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DISTURBANCE OBSERVER BASED APPROACH TO THE DESIGN OF SLIDING MODE CONTROLLER FOR HIGH PERFORMANCE POSITIONING SYSTEMS

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Abstract: Disturbance observer(DOB) based control is one of the most popular methods in the field of high performance position control. In this paper, a sliding mode controller is designed in the DOB framework. To do this, a generalized disturbance attenuation framework named robust internal-loop compensator(RIC) is introduced and it is shown that sliding mode controller based on Lyapunov redesign can be analyzed in the RIC framework. Using the structural characteristics of RIC, DOB property and performance of the sliding mode control system are analyzed. Through experiments using a high performance positioning system, the validity of the proposed method is shown.

Keywords: Disturbance observer, Robust internal-loop compensator, Sliding mode control

1. INTRODUCTION

Main difficulties of the motion control for high performance positioning systems are parametric uncertainties, unmodeled dynamics, specifically nonlinear friction, and external disturbances. For the robust motion controller to compensate these uncertainties, it is basically assumed that the uncertain disturbances satisfy matching condition. Hence, disturbance is defined as sum of external disturbance signals and all possible signals due to the differences between the actual plant and model such as modeling uncertainty and parameter variations. Thus, the actual plant with the disturbance compensator can be regarded as nominal model if the disturbance is cancelled out well. Many kinds of robust controller such as disturbance observer(DOB)(Ohnishi, 1987) and adaptive robust controller(Yao *et al.*, 1997) have been developed under the matching condition. Recently, a generalized disturbance compensating framework based on the Lyapunov redesign, named robust internal-loop compensator(RIC), was proposed to show the inherent structural equivalence of the robust disturbance compensating controllers(Kim and Chung, 2001*a*), and unified analysis and design were performed(Kim and Chung, 2001*b*; Kim *et al.*, 2002).

In this paper, a sliding mode controller is designed in the DOB framework, where a RIC is incorporated. Sliding mode control is theoretically based on the variable structure system and aims at driving the state trajectory of the system onto a switching surface in the state space and maintaining the trajectory on this surface. Specifically, a general sliding mode controller is derived and a stabilizing control input is designed based on Lyapunov redesign, where a RIC which stems from the internal model following control scheme, is employed to obtain a control input to cancel out the uncertainties and disturbances without chattering. Then, by using the structural characteristics of the RIC, DOB characteristics and the performance of the closed-loop system with sliding mode controller according to the controller gains are analyzed.

2. DOB BASED APPROACH

DOB based controller design is one of the most popular methods in the field of motion control. It is well known that DOB makes a system robust using Q-filter which cuts off the disturbance in low frequency region. From the DOB structure of Fig. 1, the output y can be expressed as follows:

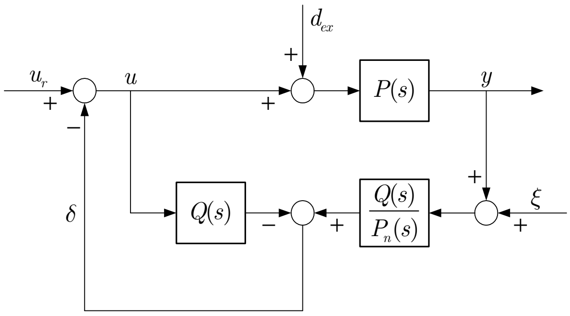


Fig. 1. Disturbance observer

$$y = \frac{P(s)}{\mathcal{X}(s)}\left[P_n(s)u_r + P_n(s)\{1 - Q(s)\}d_{ex} - Q(s)\xi\right], \quad (1)$$

where $\mathcal{X}(s) = P_n(s) + [P(s) - P_n(s)]Q(s)$, u_r is the reference control input, d_{ex} is the external disturbance, and ξ is the measurement noise. Below the cutoff frequency of $Q(s)$, $|\,Q(j\omega)\,| \approx 1$ is achieved. Hence the behavior of real plant $P(s)$ is to be the same as given nominal model $P_n(s)$. On the other hand, above the cutoff frequency of $Q(s)$, $|\,Q(j\omega)\,| \approx 0$ is achieved. Hence high frequency measurement noise is attenuated. Therefore, the most important parameter of the DOB design is the low-pass filter Q, and the main concern is the tradeoff between making $|\,Q(j\omega)\,|$ small and $|\,1 - Q(j\omega)\,|$ small. In this paper, unlike the typical design method of DOB, a systematic DOB design method is proposed in the RIC framework, which will be shown in next section.

3. ROBUST INTERNAL-LOOP COMPENSATOR

Figure 2 shows a compensated feedback system with prefilter $F(s)$ to make $P(s)$ with d_{ex} behave like a given reference model $P_m(s)$. y_r is a reference model output, u is a control input, $K(s)$ is a feedback compensator. From Fig. 2, the sensitivity and complementary sensitivity functions are obtained as follows:

$$S(s) = \frac{1}{1 + L(s)}, \quad T(s) = \frac{L(s)}{1 + L(s)} \quad (2)$$

where $L(s) = P(s)K(s)$. Hence it can be easily seen that the effect of d_{ex} and ξ on y are only determined by $P(s)$ and $K(s)$. That is, $P_m(s)$ and $F(s)$ do not have an effect on $S(s)$ and $T(s)$ of Fig. 2. The transfer function from y_r to y is given by $T_{y_r y}(s) = F(s)\left[\frac{L(s)}{1+L(s)}\right]$. Thus, $F(s)$ is just used to make $T_{y_r y}(s) = 1$. Therefore, the design objective is to design $K(s)$ and $F(s)$ so that the specified robustness and performance are achieved under the parametric uncertainty and disturbance condition. Based on Fig. 2, one of the best candidates for $F(s)$ can be chosen as follows:

$$F(s) = \left[\frac{L_m(s)}{1 + L_m(s)}\right]^{-1} = \frac{1}{G_{L_m}(s)} \quad (3)$$

where $L_m(s) = P_m(s)K(s)$ and $G_{L_m}(s)$ is a reference closed-loop system. Thus $|\,T_{y_r y}(j\omega)\,| \approx 1$ can be

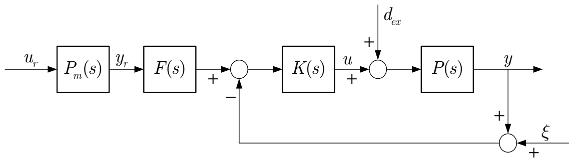


Fig. 2. Compensated feedback system with prefilter

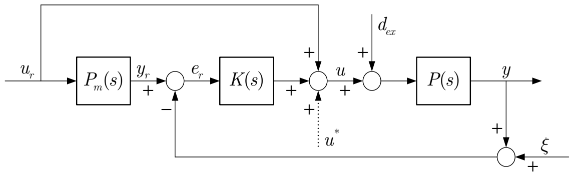


Fig. 3. Robust internal-loop compensator structure

achieved. Prefilter $F(s)$, in a crude way, approximates PD type control. Therefore this enhances transient performance and leads the phase of the unity feedback system in Fig. 2. Alternatively, Fig. 2 with (3) can be equivalently transformed into Fig. 3. In this figure, the model following error is defined as $e_r = y_r - (y + \xi)$. Then, the control input has the form of

$$u = u_r + K(s)\,e_r + u^*, \quad (4)$$

where u^* is an optional control input to compensate nonlinear disturbances. In this paper, the structure in Fig. 3 with the control input (4) is defined as robust internal-loop compensator(RIC). From Fig. 3, y can be expressed in terms of u_r, d_{ex}, and ξ:

$$y = \frac{P(s)}{1 + L(s)}\left[\{1 + L_m(s)\}u_r + d_{ex} - K(s)\xi\right]. \quad (5)$$

As a result, $S(s)$ and $T(s)$ in (2) are obtained as before. Therefore, if $K(s)$ is designed in optimal sense, the specified robustness and performance can be achieved for the system in the presence of uncertainties and disturbances. And also, this makes the $P(s)$ behave like $P_m(s)$.

If $P_m(s)$ and $F(s)$ of RIC are chosen as follows:

$$P_m(s) = P_n(s), \quad F(s) = \frac{1}{Q(s)}, \quad (6)$$

then $Q(s)$ is given by the reference closed-loop system. After recalculating this equation for $K(s)$, if $K(s)$ is substituted into Fig. 3, then DOB in Fig. 1 is obtained. This means that if $K(s)$ is designed for $P_m(s)$ in order to satisfy a given criterion, optimal Q-filter is systematically designed which has the optimality under the given specific conditions, because the transfer function of the feedback system with $P_m(s)$ and $K(s)$ is $Q(s)$. The disturbance attenuation characteristics of the designed system can be easily analyzed based on $Q(s)$ and also the estimated disturbance δ of DOB can be reformulated as follows($u^* = 0$):

$$\delta = -K(s)e_r. \quad (7)$$

Note here that (7) can be interpreted as a control input based on Lyapunov redesign. Using this control framework, the sliding mode controller is designed in the next section.

4. SLIDING MODE CONTROL BASED ON LYAPUNOV REDESIGN

4.1 *Sliding Mode Control*

Consider a system which can be expressed as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A}_1(t) \\ \boldsymbol{A}_2(t) \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_2(t) \end{bmatrix} (\boldsymbol{u} + \boldsymbol{d}_{ex}) \tag{8}$$

where $\boldsymbol{x} = \left[\boldsymbol{x}_1^T \ \boldsymbol{x}_2^T\right]^T$ is the state vector, $\boldsymbol{x}_1 \in \mathbb{R}^{n-m}$, $\boldsymbol{x}_2 \in \mathbb{R}^m$, $\boldsymbol{A}_1 \in \mathbb{R}^{(n-m)\times n}$, $\boldsymbol{A}_2 \in \mathbb{R}^{m\times n}$, $\boldsymbol{B}_2 \in \mathbb{R}^{m\times m}$, $\boldsymbol{u} \in \mathbb{R}^m$ is the control input, and $\boldsymbol{d}_{ex} \in \mathbb{R}^m$ is the unknown bounded external disturbance. The objective of the sliding mode control is to drive the states of the system into the set $\mathcal{S}$ defined by

$$\mathcal{S} = \{\boldsymbol{x} : \boldsymbol{\varphi}(t) - \boldsymbol{\sigma}_a(\boldsymbol{x}) = \boldsymbol{\sigma}(t,\boldsymbol{x}) = \boldsymbol{0}\} \tag{9}$$

where $\boldsymbol{\varphi}(t)$ is the time dependent part of the sliding function $\boldsymbol{\sigma}(t,\boldsymbol{x})$ and contains reference inputs to the controlled plant. $\boldsymbol{\sigma}_a(\boldsymbol{x})$ represents the state dependent part of $\boldsymbol{\sigma}(t,\boldsymbol{x})$, which can be expressed as $\boldsymbol{\sigma}_a = \boldsymbol{G}_1\boldsymbol{x}_1 + \boldsymbol{G}_2\boldsymbol{x}_2$ where $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ have appropriate dimension.

In order to derive the control algorithm, let's consider the Lyapunov function which has the form of

$$V = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{\sigma}. \tag{10}$$

Differentiating (10) with respect to time yields

$$\dot{V} = \boldsymbol{\sigma}^T\dot{\boldsymbol{\sigma}}. \tag{11}$$

However, it is desired for $\dot{V}$ to be $\dot{V} = -\boldsymbol{\sigma}^T\boldsymbol{D}\boldsymbol{\sigma}$ where $\boldsymbol{D}$ is a positive-definite matrix so that $\dot{V}$ will be negative definite, and this will ensure stability of the system based on Lyapunov stability. By using (11), the following equation can be obtained,

$$\boldsymbol{D}\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}} = 0. \tag{12}$$

From (9), the derivative of the sliding function can be expressed as

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\varphi}} - \boldsymbol{G}_1\boldsymbol{A}_1\boldsymbol{x} - \boldsymbol{G}_2(\boldsymbol{A}_2\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{u} + \boldsymbol{B}_2\boldsymbol{d}_{ex}). \tag{13}$$

To make $\dot{\boldsymbol{\sigma}}$ be zero, from (13), the equivalent control input can be obtained as

$$\boldsymbol{u}_{eq} = (\boldsymbol{G}_2\boldsymbol{B}_2)^{-1}\left[\dot{\boldsymbol{\varphi}} - \boldsymbol{G}_1\boldsymbol{A}_1\boldsymbol{x} - \boldsymbol{G}_2\boldsymbol{A}_2\boldsymbol{x}\right] - \boldsymbol{d}_{ex}. \tag{14}$$

By substituting (12) into (13), the control input is now obtained as

$$\boldsymbol{u} = \boldsymbol{u}_{eq} + (\boldsymbol{G}_2\boldsymbol{B}_2)^{-1}\boldsymbol{D}\boldsymbol{\sigma}. \tag{15}$$

However, since the parameters of the real system and external disturbances cannot be exactly known, it is not possible to design the equivalent control input such as (14). Hence, the stability of the whole closed-loop system designed based on Lyapunov function of (10) is not guaranteed. Therefore a design method of sliding mode controller based on Lyapunov redesign is proposed to overcome this problem.

4.2 *Disturbance Attenuation*

The Lyapunov redesign starts from taking a reference model of the system (8) as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A}_{m1}(t) \\ \boldsymbol{A}_{m2}(t) \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{m2}(t) \end{bmatrix} \boldsymbol{u}. \tag{16}$$

A stabilizing feedback controller is to be designed by using this reference model. Note that (8) can be rewritten in terms of $\boldsymbol{A}_{m1}$, $\boldsymbol{A}_{m2}$, and $\boldsymbol{B}_{m2}$ as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A}_{m1}(t) \\ \boldsymbol{A}_{m2}(t) \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{m2}(t) \end{bmatrix} [\boldsymbol{u} + \boldsymbol{d}_{eq}(t,\boldsymbol{x},\boldsymbol{u})] \tag{17}$$

where $\boldsymbol{d}_{eq} = \boldsymbol{B}_{m2}^{-1}(t)\left[\widetilde{\boldsymbol{A}}_2(t)\,\boldsymbol{x} + \widetilde{\boldsymbol{B}}_2(t)\,\boldsymbol{u} + \boldsymbol{B}_2(t)\,\boldsymbol{d}_{ex}\right]$ with $\boldsymbol{A}_{m1} = \boldsymbol{A}_1$, $\widetilde{\boldsymbol{A}}_2 = \boldsymbol{A}_2 - \boldsymbol{A}_{m2}$ and $\widetilde{\boldsymbol{B}}_2 = \boldsymbol{B}_2 - \boldsymbol{B}_{m2}$. Next, let us design a feedback control law $\boldsymbol{u} = \boldsymbol{u}_r(t,\boldsymbol{x})$ such that the origin of the reference closed-loop system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A}_{m1}(t) \\ \boldsymbol{A}_{m2}(t) \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{m2}(t) \end{bmatrix} \boldsymbol{u}_r(t,\boldsymbol{x}) \tag{18}$$

is uniformly asymptotically stable. The derivative of the sliding function for (18) is obtained as

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\varphi}} - \boldsymbol{G}_1\boldsymbol{A}_{m1}\boldsymbol{x} - \boldsymbol{G}_2(\boldsymbol{A}_{m2}\boldsymbol{x} + \boldsymbol{B}_{m2}\boldsymbol{u}_r). \tag{19}$$

Hence, the sliding mode control input which can stabilize (18) is given by

$$\begin{aligned} \boldsymbol{u}_r = {} & (\boldsymbol{G}_2\boldsymbol{B}_{m2})^{-1}[\dot{\boldsymbol{\varphi}} - \boldsymbol{G}_1\boldsymbol{A}_{m1}\boldsymbol{x} - \boldsymbol{G}_2\boldsymbol{A}_{m2}\boldsymbol{x}] \\ & + (\boldsymbol{G}_2\boldsymbol{B}_{m2})^{-1}\boldsymbol{D}\boldsymbol{\sigma}. \end{aligned} \tag{20}$$

And assume that with $\boldsymbol{u} = \boldsymbol{u}_r(t,\boldsymbol{x}) + \boldsymbol{v}$, the uncertain term $\boldsymbol{d}_{eq}$ satisfies the inequality

$$\|\,\boldsymbol{d}_{eq}(t,\boldsymbol{x},\boldsymbol{u}_r + \boldsymbol{v})\,\| \le \rho(t,\boldsymbol{x}) + k\,\|\,\boldsymbol{v}\,\|, \tag{21}$$

where $0 \le k < 1$, $\rho : [0,\infty) \times \mathbb{D} \to \mathbb{R}$ is a nonnegative continuous function. Our goal is to show that with the knowledge of the Lyapunov function V, the function ρ and the constant k in (21), an additional feedback control $\boldsymbol{v} = \boldsymbol{\gamma}(t,\boldsymbol{x})$ called as Lyapunov redesign can be designed such that the overall control

$$\boldsymbol{u} = \boldsymbol{u}_r(t,\boldsymbol{x}) + \boldsymbol{\gamma}(t,\boldsymbol{x}) \tag{22}$$

stabilizes the actual system (17) in the presence of the uncertainty.

Now, let us apply the control $\boldsymbol{u} = \boldsymbol{u}_r(t,\boldsymbol{x}) + \boldsymbol{v}$ to the the system (17). Then, the closed-loop system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{A}_{m1}(t) \\ \boldsymbol{A}_{m2}(t) \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{m2}(t) \end{bmatrix} (\boldsymbol{u}_r + \boldsymbol{v} + \boldsymbol{d}_{eq}) \tag{23}$$

is a perturbation of the reference closed-loop system (18). Hence, the derivative of V along the trajectories of (23) is obtained as

$$\dot{V} = -\boldsymbol{\sigma}^T\boldsymbol{D}\boldsymbol{\sigma} + \boldsymbol{w}^T\boldsymbol{v} + \boldsymbol{w}^T\boldsymbol{d}_{eq} \tag{24}$$

where $\boldsymbol{w}^T = -\boldsymbol{\sigma}^T\boldsymbol{G}_2\boldsymbol{B}_{m2}$. Consequently, it is possible to choose $\boldsymbol{v}$ to cancel the effect of $\boldsymbol{d}_{eq}$ on $\dot{V}$.

Since (18) is a closed-loop equation for the reference system (16) with the control input $\boldsymbol{u} = \boldsymbol{u}_r(t, \boldsymbol{x})$, this equation can be rewritten as

$$\dot{\boldsymbol{x}}_r = \begin{bmatrix} \boldsymbol{A}_{m1}(t) \\ \boldsymbol{A}_{m2}(t) \end{bmatrix} \boldsymbol{x}_r + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{m2}(t) \end{bmatrix} \boldsymbol{u}_r(t, \boldsymbol{x}) \tag{25}$$

where $\boldsymbol{x}_r = \begin{bmatrix} \boldsymbol{x}_1^T & \boldsymbol{x}_2^T \end{bmatrix}^T$. For the model following error, $\boldsymbol{e}_r = \boldsymbol{x}_r - \boldsymbol{x}$, (23) is given by

$$\dot{\boldsymbol{e}}_r = \begin{bmatrix} \boldsymbol{A}_{m1}(t) \\ \boldsymbol{A}_{m2}(t) \end{bmatrix} \boldsymbol{e}_r - \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_{m2}(t) \end{bmatrix} (\boldsymbol{v} + \boldsymbol{d}_{eq}). \tag{26}$$

Therefore, $\boldsymbol{v}$ can be designed as

$$\boldsymbol{v} = \boldsymbol{K}\boldsymbol{e}_r \tag{27}$$

where $\boldsymbol{K} \in \mathbb{R}^{m \times n}$.

5. SLIDING MODE CONTROL FOR HIGH PERFORMANCE POSITIONING SYSTEMS

In this section, a high performance positioning system is considered as one example of the specific applications. The equation of motion for this system can be written as follows:

$$J\ddot{x} + B\dot{x} + F_r(\dot{x}) - d_{ex} = u \tag{28}$$

where J is the inertia, B is the damping coefficient, u is the control input, x is the output, $F_r(\dot{x})$ is the friction term, and d_{ex} is the uncertain disturbance. The state-space representation for this equation is

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -B/J \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} [u - F_r(x_2) + d_{ex}] \tag{29}$$

where $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $x_1 = x$, $x_2 = \dot{x}$. Let $e = x_d - x$ be the tracking error, where x_d is a desired trajectory, and let's define a time-varying function $\mathcal{S}(t)$ in the state-space $\mathbb{R}^n$ by the scalar equation $\sigma(\boldsymbol{x}, t) = 0$, where $\sigma = \left(\frac{d}{dt} + \Lambda\right)^{n-1} e$ and Λ is a strictly positive constant. For instance, since the system order n is 2 in (28), the sliding mode function is expressed as $\sigma = \dot{e} + \Lambda e$.

Now, in order to design a sliding mode controller structurally in the RIC framework, consider the following reference model for (29):

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -B_m/J_m \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1/J_m \end{bmatrix} u \tag{30}$$

where J_m and B_m are the nominal values of J and B respectively. Hence (28) can be rewritten in terms of the reference model parameters J_m and B_m of (30)

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -B_m/J_m \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1/J_m \end{bmatrix} (u + d_{eq}) \tag{31}$$

where $d_{eq} = (J_m - J)\,\dot{x}_2 + (B_m - B)\,x_2 - F_r(x_2) + d_{ex}$. And the sliding mode control input which can stabilize (30) can be obtained by (20),

$$u_r = J_m(\ddot{x}_d + \Lambda\dot{e}) + B_m\dot{x} + (J_m D)\,\sigma. \tag{32}$$

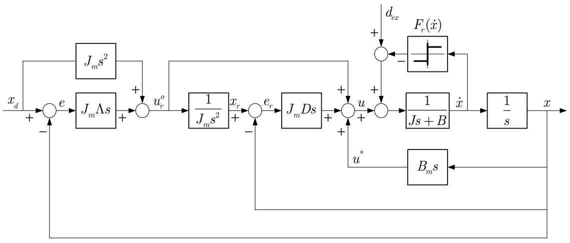


Fig. 4. Sliding mode controller in the RIC framework

Next, let us choose the reference state variable as $x_r = x_d + \Lambda \int_0^t e\, d\tau$. Hence the following equation is arranged:

$$e_r = e + \Lambda \int_0^t e\, d\tau = \int_0^t \sigma\, d\tau. \tag{33}$$

Therefore, from (4) and (27), the RIC-based sliding mode control input is formulated as

$$u = J_m(\ddot{x}_d + \Lambda\dot{e}) + B_m\dot{x} + (J_m D)\,\sigma + K(s)\,e_r. \tag{34}$$

Here, note that $v = K(s)e_r$ designed based on Lyapunov redesign can be represented by using σ. That is, since $(J_m D)\,\sigma = (J_m D)\,\dot{e}_r$ is satisfied, the design of v can be covered by this equation. Therefore, the sliding mode control input can be represented as

$$u = u_r^o + K(s)\,e_r + u^* = J_m\ddot{x}_r + (J_m D)\,\dot{e}_r + B_m\dot{x} \tag{35}$$

where u_r^o is the reference state dependent part of u_r. And the last term of (35) can be interpreted as the additional control input for dynamic compensation of unknown parameters. Fig. 4 shows the proposed structure of sliding mode controller in the RIC framework.

From (35), $P_m(s)$ and $K(s)$ of RIC are obtained as

$$P_m(s) = \frac{1}{J_m s^2}, \qquad K(s) = (J_m D)s. \tag{36}$$

By substituting these into (6), $Q(s)$ is obtained as

$$Q(s) = \frac{D}{s + D}. \tag{37}$$

Therefore, it can be known that below the cutoff frequency (D rad/s), the disturbances are attenuated. From (35), as shown in Fig. 4, the external-loop controller and the feedforward compensator are given by

$$C(s) = (J_m\Lambda)\,s, \quad C_{ff}(s) = J_m s^2. \tag{38}$$

Since the feedforward compensator satisfies $C_{ff}(s) = 1/P_m(s)$, the following inequality can be obtained:

$$|\,e(j\omega)\,| \leq \frac{1}{|\,1 + D/(j\omega)\,|} \times |\,W(j\omega)\,| \tag{39}$$

where $|\,W(j\omega)\,| = \left| \frac{P_m(j\omega)d_{eq}(j\omega)}{1+P_m(j\omega)C(j\omega)} \right|$. Therefore, if the gain Λ of $C(s)$ is fixed and the magnitude of D of (36) is increased N times, then it can be roughly said that the magnitude of error is reduced by the factor of $|\,[1 + D/(j\omega)]/[1 + (N \times D)/(j\omega)]\,|$. Specifically, when D is large enough or the system is operated in low frequency range, it can be predicted that if D is increased by N times, the error will be reduced to its $1/N$, approximately.

To make the steady state error of the high-accuracy positioning system, integral control can be considered. Let $\int_0^t e(\tau)\, d\tau$ be the variable of interest to design the sliding mode controller in which the integral control is included. The system of (31) is now third-order relative to this variable. Hence, the sliding function is expressed as $\sigma = \dot{e} + 2\Lambda e + \Lambda^2 \int_0^t e\, d\tau$. And then, $\varphi = \dot{x}_d + 2\Lambda x_d + \Lambda^2 \int_0^t x_d\, d\tau$ and $\sigma_a = x_2 + 2\Lambda x_1 + \Lambda^2 \int_0^t x_1\ d\tau$. Therefore, the sliding control input for the system of (31) is obtained as

$$u = J_m(\ddot{x}_d + 2\Lambda\dot{e} + \Lambda^2 e) + B_m\dot{x} + (J_m D)\sigma. \quad (40)$$

If the reference state variable is chosen as $\dot{x}_r = \dot{x}_d + 2\Lambda e + \Lambda^2 \int_0^t e\, d\tau$, then this can be rewritten as following equation in the RIC framework:

$$u = J_m\ddot{x}_r + (J_m D)\,\dot{e}_r + B_m\dot{x}. \quad (41)$$

Consequently, the sliding mode controller with integral control and the controller without integral control are the same in the sense of controller structure. That is, the disturbance attenuation property described by (37) and the inequality equation for the error described by (39) can be satisfied regardless of integral control. Since an integral term is added to the reference state variable and a proportional term is added to the feedback controller of (38), only the internal dynamic equation about the error is expressed differently from the case without integral control. Therefore, if the magnitude of D is increased N times, when D is large enough or the system is operated in low frequency range, the feature that the magnitude of error is reduced to its $1/N$ approximately is not changed.

If Λ is given as B_m/J_m, in the RIC framework, (40) can be rewritten as

$$u = (J_m s^2 + B_m s)\, x_r + [(J_m s + B_m)D]\, e_r \quad (42)$$

where x_r is defined by $x_r = x_d + \Lambda \int_0^t e\, d\tau$. Thus, the reference model and RIC controller are obtained as

$$P_m(s) = \frac{1}{J_m s^2 + B_m s}, K(s) = (J_m s + B_m)\, D. \quad (43)$$

And $Q(s)$ has the form of

$$Q(s) = \frac{D}{s+D}. \quad (44)$$

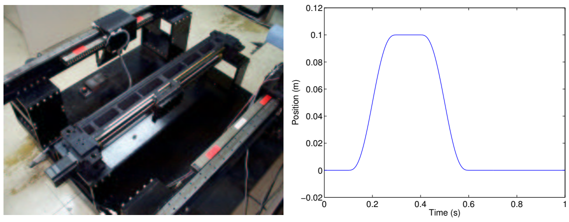
Although the reference model is selected as a different form, thanks to the structural characteristics of the proposed controller, it can be seen that the $Q(s)$ is the same as (37). From (42), the external-loop controller and the feedforward compensator are given by

$$C(s) = (J_m s + B_m)\,\Lambda,\ C_{ff}(s) = J_m s^2 + B_m s. \quad (45)$$

Since the feedforward compensator satisfies $C_{ff}(s) = 1/P_m(s)$, in this case also, the inequality equation of (39) is satisfied as before.

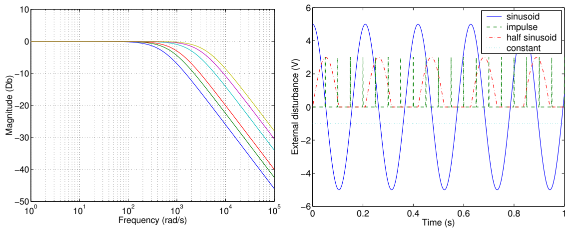
6. EXPERIMENTAL RESULTS

The system considered in this paper is a high-accuracy positioning system shown in Fig. 5 (a). The proposed



(a) Test bed (b) Desired trajectory

Fig. 5. Test bed and desired trajectory graph



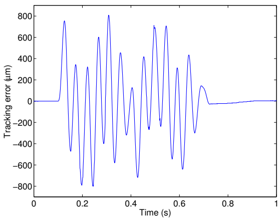
(a) Q by D gain (b) external disturbance

Fig. 6. Q determined by D and external disturbance

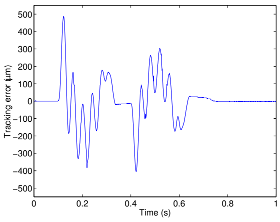
RIC-based tracking controller in (41) is used to stabilize the whole system and track the desired position accurately, where J_m is 0.01, B_m is 0.5, Λ is 150, and D is the gain which should be designed to meet the given performance specifications. The 5th order polynomial function is used to specify the position, velocity, and acceleration at the beginning and end of path. Fig. 5 (b) shows the desired trajectory graph. The control frequency is set to 1 kHz, and the position is measured by rotary encoder, whose resolution is 3.125 μm at the rectilinear motion.

From (41), $P_m(s)$ and $K(s)$ are given by (36) and $Q(s)$ is obtained as (37). For the performance tuning, only D is changed to 500, 750, 1000, 2000, 3000, and 4000. The change of Q function by D is shown in Fig. 6 (a), and resulting trajectory errors are shown in Fig. 7. Fig. 7 (a) shows the initial result for $D = 500$. By increasing D to 750, the result shown in Fig. 7 (b) is obtained. By continuously increasing D, it can be known that the maximum magnitude of resulting error is reduced having the predicted performance law.

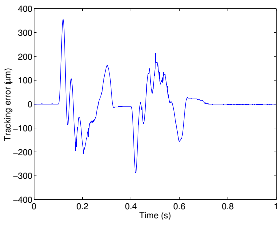
To verify the rate of error reduction for the system when there are external disturbances, the disturbance signals shown in Fig. 6 (b) are added to the control input. Fig. 8 shows the experimental result with external disturbances. D is set as the same value in the experiment without external disturbance. The maximum magnitude of resulting error with and without disturbances for each D gain is shown in Table 1, which shows that if D is N times, the error is reduced to its $1/N$, approximately. Here, note that since (39) is an inequality condition, the reduction ratio may be somewhat larger than $1/N$. And since D is the cutoff frequency of Q as shown in (37), if it is larger than 4000, the error signal is too noisy to verify the reduction ratio. That is, as D gain becomes larger, the cutoff frequency which indicates the disturbance attenuation ability becomes higher.



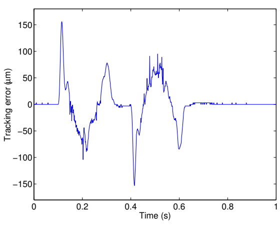
(a) $D = 500$



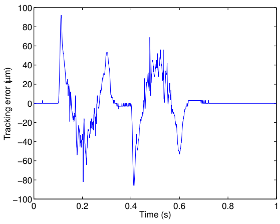
(b) $D = 750$



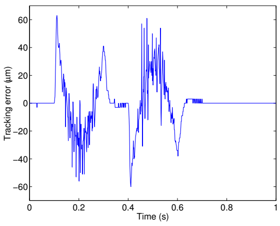
(c) $D = 1000$



(d) $D = 2000$



(e) $D = 3000$



(f) $D = 4000$

Fig. 7. Tracking error by D gain of RIC controller

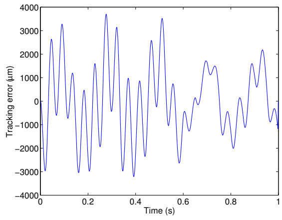
Table 1. e_{max} of tracking error by D

D	e_{max} w/o d_{ex}	e_{max} with d_{ex}
500	8.09×10^{-4} m	3.70×10^{-3} m
750	4.88×10^{-4} m	2.05×10^{-3} m
1000	3.75×10^{-4} m	1.56×10^{-3} m
2000	1.56×10^{-4} m	7.88×10^{-4} m
3000	9.20×10^{-5} m	5.13×10^{-4} m
4000	6.30×10^{-5} m	3.78×10^{-4} m

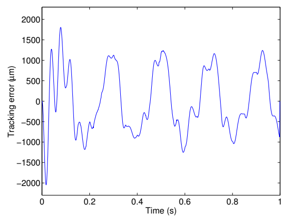
Fig. 9 (a) shows the control input and the external disturbance in the case of Fig. 8 (f), and Fig. 9 (b) shows the sum of the control input and the external disturbance, which is compared with the control input signal in the case of Fig. 7 (f). From this comparison, it can be known that the proposed method successfully compensates external disturbances and could produce high-accuracy motions.

7. CONCLUSION

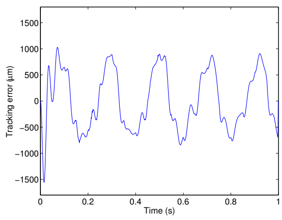
A sliding mode controller for high performance positioning systems was designed in the DOB framework. By analyzing the structural characteristics of RIC and adopting the sliding mode controller into the RIC structure, disturbance attenuation characteristics was shown and the performance was analyzed. The proposed design of sliding mode controller provides a systematic approach to tune the controller parameters for performance specification and robust stability. Through experiments using the high-accuracy positioning system used in the semiconductor chip mounting devices, the validity of the proposed method was verified.



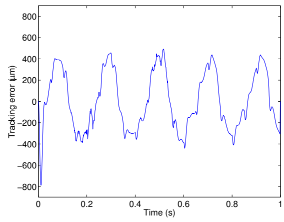
(a) $D = 500$



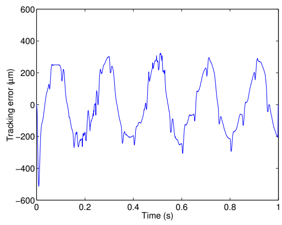
(b) $D = 750$



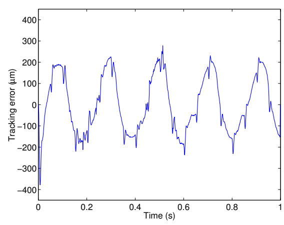
(c) $D = 1000$



(d) $D = 2000$

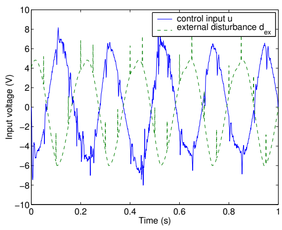


(e) $D = 3000$

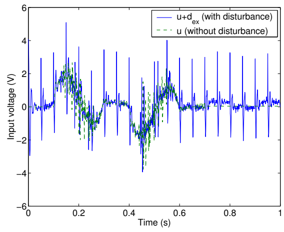


(f) $D = 4000$

Fig. 8. Tracking error of the experiment with d_{ex}



(a) u and d_{ex}



(b) $u + d_{ex}$, u of Fig. 7 (f)

Fig. 9. Control input and external disturbance

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