulum.

Keywords: Super Mechano-Systems, Pendulum Control, Projection Method, Nonlinear Control, Artificial Gravity, Virtual Gravity, Biped Walking

1. INTRODUCTION

Super Mechano-System(SMS) is, we define, a New Mechanical Systems with self-organizing capabilities of its structure and functions. The Grand-in Aid COE research project has started from April 1997 at Tokyo Institute of Technology aiming the Creation of New Functionality by the Fusion of Control and Mechanisms supported by Ministry of Education, Culture, Sports, Science and Technology (Furuta and Xu, 2001).

For the modeling of SMS which can self-organize its structure, we have to consider the system consisting of subsystems with variable constraints. Modeling the components and constraints should be described individually in this research. The project consists of several topics like Cybermechanism, Neo-Function, Concurrent Design of Mechanism and Controller. The key idea of the SMS is to design autonomously not only the objective-configured mechanisms but also the most appropriate controller. This concept is completely different from the conventional controlled mechanical systems, where controller is designed for the given system. Thus conventional approach is the sequential design of the components such structure of mechanisms, actuator and controllers.

In SMS, not only the structure of controllers but also that of systems aims to be designed concurrently adapting to the varying environment. These variable structure system can be treated under variable constraints. The aim of this project is thus to seek the fusion of mechanism and control to attain high performance of systems as a whole. One of the results about the concurrent design of the disk head has been developed by T. Iwasaki (Iwasaki, 1999) and S. Hara (Hara *et al.*, 1999), which introduce an interesting idea of the integrated system designs.

2. ROBOTICS OF SMS PROJECT

Since April 2000, Prof. Shigeo Hirose, Tokyo Institute of Technology plays the role of the leader succeeding the auhtor after his move to Tokyo Denki University. Under the leadership of the new leader many interesting robots have been developed. Many of the results are presented at TITech COE/SMS Workshops. One of the unique robots is Hirose's roller walker which walks on the rough terrain but roller skates on the flat plane shown in Figure 1 which adaptively change its structure and functions (Endo and Hirose, 1999). Similar type of the idea is used for developing mother and children type robot called Super Mechano-Colony where all wheels are independently movable (Hirose *et al.*, 2000). This is shown in Figure 2. The control of the snake like robots (Prautsch and Mita, 1999)(Matsuno and Mogi, 2000)(Date *et al.*, 2001) which firstly studied by S. Hirose has been extensively studied (Figure 3). As an example to study cooperatively by control and mechanical researchers, the acrobot type robot has been designed by S. Hirose and various control for the motion are studied (Figure 4).

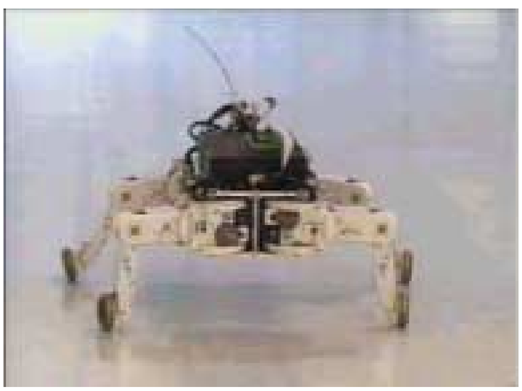

Fig. 1. Roller Walker

Fig. 2. Super Mechano-Colony Rover

The jump is studied by M. Sampei and others (Miyazaki *et al.*, 2000) and swinging is studied by M. Yamakita and others (Michitsuji *et al.*, 2001). The running and jumping are also important subject in the project and T. Mita (Ikeda *et al.*, 1999) also present the idea for designing the running robot (Figure 5). Thus the cooperation of mechanical and control engineers could develop many interesting.

Fig. 3. Snake Robot

Fig. 4. Acrobat Robot jumping to iron bar

Fig. 5. Running studied by T. Mita

3. MODELING OF CONSTRAINT SYSTEMS

This paper is only to describe a part of the project relating to the pendulum and its applications. The

Fig. 6. Robot changeable its configuration developed by Omata

author has studied the control of multiple pendulum for swing-up of single and double pendulum (Mori *et al.*, 1976)(Furuta *et al.*, 1993)(Astrom and Furuta, 1996)(Yamakita and Furuta, 1999), stabilization of hinge control pendulum said acrobot (Furuta *et al.*, 1984) and spherical pendulum (Hoshino *et al.*, 2000). The some examples of control of pendulum such as photo of stabilization of the triple spherical pendulum and the transfer of a stabilized pendulum between manipulators are shown in figures.

Fig. 7. Triple Spherical Pendulum

The modeling of the variable constraint system is considered with the constraint force and the dynamics of the individual components individually, and this internal force should be considered in the control, which can control the constraint force on the constraint structures.

One of modeling approaches for the variable constrain system is the projection method (Blajer, 1992) and it is demonstrated to model the rotating type pendulum called Furuta Pendulum (Furuta *et al.*, 1991)(Furuta *et al.*, 1993)(Yamakita and Furuta, 1999).

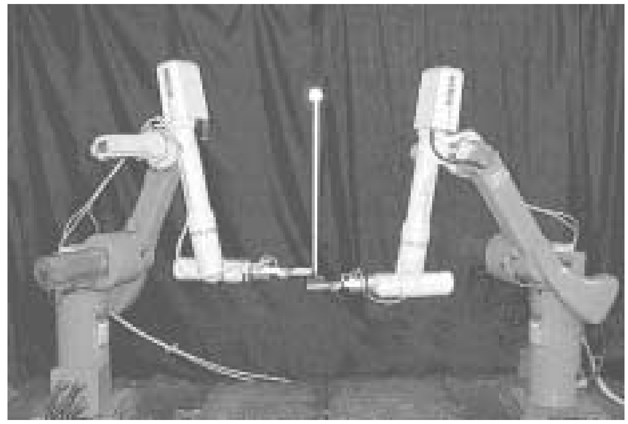

Fig. 8. Transfer of Pendulum

This system with constraints has less number of actuators than the degrees of freedom, and is the under-actuated system. The constraint forces work inside the mechanical structure, but they have not been paid much attention in the control.

This paper studies the control of pendulum and walking system taking constraint force into consideration and discusses the following items:

1. (1) The modeling of the variable constraint system by the projection method.
2. (2) The control of the pendulum by the nonlinear control taking the constraint force into consideration.
3. (3) The adaptive control of the biped walking is discussed.

3.1 *Modeling of Furuta Pendulum*

In this paper, projection method (Blajer, 1992) (Arczewski and Blajer, 1996) is applied to obtain dynamical equations of Furuta Pendulum.

To illustrate the idea of modeling for constrained systems by projection method, let us consider the modeling of Furuta pendulum which is shown in Fig. 9 .

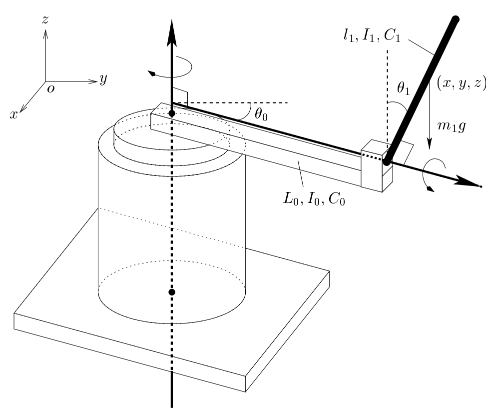

Fig. 9. Illustration of Variables (Furuta pendulum)

First, the pendulum is assumed to be able to move freely in space, in other words, Furuta pendulum is treated as a unconstrained system. With the help of Euler-Lagrange equation, it is easy to get the differential equation to describe the unconstrained augmented system

$$\dot{q}_a = v \tag{1}$$
$$M_a \dot{v} = h_a \tag{2}$$

where q_a denotes the augmented state

$$q_a = [\theta_0 \ x \ y \ z \ \theta_1]^T$$

$$M_a = \begin{bmatrix} I_0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & 0 & I_1 \end{bmatrix} \tag{3}$$

$$h_a = \begin{bmatrix} \tau_1 - C_0\dot{\theta}_0 \\ 0 \\ 0 \\ -m_1 g \\ -C_1\dot{\theta}_1 \end{bmatrix} \tag{4}$$

The parameters of the model are chosen as

I_0	1.75e-2(kgm^2)	inertia of arm around shaft
I_1	1.98e-4(kgm^2)	inertia of pendulum
L_0	0.215(m)	arm length
l_1	0.113(m)	length from pendulum-pivot to c.g.
m_1	5.38e-2(kg)	weight of pendulum
C_0	0.118(Nms)	friction of arm
C_1	8.3e-5(Nms)	friction of pendulum
g	9.8(m/s^2)	acceleration of gravity

Now, consider the pendulum attached to the arm, then not all coordinates are independent, and constraints are expressed as

$$\begin{aligned} x &= L_0 \sin\theta_0 + l_1 \sin\theta_1 \cos\theta_0 \\ y &= L_0 \cos\theta_0 - l_1 \sin\theta_1 \sin\theta_0 \\ z &= l_1 \cos\theta_1 \end{aligned}$$

The derivative of the above equation gives

$$\begin{aligned} \dot{x} &= (L_0\cos\theta_0 - l_1\sin\theta_1\sin\theta_0)\dot{\theta}_0 + l_1\cos\theta_1\cos\theta_0\dot{\theta}_1 \\ \dot{y} &= (-L_0\sin\theta_0 - l_1\sin\theta_1\cos\theta_0)\dot{\theta}_0 - l_1\cos\theta_1\sin\theta_0\dot{\theta}_1 \\ \dot{z} &= -l_1\sin\theta_1\dot{\theta}_1 \end{aligned}$$

The above constraint is written as

$$C_a v = 0$$

with

$$C_a = \begin{bmatrix} L_0c_0 - l_1s_1s_0 & -1 & 0 & 0 & l_1c_1c_0 \\ -L_0s_0 - l_1s_1c_0 & 0 & -1 & 0 & -l_1c_1s_0 \\ 0 & 0 & 0 & -1 & -l_1s_1 \end{bmatrix} \tag{5}$$

where

$$s_i = \sin\theta_i \quad c_i = \cos\theta_i \quad i = 0, 1$$

D_a and the reduced state q can be rewritten as

$$q = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$$

$$D_a = \begin{bmatrix} 1 & 0 \\ L_0c_0 - l_1s_1s_0 & l_1c_1c_0 \\ -L_0s_0 - l_1s_1c_0 & -l_1c_1s_0 \\ 0 & -l_1s_1 \\ 0 & 1 \end{bmatrix}$$

Since

$$v = D_a\dot{q}$$

$C_a, \ D_a$ satisfy

$$C_a D_a = 0$$

Then this Furuta pendulum is described as a constrained system by using the force λ written by

$$M_a\dot{v} = h_a + C_a^T\lambda_a \tag{6}$$

Multiplying D^T from the left, the following equation is derived.

$$D_a^T M_a(D_a\ddot{q} + \dot{D}_a\dot{q}) = D_a^T h_a \tag{7}$$

where the force for the constraint is

$$\lambda_a = (C_a M_a^{-1} C_a^T)^{-1}(C_a\dot{v} - C_a M_a^{-1} h_a) \tag{8}$$

The coefficient matrices of equation (7) are written as

$$D_a^T M_a D_a = \begin{bmatrix} I_0 + m_1(L_0^2 + l_1^2 s_1^2) & m_1 l_1 L_0 c_1 \\ m_1 l_1 L_0 c_1 & I_1 + m_1 l_1^2 \end{bmatrix}$$

$$D_a^T M_a \dot{D}_a = \begin{bmatrix} \frac{1}{2}m_1l_1^2\sin 2\theta_1\dot{\theta}_1 & \frac{1}{2}m_1l_1^2\sin 2\theta_1\dot{\theta}_0 - m_1l_1L_0\sin\theta_1\dot{\theta}_1 \\ \frac{1}{2}m_1l_1^2\sin 2\theta_1\dot{\theta}_0 & 0 \end{bmatrix}$$

$$D_a^T h_a = \begin{bmatrix} \tau_1 - C_0\dot{\theta}_0 \\ m_1gl_1\sin\theta_1 - C_1\dot{\theta}_1 \end{bmatrix}$$

Equation (7) is rewritten as

$$M\ddot{q} + h(q, \dot{q}) = \tau \tag{9}$$

where

$$\tau = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix}$$

4. CONTROL OF PENDULUM

4.1 *Artificial Gravity Approach*

The pendulum is modeled by the equation. The control to swing up of the pendulum has been studied by several researchers after (Mori *et al.*, 1976). Several approaches have been studied. K. J. Astrom (Astrom and Furuta, 1996)(Wiklund *et al.*, 1993) and others (Chung and Hauser, 1995) proposed to use the energy for the design of the control. The approach is to make the total energy is equal to the potential energy at the upright position. Some modification of the approach has been studied (Acosta *et al.*, 2001)(Fantoni and Lozano, 2001). M. Saeki (Saeki, 1993) used the feedback linearization and the saturating control at the singular state. The approach is still effective in practice. The other approaches is to use the idea of the artificial gravity. The term of "virtual gravity" was used by M. W. Spong (Spong, 1999) for the biped walking. The author used a similar idea of artificial gravity for the swing up the pendulum. In this approach, the control law is designed so that the controlled system is matched to the one replacing the acceleration of the gravity g by $-g$. This means that the potential energy is minimized at the up-right position. The shaping of the potential function equivalent to potential energy approach was proposed by M. Takegaki and S. Arimoto (Takegaki and Arimoto, 1981). The similar approach has been used in (Chung and Hauser, 1995). In order to control the arm position by the artificial gravity approach, we have to tilt the motor base around the y axis about γ, then h of (4) is replaced by

$$h_\gamma = \begin{bmatrix} \tau - C_0\dot{\theta}_0 \\ m_1 g \sin\gamma \\ 0 \\ -m_1 g \cos\gamma \\ -C_1\dot{\theta}_1 \end{bmatrix} \tag{10}$$

and $D^T h_\gamma$ is written by

$$D^T h_\gamma = \begin{bmatrix} \tau - C_0\dot{\theta}_0 + m_1 g \sin\gamma(L_0 c_0 - l_1 s_1 s_0) \\ l_1 c_1 c_0 m_1 g \sin\gamma + l_1 s_1 m_1 g \cos\gamma - C_1\dot{\theta}_1 \end{bmatrix} \tag{11}$$

The above operation is corresponding the direction of the gravity not to the upward but to given direction tilted the angle to γ around y axis. The control law is designed that the acceleration of the arm is firstly chosen so that the model is matched to the one replacing the acceleration of the gravity g by $-g$ in the tilted model given above and choosing the damping coefficient appropriately.

4.2 *Nonlinear Control*

In this section, more direct way to it presents the optimal nonlinear control minimizing the criterion function

$$J = \int_0^\infty (x^T Q(x)x + 2x^T S(x)\tau + \tau^T R(x)\tau)dt \tag{12}$$

where

$$x^T = [q^T,\ \dot{q}^T]$$

and the mathematical model should be written as

$$\frac{d}{dt}x = A(x)x + B(x)\tau \tag{13}$$

The optimal control law is given by

$$\tau = -R(x)^{-1}(B(x)^T P(x) + S(x)^T)x \tag{14}$$

where $P(x)$ is the positive definite solution of

$$A^T(x)P + PA(x) + Q(x) - (PB(x) + S(x)) \\ \times R(x)^{-1}(B(x)^T P + S(x)^T) = 0$$

satisfying (Lu and Doyle, 1993)

$$x^T \frac{\partial p_i}{\partial x_j}(x) = x^T \frac{\partial p_j}{\partial x_i}(x)$$

for all $x, i, j = 1, 2, \cdots, n$ and

$$P(x) = [p_1(x), p_2(x), \cdots, p_n(x)]$$

Controlling the system with the constraint force we have not paid attention on the constraint force. But many situations, we have to pay attention also on the constraint force. So in the previous example the constraint force of λ should be taken into account in the criterion function also in the design of the control system. Since

$$C_a\dot{v} + \dot{C}_a v = 0$$

The λ can be written as

$$\lambda = (C_a M_a^{-1} C_a^T)^{-1}(-\dot{C}_a v - C_a M_a^{-1} h_a) \\ = (C_a M_a^{-1} C_a^T)^{-1}(-\dot{C}_a D_a \dot{q} - C_a M_a^{-1} h_a) \tag{15}$$

where

$$\dot{C}_a = \begin{bmatrix} (-L_0 s_0 - l_1 c_0 s_1)\dot{\theta}_0 - l_1 s_0 c_1 \dot{\theta}_1 & 0 & 0 & 0 & -l_1(s_0 c_1 \dot{\theta}_0 + c_0 s_1 \dot{\theta}_1) \\ (-L_0 c_0 + l_1 s_0 s_1)\dot{\theta}_0 - l_1 c_0 c_1 \dot{\theta}_1 & 0 & 0 & 0 & -l_1(c_0 c_1 \dot{\theta}_0 - s_0 s_1 \dot{\theta}_1) \\ 0 & 0 & 0 & 0 & -l_1 c_1 \dot{\theta}_1 \end{bmatrix}$$

$$C_a M_a^{-1} C_a^T =$$

$$\begin{bmatrix} I_0^{-1}(L_0c_0-l_1s_0s_1)^2+I_1^{-1}l_1^2c_0^2c_1^2+m_1^{-1} & I_0^{-1}(L_0c_0-l_1s_0s_1)(-L_0s_0-l_1c_0s_1)-I_1^{-1}l_1^2s_0c_0c_1^2 & -I_1^{-1}l_1^2c_0s_1c_1 \\ I_0^{-1}(L_0c_0-l_1s_0s_1)(-L_0s_0-l_1c_0s_1)-I_1^{-1}l_1^2s_0c_0c_1^2 & I_0^{-1}(-L_0s_0-l_1c_0s_1)^2+I_1^{-1}l_1^2s_0^2c_1^2+m_1^{-1} & I_1^{-1}l_1^2s_0s_1c_1 \\ -I_1^{-1}l_1^2c_0s_1c_1 & I_1^{-1}l_1^2s_0s_1c_1 & I_1^{-1}l_1^2s_1^2+m_1^{-1} \end{bmatrix}$$

$$\dot{C}_a D_a =$$

$$\begin{bmatrix} (-L_0s_0-l_1c_0s_1)\dot{\theta}_0-l_1s_0c_1\dot{\theta}_1 & -l_1(s_0c_1\dot{\theta}_0+c_0s_1\dot{\theta}_1) \\ (-L_0c_0+l_1s_0s_1)\dot{\theta}_0-l_1c_0c_1\dot{\theta}_1 & -l_1(c_0c_1\dot{\theta}_0-s_0s_1\dot{\theta}_1) \\ 0 & -l_1c_1\dot{\theta}_1 \end{bmatrix}$$

Taking account of the constraint force λ, the criterion function shall be written as

$$J = \int_0^\infty (x^T Q_x(x)x + \bar{\lambda}^T Q_\lambda \bar{\lambda} + \tau^T \tau)dt \quad (16)$$

where $\bar{\lambda}$ is considered the constraint force after eliminating the effect of the gravity, then the above equation is rewritten as

$$J = \int_0^\infty (x^T Q(x)x + 2x^T S(x)\tau + \tau^T R(x)\tau)dt (17)$$

where

$$Q(x) = Q_x + (C_a M_a^{-1} W_1 + \dot{C}_a D_a W_2)^T \times Q_\lambda (C_a M_a^{-1} W_1 + \dot{C}_a D_a W_2)$$

$$S(x) = (-(C_a M_a^{-1} C_a^T)^{-1}(C_a M_a^{-1} W_1 + \dot{C}_a D_a W_2))^T Q_\lambda a$$

$$R(x) = a^T Q_\lambda a + 1$$

$$a = -(C_a M_a^{-1} C_a^T)^{-1} C_a M_a^{-1} [1\ \ 0\ \ 0\ \ 0\ \ 0]^T$$

$$W_1 = \begin{bmatrix} 0 & 0 & -C_0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Riccati equation may not give the optimal control law in the neighbourhood of the singularity, i.e., uncontrollable state. But as we shall see in the next section, the control law is giving the swing up under the saturating control. The approach tells that if the computational power is available, the receding horizon control and the time-optimal control may be used (Xu *et al.*, 2001). The variable structure control with sliding sector for a linear systems (Furuta and Pan, 2000) can also be extended to the nonlinear control. The receding horizon approach for a discrete-time nonlinear system can be developed from that of linear systems (Furuta and Wongsaisuw, 1995).

4.3 *Simulation of Swing-up by Nonlinear Control*

In this section, the simulation result of the swing-up by the nonlinear control is presented. For this system control the criterion is chosen as

$$Q(x) = diag(1 + 5000/(1 + e^{(10(\theta_1 - \pi/9))}), 3000(2 - \cos\theta_1), 1, 1 + 1000/(1 + e^{(10(\theta_1 - \pi/6))}))$$

Based on the approach presented by the previous section, the swing-up of the single pendulum is achieved with and without considering the constraint force in the criterion function, where the angles of the arm and pendulum are shown in the Figure 10 and 11. The constrained force λ at the center of the gravity is shown, and this can be also taken into consideration into the criterion function. The constraint force λ is shown in Figure

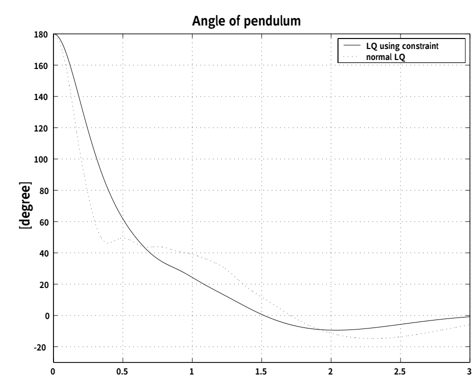

Fig. 10. Pendulum Angle (Furuta pendulum)

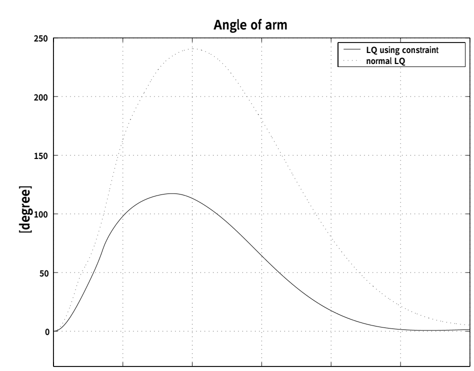

Fig. 11. Arm Angle (Furuta pendulum)

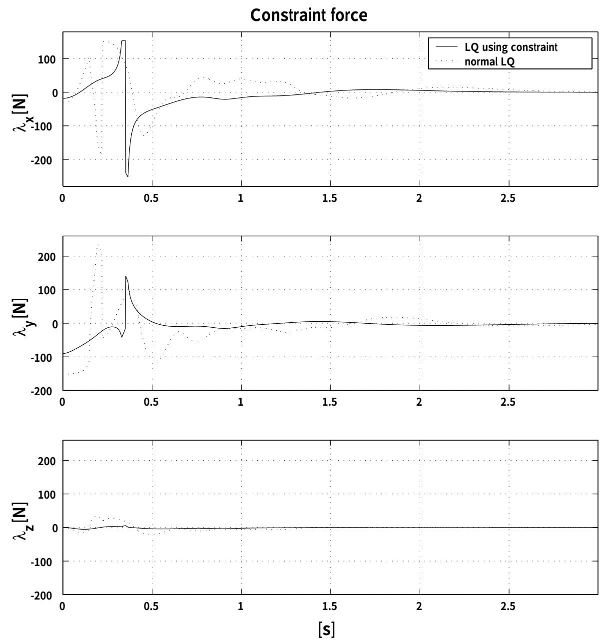

Fig. 12. Constraint Force (Furuta pendulum)

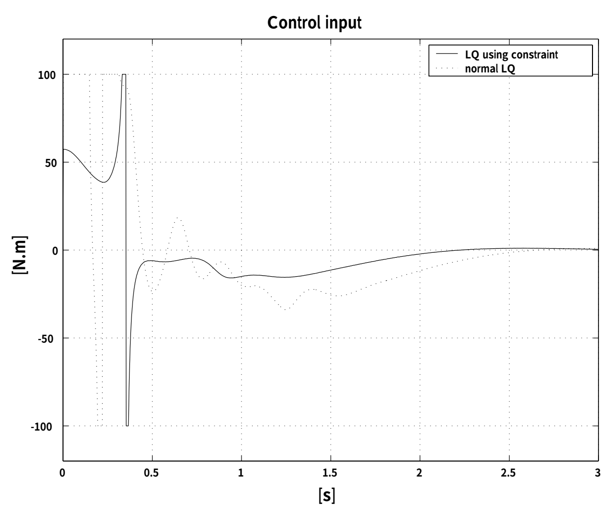

Fig. 13. Control Input (Furuta pendulum)

12. The solid lines in the figures shows the results due to the criterion taking the constraint into the consideration and the dotted line shows one given without considering the constraint forces in the criterion function.

4.4 *Simulation of Swing-up of Double Pendulum*

The similar approach can be applied for the swing-up of the double pendulum (Suzuki *et al.*, In preparation), which may be the first effective way to design the swing-up control of double pendulum. The results show that a discrete-time control law is determined from a single criterion function different from switching several control strategies used before (Yamakita and Furuta, 1999). The discrete-time nonlinear quadratic criterion considered is

$$Q(x) = diag(1, 10^4(2-0.9\cos\theta_1), 10^4 f(\theta_1), 1$$
$$, \ 10^3(2-\cos\theta_1) + 10^4/(1+e^{10(|\theta_1|-\pi/6)}), 10^2 f(\theta_1))$$
$$f(\theta_1) = \begin{cases} (1+\sin|\theta_1|) & |\theta_1| > \pi/2 \\ 2 & |\theta_1| \le \pi/2 \end{cases}$$
$$R = 2(1+\cos\theta_1)$$
$$S(x) = diag(0.1, 0.1, 0, 0.1, 0.1, 0)$$
$$x = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 & \dot{\theta}_0 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}^T$$

The simulation results from the pendant to upright position are shown in Fig. 14.

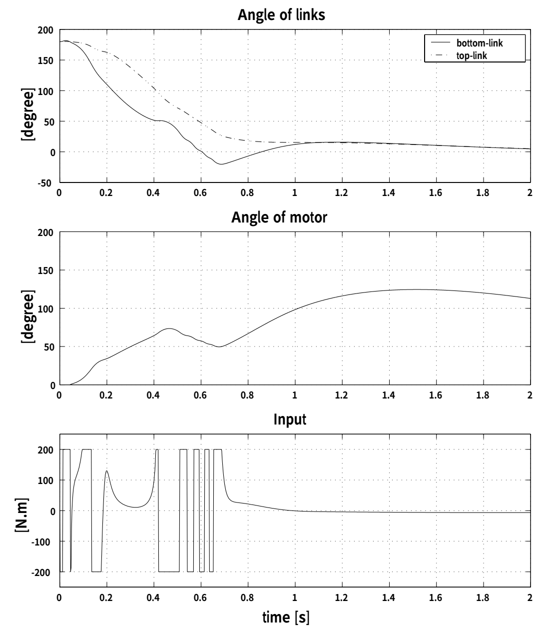

Fig. 14. Pendulum Angle

5. MODELING AND CONTROL OF WALKING SYSTEM

5.1 *Modeling of Walking System*

The modeling of the walking system used by Yamakita and Asano shown in Figure 15 is modeled by using the same idea of the previous section: projection approach. By choosing the augmented state variables as

$$q_a = [\theta_1, x_1, z_1, x_H, z_H, \theta_2, x_2.z_2.\theta_3, x_3, z_3]^T$$

where $[x_1, z_1]^T, [x_H, z_H]^T, [x_2, z_2]^T, [x_3, z_3]^T$ are coordinates of the centers of gravity at the stance leg, the hip, the thigh and shank. The masses at these places are m_1, m_H, m_2, m_3 and $\theta_1, \theta_2, \theta_3$ are angles of stance leg, thigh, shank leg with respect to the vertical line. It is related to the state $q = [\theta_1, \theta_2, \theta_3]^T$ as

$$\dot{q}_a = D_a \dot{q}$$

where

$$D_a = \begin{bmatrix} 1 & 0 & 0 \\ a_1c_1 & 0 & 0 \\ -a_1s_1 & 0 & 0 \\ l_1c_1 & 0 & 0 \\ -l_1s_1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1c_1 & -b_2c_2 & 0 \\ -l_1s_1 & b_2s_2 & \\ 0 & 0 & 1 \\ l_1c_1 & -l_2c_2 & -b_3c_3 \\ -l_1s_1 & l_2s_2 & b_3s_3 \end{bmatrix} \qquad (18)$$

where l_1, l_2, l_3 are stance leg, thigh and shank lengths, a_1, a_2, a_3 and b_1, b_2, b_3 are lower and upper parts of stance leg, thigh, and shank from the tips to the center of gravities. In this subsection the gravity is considered as usual working in "z" direction, then we can write

$$M_a = diag(I_1, m_1, m_1, m_H, m_H, I_2, m_2, m_2, I_3, m_3, m_3)$$

$$h_a = [\tau_1, 0, -m_1g, 0, -m_Hg, \tau_2, 0, -m_2g, \tau_3, 0, -m_3g]^T$$

where τ_1, τ_2, τ_3 are equivalent torque applied at the center of gravities of the stance leg, thigh and shank. The constraint of the system is described by

$$C_a\dot{q}_a = 0$$

and the system is described by

$$M_a\ddot{q} = h_a + C_a^T\lambda_a$$

where

$$C_aD_a = 0$$

The constraint force λ might be taken into consideration in the design of the walking systems in the future. The dynamic model of the system is given by

$$D_a^T M_a(D_a\ddot{q} + \dot{D}_a\dot{q}) = D_a^T h_a$$

This dynamic model just shows the case while stance leg and swing leg are keeping their roles.

5.2 *Control of Walking System based on Artificial Gravity*

The control of a biped robot has received the attention again for applying control theory. Linearization is applied by M. W. Spong (Spong *et al.*, 2000) and stabilization of a and stabilization of zero dynamics of appropriately chosen controlled

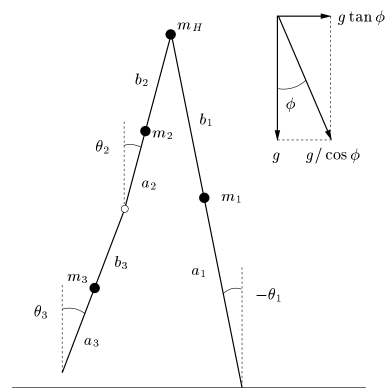

Fig. 15. Parameters of Walking Machine (Yamakita and Asano)

variables is studied by J. W. Grizzle (Grizzle *et al.*, 2001)(Grizzle, 2001)(Cambrini *et al.*, 2000). This section is entirely due to the work presented by H. Ohta (Ohta *et al.*, 2001), M. Yamakita and F. Asano(Asano and Yamakita, 2001). The main objective of the session is to use the gait of the passive walking (McGeer, 1990)(Goswami *et al.*, 1996) on the descending slope for the walking on the flat plane. The walking model is firstly derived by using the idea of the constraint including the collision phase. The walking model considered is the quadrupeds model shown in the following Figure 16.

Fig. 16. Walking Machine

The inner two legs are connected and move together, and the other two legs also move simultaneously. The walking is thus restricted in the sagittal plane. Legs are connected to the hip. Knee joints are free for foward swing and the hip has the actuator. In the walking cycle, the stance leg is kept straight in the swinging phase and at the collision phase the heel strikes. From the dynamic model of the previous section, the dynamic model for the reduced state on the flat ground is given by

$$M(q)\ddot{q} + h(q, \dot{q}, 0) = \tau + \tau_c + \tau_I \delta(t - t_I)$$
$$J_c \dot{q} = 0$$
$$J_I \dot{q} = 0,\ t \in (t_I-, t_I+)$$

where τ_c is the generalized force due to the constraint for the relation of shank and tigh, τ_I is the constraint force t_I denotes the time moments of collision. In the swinging phase the system is described by

$$M(q)\ddot{q} + Z(q)h(q, \dot{q}, 0) = Z(q)\tau - J_c^T X(q)^{-1} \dot{J}_c \dot{q}$$
$$Z(q) = I - J_c^T X(q)^{-1} J_c M(q)^{-1}$$
$$X(q) = J_c M(q)^{-1} J_c^T$$

This dynamic model is quite similar to the triple inverted pendulum. At the collision phase, the velocity after the collision $q_+$is related to one before the collision q_- as

$$\dot{q}_+ = \{I - M(q)^{-1}(I - J_c^T X^{-1} J_c M(q)^{-1}) J_I^T Y^{-1} J_I\} \dot{q}_-$$
$$Y(q) = J_I(I - J_c^T X(q)^{-1} J_c M(q)^{-1}) M(q)^{-1} J_I^T$$
$$X(q) = J_c M(q)^{-1} J_c^T$$

The walking on the ground level can be controlled by imitating the passive walking. During the walking phase, the dynamic model is written as

$$M(q)\ddot{q} + E(q, \dot{q}, 0) = Z(q)\tau \qquad (19)$$

where

$$E(q, \dot{q}, 0) = Z(q)h(q, \dot{q}, 0) - J_c^T X(q) J_c \dot{q} \qquad (20)$$

If the walking robot is placed in the environment with the artificial gravity $(g \tan\phi, -g)$ in the direction of (x, z), then

$$h_a = [\tau_1, m_1 g \tan\phi, -m_1 g, m_H g \tan\phi, -m_H g \tan\phi,$$
$$-\tau_2, m_2 g \tan\phi, -m_2 g, \tau_3, m_3 g \tan\phi, -m_3 g]^T$$

is used for autonomous walking of the dynamic model. The dynamic model is controlled by the input v

$$M(q)\ddot{q} + E(q, \dot{q}, \phi) = Z(q)v \qquad (21)$$

So if we choose the input torque τ for the ground level walking robot as

$$\tau = v + Z(q)^{-1}(E(q, \dot{q}, 0) - E(q, \dot{q}, \phi)) \qquad (22)$$

then the dynamic model on the level ground behaves samilar to one in the artificial gravity filed. So by choosing input v appropriately for stabilization, the walking model on the level ground is same as one on the slope with the angle γ. For the biped case, M. W. Spong named the approach as the virtual gravity compensation.

6. CONCLUSION

A basic idea to approach Super Mechano-System is presented from the viewpoint of the control of pendulum. Several researches on SMS are now under going at Tokyo Institute of Technology, such as running, swimming, and swinging robots. Besides them, the cyber mechanism for demining is now under development. As the purpose of this project, new discipline is expected to be founded.

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