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MODELLING OF A THERMOSTATIC VALVE WITH HYSTERESIS EFFECTS

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Abstract: This paper presents a model of a thermostatic valve based on first principles and where the hysteresis effect is modelled using an adaptive model for friction compensation. The grey box modelling approach is applied, i.e. both physical interpretation and statistical methods are used to build and validate the suggested model. The model performance is illustrated using empirical data and issues concerning the modelling of hysteresis in valves are discussed.

Keywords: Thermostatic Valve, Hysteresis effects, Friction, Grey box modelling, Statistical methods.

1. INTRODUCTION

The introduction of thermostatic valves in residential buildings during the last decades has significantly reduced the energy consumption and improved the thermal comfort. Today the thermostatic valve is a standard component in most water based heating systems. Thus, it is important to develop adequate models, e.g. for design of control strategies and product development.

In this study a model for a thermostatic valve is presented. A major difficulty in the modelling thermostatic valves is the highly non-linear effect from hysteresis. We propose to use a dynamic model based on physical interpretation and first principles (force balances) in combination with an adaptive friction model to deal with this problem. The resulting model is characterized as a grey box model due to the combination of using first principles and empirical methods. The unknown terms of the model are estimated using statistical methods and experimental data, whereas the model is validated using both physical interpretation and statistical methods.

2. THE MODELLING OF THERMOSTATIC VALVES

The thermostatic valve considered in this study is typically used for temperature control of small hot water cylinders (e.g. storage tanks) or heat exchangers in radiator heating systems. It is a self-acting thermostatic valve, where a gas ampoule in the thermostat will increase/decrease the pressure on the valve cone as the surrounding temperature increases/decreases and force the valve to close/open.

There exist extensive literature on mathematical models of both thermostats and valves separately as well as models of thermostatic valves, see e.g. (Hansen, 1997; Bourdouxhe *et al.*, 1998; Zou *et al.*, 1999; Kayihan, 2000). The purpose of these models is primarily to optimize the control of heating and cooling systems. The mathematical modelling, however, is not straightforward due to strong non-linear factors such as hysteresis, and as a result the prediction performance might be poor. The purpose of this study is to develop a model that is directly physical interpretable and able to handle the non-linearities.

To obtain an adequate physical model we propose to use the grey box modelling approach in modelling the thermostatic valve. The grey box modelling approach is characterized by using both prior knowledge, such as physical laws, as well as information from empirical data in the identification procedure. The advantage of this approach is that the model may be given a physical interpretation as well as non-linear effects may be more effectively described compared to a black box model. On the other hand, the use of empirical data provides for more adequate models compared to a pure deterministic or white-box model. The grey box method will not be discussed in detail in this paper. For a discussion on grey box modelling approaches, see (Tulleken, 1992; Ljung and Glad, 1994; Melgaard, 1994). For grey box applications of components in heating systems, see e.g. (Jonsson and Palsson, 1994; Weyer *et al.*, 2000) for heat exchanger modelling, (Madsen and Holst, 1995) for modelling building heat dynamics, and (Gordon *et al.*, 2000) for modelling of air conditioners.

3. THE MODEL FORMULATION

In this section a model of the valve will be proposed. The model is based on physical interpretation and first principles. A simplified sketch of the thermostatic valve is shown in Fig. 1, emphasizing the mechanical parts. Arrows indicate the direction of the forces, which influences on the mechanical parts. Note that only forces that are considered significant are sketched. These are in particular the force from the thermostat (via the spring), friction in the valve and the forces due to the differential pressure as well as the velocity of the liquid through the valve opening.

3.1 Formulation of the force balance

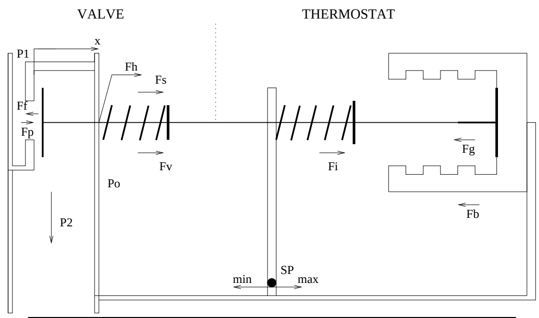
In the following each of the considered forces that is assumed to affect the valve will be formulated in mathematical terms. Based on this a force balance will be proposed. It should be emphasized that the sign of the various forces are indicated by the arrows in Fig. 1, assuming that the forces are positive when considered from the right. The force balance is given as the sum of the considered forces:

$$-F_f + F_p + F_s - F_g + F_v + F_i - F_b \pm F_h = 0, \quad (1)$$

where the terms are explained in Fig. 1. In the following each term will be considered more closely. The flow forces, F_f can be interpreted as a spring force $K_f \Delta p_k$ that are trying to close the valve:

$$F_f = K_f \cdot x \cdot \Delta p_k, \quad (2)$$

where Δp_k is the differential pressure at the cone, K_f is a constant and x is the valve opening. The forces



p_0	Atmospheric pressure
p_1	Pressure at inlet
p_2	Pressure at outlet
SP	Fix point
x	Valve opening
F_f	Flow forces
F_p	Differential pressure at valve cone
F_s	Differential pressure at stem/spindle
F_h	Friction
F_v	Valve spring
F_i	Setting spring
F_g	Differential pressure at bellows
F_b	Bellows spring

Fig. 1. Sketch of the thermostatic valve. The shaded area surrounding p_q indicates the gas chamber, while the arrows at p_1 and p_2 indicates the upstream and downstream pressure. The nomenclature for the various parameters are given in the table below the sketch.

due to the differential pressure are defined from the area on which they are affecting:

$$F_p = A_k(p_1 - p_2) = A_k \cdot \Delta p_k. \quad (3)$$

$$F_s = A_s(p_1 - p_0) = A_s \cdot \Delta p_s. \quad (4)$$

$$F_g = A_b(p_g - p_0) = A_b \cdot \Delta p_b(T_g). \quad (5)$$

A_k, A_s and A_b are the area of the cone, the spindle and the bellows, respectively. The differential pressure is dependent of the temperature of the gas inside the thermostat, T_g, which determines the pressure of the gas p_b. The spring forces, i.e. the forces from the valve spring, the setting spring and the bellows spring, are defined from Hooks law, which is a linear function of the valve position:

$$F_v = F_{v0} - C_v \cdot x. \quad (6)$$

$$F_i = F_{i0} - C_i \cdot x. \quad (7)$$

$$F_b = F_{b0} + C_b \cdot x. \quad (8)$$

$F_{v0,i0,b0}$ and $C_{v,i,b}$ are constants. The force due to friction, $\pm F_h$, is working against the velocity of the valve and hereby a cause to hysteresis. Since it is not straight-forward to define this friction force, an adaptive model will be introduced in Section 3.2 to model for the friction force. If the valve opening is set equal to $x = 0$ in (10), four fix-point values (SP) are given as:

$$A_k \cdot \Delta p_{k,SP} + A_s \cdot \Delta p_{s,SP} + F_{v0} + F_{i,SP} - A_b \cdot \Delta p_b(T_{g,SP}) - F_{b0} = 0. \quad (9)$$

In this expression the force from fiction is neglected. The fix-point values are constant and depend only on the actual calibration (set-points) of the thermostatic valve. Now, the force balance can be written as Eq. (10):

$$- K_f \cdot x \cdot \Delta p_k + A_k \cdot \Delta p_k + A_s \cdot \Delta p_s - A_b \cdot \Delta p_b(T_g) + \big(F_{v0} - C_v \cdot x\big) + \big(F_{i0} - C_i \cdot -x\big) - \big(F_{b0} + C_b \cdot x\big) \pm F_h = 0. \quad (10)$$

By isolating the valve position x we obtain:

$$x = \frac{A_k\big(\Delta p_k - \Delta p_{k,SP}\big) + A_s\big(\Delta p_s - \Delta p_{s,SP}\big)}{C_v + C_i + C_b + K_f \Delta p_k} - \frac{A_b\big(\Delta p_b(T_g) - \Delta p_b(T_{g,SP})\big) \pm F_h}{C_v + C_i + C_b + K_f \Delta p_k}. \quad (11)$$

In the following the modelling of the hysteresis force $\pm F_h$ will be discussed.

3.2 Models of hysteresis

This section discusses the modelling of the hysteresis force, $\pm F_h$. Hysteresis is a phenomenon that appears in many mechanical and electrical systems, including thermostatic valves. The hysteresis force is characterized by the fact that the valve position is a function of local phenomena such as the gradient of the velocity of the stem. The hysteresis effect is a significant factor in the modelling of thermostatic valves and it is considered important to account for this in the model which objective is for prediction and control purposes.

A detailed model is the *Dahl model*. It is a commonly used model for describing forces arising from friction F_h and described in (Dahl, 1968). The model states the relationship between the stretch and strain of a given material. The Dahl model was originally developed for simulation of control systems with friction. The model may however be used for other applications, e.g. in adaptive friction compensation, see (Olsson *et al.*, 1997). This is also the purpose of the Dahl model in this study. Let the friction force be defined as F, the Coulomb's friction force as F_c and the displacement as x. Dahl's model has then the form:

$$\frac{dF}{dx} = \sigma\Big(1 - \frac{F}{F_c} sgn(v)\Big)^{\alpha}, \quad (12)$$

where σ denotes the elasticity, and α is the slope of the movement. A sketch of the model is shown in Fig. 2. It is seen that the friction force is solely defined

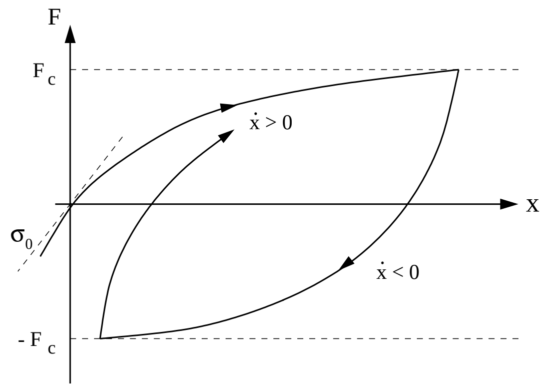


Fig. 2. Sketch of the friction force F as a function of the dissaplacement x.

from the length of the displacement and its direction, $sgn(v)$. Hereby the possibility of static friction nor the Stribeck effect is not considered in the model (Olsson *et al.*, 1997). In Fig. 2 it is shown that as the displacement increase the friction force will equal the Coulomb's friction force F_c, and thus the maximal friction force for both negative and positive displacement, i.e. $|F| \leq F_c$. By changing the direction of the movement the curve is flipped over and the maximal friction force is then $-F_c$. The slope for $F = 0$ is indicated as σ_0.

The Dahl model can be reformulated for the time domain, see (Olsson *et al.*, 1997):

$$\frac{dF}{dt} = \frac{dF}{dx}\frac{dx}{dt} = \frac{dF}{dx} v = \sigma\Big(1 - \frac{F}{F_c} sgn(v)\Big)^{\alpha} v, \quad (13)$$

which results in a first order non-linear differential equation.

Since Dahl's model directly gives the friction force as a function of the displacement it seems obvious to use the the valve position as x. This gives some difficulties, however, since the purpose of the model is to predict the valve position, but then one would not know the friction force before the valve position is known. There is a clear contradiction in using F_h to determine the valve position x and at the same time know the value of F_h. To overcome this problem we have modified Dahl's model to the valve problem by letting the gas temperature T_g be the parameter which determines the friction force. It is seen from empirical data in Section 4 that it is mainly the gas temperature of the valve, T_g, that can be correlated to the effects of hysteresis. By determining a friction force using Dahl's model and T_g it is possible to model the hysteresis of the valve. Hereby the adaptive model of the friction force becomes:

$$\frac{dF_h}{dt} = v\sigma\big(1 - \frac{F_h}{F_{h,max}} sgn(v)\big)^{\alpha}, \quad (14)$$

where $F_{h,max}$ is the maximum of the friction force and $v = \frac{dT_g}{dt}$ is the gradient of the gas temperature.

In (Olsson *et al.*, 1997) it is argued that the slope α may be set equal to 1 and this will be assumed in the following.

In summary the steady-state model for the valve position can be formulated as a continuous time state space model, combining the dynamic friction model Eq. (14) with the force balance for the valve position Eq. (11). By introducing noise terms, the state space formulation becomes:

$$\frac{dF_h}{dt} = v\sigma\big(1 - \frac{F_h}{F_{h,max}} sgn(v)\big) + dw_t. \qquad (15)$$

$$x = \frac{A_k\big(\Delta p_k - \Delta p_{k,SP}\big) + A_s\big(\Delta p_s - \Delta p_{s,SP}\big)}{C_v + C_i + C_b + K_f \Delta p_k} - \frac{A_b\big(\Delta p_b(T_g) - \Delta p_b(T_{g,SP})\big) \pm F_h}{C_v + C_i + C_b + K_f \Delta p_k}. \qquad (16)$$

Here dw_t is assumed to be a standard Wiener process and e_k is assumed to be a sequence of independent normally distributed variables. The introduction of the noise terms indicates that the model is an approximation to the system and that noise and unmodelled input may affect the system. Furthermore, the assumption about the noise process makes it possible to estimate unknown parameters in the model using a Maximum Likelihood method (Melgaard, 1994). The results of applying the model Eq. (15-16) and estimating the unknown parameters will be presented in Section 5.

4. THE EXPERIMENT AND THE DATA

This section presents empirical data that will be used for the subsequent parameter estimation and model validation. An experimental setup has been established in order to obtain a steady state characteristic of the valve position and to investigate the influence from the assumingly significant factors. Basically, the valve is put down in a small tank filled with water, mimicking the surrounding temperature, and the following variables are measured:

- The flow, q through the valve $[kg/h]$.
- The temperature of the surroundings, T_{tank} (water tank) $[^oC]$.
- Temperature of the water, T_{med}, through the valve $[^oC]$.
- The static pressure, p_s $[bar]$.
- The differential pressure, p_{dif} $[mbar]$.

The surrounding temperature (temperature of the water tank) may only be varied slowly so that the temperature in the valve can be assumed to be in steady state. Due to the slow variations it is assumed that the temperature of the gas in the thermostatic valve equals the temperature of the surroundings, i.e. the temperature of the water tank:

$$T_g = T_{tank}. \qquad (17)$$

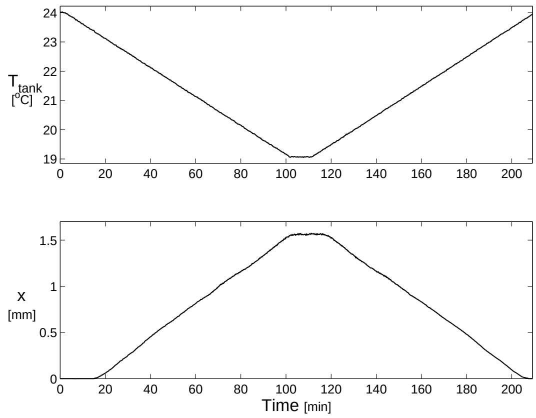


Fig. 3. Experimental data from run 1.

For the subsequent model identification only the temperature T_{tank} has been purposely varied by introducing steps and the resulting flow has been measured. Note that the variables p_s and p_{dif} here are kept constant, while T_{med} is assumed not to influence on the valve position since this is totally dominated by T_{tank}.

Two experimental runs have been applied to the experimental setup in order to collect data. The experimental runs are characterized by the excitation (steps) in the water temperature in order to determine the response on the valve position. During such an experiment the valve will be both closed and almost fully open. Plots of the collected data (time series) from two experimental runs are shown in Fig. 3 and 4. In Fig. 5 a phase diagram of the valve opening and the temperature of the valve is shown. The hysteresis effect is seen very clearly.

The uncertainties of the measured data used for model identification are listed in Tab. 1. The uncertainty in the flow and the various pressures are given relatively and the uncertainty of the temperatures are in absolute units.

5. RESULTS

The measured variables listed in Tab. 1 together with the model Eq. (15-16) has been applied and the un-

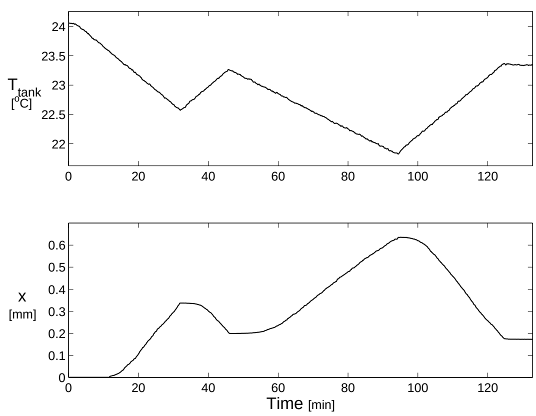


Fig. 4. Experimental data from run 2.

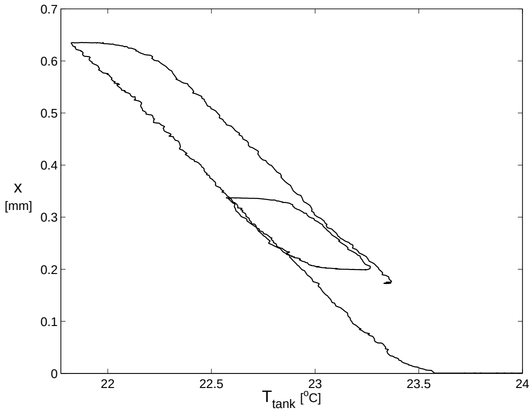


Fig. 5. Phase diagram of valve opening vs. valve temperature (run 2).

Table 1. Measurement uncertainties

Variable	Symbol	Uncertainty σ	Unit
flow	q	$0.015 \cdot q$	kg/h
surrounding temperature	T_{tank}	0.05	^oC
differential pressure	p_{dif}	$0.015 \cdot p_{dif}$	$mbar$
static pressure	p_s	$0.015 \cdot p_s$	bar

known parameters have been estimated. Note that the pressure of the thermostat gas, $\Delta p_b(T_g)$, has been calculated using a higher order polynomial for the relation between $\Delta p_b(T_g)$ and T_g. Also, it should be noted that the valve position x is calculated by a formula (table values) for the relation between the flow, differential pressure and the corresponding valve position.

The parameters of the model that are known a priori are listed in Tab. 2. The remaining parameters are unknown and have been estimated. First, the parameters listed in Tab. 3 are found using data from a preliminary experiment where the temperature is constant, i.e. the hysteresis term $\pm F_h$ is set to zero. These parameters are determined using the least squares criterion. Finally, the estimated parameters of the parameters for the hysteresis function are found using the ML method, keeping the remaining parameters of Tab. 2 and 3 constant. These estimated parameters of the hysteresis function are listed in Tab. 4.

Table 2. Known parameter values

Parameter	Value	Unit	Description
A_b	1032	mm^2	Area of Bellows
C_v	1.8	N/mm	Valve spring
C_i	13.7	N/mm	Setting spring
C_v	3.2	N/mm	Bellows spring

Table 3. Estimated parameter values found by least squares

Parameter	Value	Unit	Description
A_s	2.45	mm^2	Area of spindle
A_k	52.0	mm^2	Area of cone
K_f	99.7	mm	Flow spring

As a first check the estimated model parameters are considered. Compared to physical interpretation the

Table 4. Estimated parameters found by ML

variable	estimate	std.dev.
σ	6.41	0.589
$F_{h,max}$	1.31	0.067
$p_{b,set}$	0.0776	$0.892 \cdot 10^{-3}$

estimates seem very reasonable indicating that the model formulation is realistic from a physical point of view. From an empirical point of view, a comparison between the measured and modelled output is shown in Fig. 6. It is seen that the model is able to predict the system quite accurately.

The model residuals shows no systematic patterns, and this is supported by the estimated autocorrelation function for the model residuals. The test in the estimated autocorrelation function indicates a satisfactory model fit since no information is left in the model residuals, see e.g. ((Holst *et al.*, 1992)). As another comparison the estimated standard deviation of the residuals is $\hat{\sigma}_\epsilon = 2.4 \cdot 10^{-3} mm$ which corresponds to a relative uncertainty of 1%. This is less than the measurement uncertainty, listed in Tab. 1. Furthermore the model may be interpreted physically. Finally, a cross validation study using the model estimated model on the independent experimental data from run 1 is shown in Fig. 7. It is seen that the model is still able to predict the system with high accuracy.

6. SUMMARY AND DISCUSSION

A grey box model of a thermostatic valve has been presented. The model is based on physical interpretation and statistical methods has been used to estimate and validate the model. Hysteresis is a phenomena that makes the modelling of thermostatic valves difficult at least if the approach is solely based on empirical data. To overcome this problem an adaptive model for friction was applied to overcome for the hysteresis effect.

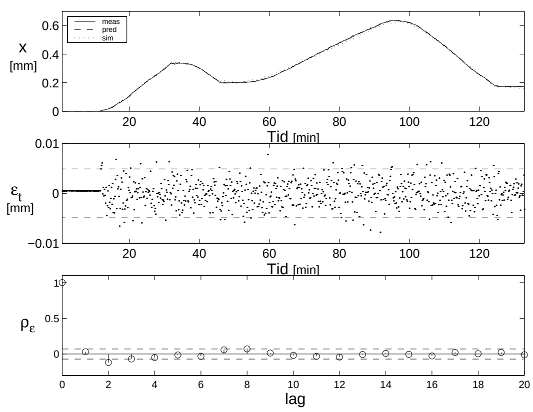


Fig. 6. Estimated model performance. Top: Measured, predicted and simulated model output. Middle: Model residuals. Bottom: Estimated autocorrelation function for the residuals.

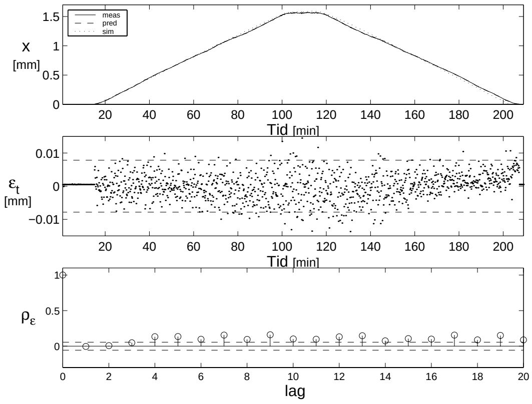


Fig. 7. Cross validation of the model. Top: Measured, predicted and simulated model output. Middle: Model residuals. Bottom: Estimated autocorrelation function for the residuals

The combination of physical knowledge and statistical methods implies that the model can be interpreted physically and that non-linear effects, such as hysteresis, can be handled effectively. The ability of the model to handle non-linearities is important for control purposes, where accurate predictions are necessary. It could be argued, that more black-box oriented models could do the job just as well. However, the physical interpretation is important for product development, extrapolation with the model or applying the model for other types of thermostatic valves. This could be done since all the model parameters can be related to physical quantities.

If the model should be applied for system analysis, e.g. in interaction with a building, it might be necessary to model the heat dynamics of the thermostat as well. However, the model for the valve position would probably still be valid. Preliminary results show that the model presented in this study is still useful and adequate. The results of the dynamic model will be given in a subsequent paper.

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