COMBINED CONTROL BY SPACE ROBOTIC MODULE WITH USING MANIPULATOR'S MOBILITY

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Abstract: An approach of designing of combined energy-economic control of a freeflying robotic module, used e.g. as a transport unit at its flight near by a manned orbital station is considered. The fuel consumption economy for the control of the configuration is achieved by stabilising the module's main body angular position with the help of a motion "exchange" between the module's main body's and the manipulator's links activated by their arm actuators. The control algorithms are suggested and the dynamics of the operation of all interacting subsystems module's for the combined control are investigated. A mathematical simulation was carried out taking into account the nonlinear properties of Lagrange module's equations and the nonlinear characteristics of the control subsystem's elements. *Copyright* © 2002 IFAC

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1. INTRODUCTION.

The free-flying space robotic module (SRM) (Putz, 1999), (Kirchhoff, et al., 2000) is a relatively new type of small spacecrafts moving in the manned orbital space station (SS) neighbourhood and intending for different purposes of its servicing. This spacecraft consists of rigid platform (main body) to which one or more multilink manipulators are attached via hinges. The manipulators are intended for grasping payloads which may be either rigid or flexible bodies. In particular the SRM are to have the possibilities to execute the following operations: -Launching and servicing small space vehicles from the SS; -servicing experiments which are performed outside the SS; -its inspection and repair operations of the station's outside elements, -retrieving of payloads including cosmonauts separated from the station in the case of emergency; -remotely controlled space structures assembly in the SS' neighbourhood etc. As an example in Fig.1

a typical SRM configuration is shown in the mode of payload transportation. Taking into account the

above mentioned variety of SRM's operations representing a complicated rather object from control point of view is presented in the following. For the most modes of SRM's operations it must be considered as a complex mechanical structure (multilink



manipulators on a moving base) influenced by many nonlinear control and disturbance actions. In such case the SRM is characterized by an essential variation of its mass-inertia characteristics. It comprises not only by the different masses and dimensions of payloads but also by the position of the manipulators' links and payload with respect to the main body. One of the SRM's peculiarities is the mobility of its main body during manipulators operation. Other SRM's peculiarities are the multiple recurrent dynamic operations (different manoeuvres near by SS, docking with SS, zero-length launches and so on), the inconsistency of the requirements to their execution, and the necessity of the realization of the transient processes' monotony near by SS. All these peculiarities lead to the statement of the new tasks in control of spacecrafts. Among others, key tasks are the technical controllability, the optimal configuration of SRM during its flight with and without different payloads, the different types of payloads' grasping, the concept of SRM's safety manoeuvring near by SS, the anti-resonance control of SRM for transportation of highly flexible payloads and the task of energyeconomic control design, which is very important.

In this paper the new approach to the energyeconomic attitude control design during the motion of the SRM into the safety corridors (Kirchhoff, *et al.*, 2000) using the manipulators' links mobility for attitude control is considered in the class of combined control (Razygraev, 1977).

2. MATHEMATICAL MODEL OF SRM AS ELECTROMECHANICAL SYSTEM

The task of energy-economic attitude control is considered for the example of flat motion of the SRM with a single manipulator. For this case the SRM's mathematical model represented in the Lagrange form (Rutkovsky, *et al.*, 1999), (Rutkovsky and Sukhanov, 2000) is the following

$$A(q)\ddot{q} + H\dot{q} + Bq = F_u(t) - \sum_{s=1}^n \left[\dot{q}^T D_k(q) \dot{q} \right] e_k, \quad (1)$$

where

 $q = (q_1 = X_0, q_2 = Y_0, q_3 = \vartheta, q_4 = \alpha_1, q_5 = \alpha_2, q_6 = \alpha_3)$ is generalized coordinates vector. The first components q_1, q_2, q_3 or $q^0 = (X_0, Y_0, \vartheta)^T$ we shall consider as independent generalized coordinates, which define the main body position in the inertial coordinate system. The components q_4, q_5, q_6 , or $q^\alpha = (\alpha_1, \alpha_2, \alpha_3)^T$ define the position of manipulator's links; A(q) is symmetric matrix of inertia coefficients; H, B - are constant $(n \times n)$ matrices, which are introduced when we take into consideration the dissipative and elastic properties of the module; $F_u(t)$ is column vector of

control forces;
$$\sum_{s=1}^{n} \left[\dot{q}^{T} D_{k}(q) \dot{q} \right] e_{k} = - \left(Q_{k}^{Cor} + Q_{k}^{Cf} + Q_{k}^{add} \right)$$

is matrix of Coriolis and centrifugal forces; e_k – is *n*-dimensional unit vector with the *k* -th nonzero row. Assuming payload as a rigid body and representing it by a point mass a conventional simplified model can be obtained from the equation (1) in the form

$$\begin{aligned} a_{\chi_{0}\chi_{0}}(q^{r})\ddot{X}_{0} + a_{\chi_{0}\chi_{0}}(q^{r})\ddot{Y}_{0} + a_{\chi_{0}\eta}(q^{r})\ddot{\Theta} + a_{\chi_{0}\eta}(q^{r})\ddot{\alpha} + a_{\chi_{0}\mu_{2}}(q^{r})\ddot{\alpha}_{2} = F_{x} + F_{x}^{d}, \\ a_{\chi_{0}\chi_{0}}(q^{r})\ddot{X}_{0} + a_{\chi_{0}\chi_{0}}(q^{r})\ddot{Y}_{0} + a_{\chi_{0}\eta}(q^{r})\ddot{\Theta} + a_{\chi_{0}\mu_{2}}(q^{r})\ddot{\alpha}_{2} = F_{y} + F_{y}^{d}, \\ a_{\chi_{0}}(q^{r})\ddot{X}_{0} + a_{\chi_{0}}(q^{r})\ddot{Y}_{0} + a_{\eta_{0}\eta}(q^{r})\ddot{\Theta} + a_{\eta_{0}\mu_{2}}(q^{r})\ddot{\alpha}_{2} = M_{y} + M_{y}^{d}, \end{aligned}$$

$$\begin{aligned} (2) \\ a_{\chi_{0}}(q^{r})\ddot{X}_{0} + a_{\chi_{0}}(q^{r})\ddot{Y}_{0} + a_{\eta_{0}\eta}(q^{r})\ddot{\Theta} + a_{\eta_{0}\mu_{2}}(q^{r})\ddot{\alpha}_{2} = M_{y} + M_{y}^{d}, \end{aligned}$$

$$\begin{aligned} a_{\chi_{0}}(q^{r})\ddot{X}_{0} + a_{\eta_{0}}(q^{r})\ddot{Y}_{0} + a_{\eta_{0}\eta}(q^{r})\ddot{\Theta} + a_{\eta_{0}\mu_{2}}(q^{r})\ddot{\alpha}_{2} = M_{y} + M_{y}^{d}, \end{aligned}$$

$$\begin{aligned} a_{\chi_{0}}(q^{r})\ddot{X}_{0} + a_{\chi_{0}}(q^{r})\ddot{Y}_{0} + a_{\eta_{0}\eta}(q^{r})\ddot{\Theta} + a_{\eta_{0}\mu_{2}}(q^{r})\ddot{\alpha}_{2} = M_{y} + M_{y}^{d}, \end{aligned}$$

where: $F_X^d, F_Y^d, M_{\vartheta}^d$ - are external disturbances; coefficients a_{kj} are obtained in (Rutkovsky and Sukhanov, 2000).

Usually in robotics DC-motors with separate excitation and with worm train are used as the arm actuators. As the first approximation (without taking into account of motor's response time, backlashes, forces of dry friction etc) they can be described for the any *i*-th joint (further the index *i* will be omitted) by the following equations:

$$k_{m}M_{m}(t) = \frac{1}{k_{b}}u(t) - \omega_{m}(t), \ J_{m}\dot{\omega}_{m}(t) = M_{m}(t) + M_{Lm}(t),$$
(3)
$$\omega_{m}(t) = \omega(t)i_{g} = \dot{\alpha}(t)i_{g}, \ M_{L}(t) = M_{Lm}(t)i_{g}, \ M_{\alpha}(t) = i_{g}M_{m}(t),$$

where: $M_m(t)$ is the DC motor's torque, α_m is the DC-motor's output, u(t) - is the DC-motor's input, $\omega_m(t) = \dot{\alpha}_m$ - is the rate of DC-motor's shaft, $M_{Lm}(t)$ - is the moment of loading which is defined by the time lag of all subsequent after the *i*-th manipulator's links and joints and payload, i_g is the gear ratio of the gearbox. $\omega = \dot{\alpha}, \alpha, M_\alpha(t)$ are the outputs of the arm actuator, $M_L(t)$ is the reduced to the shaft moment of loading; other coefficients are defined in (Yurevich, *et al.*, 1984).

The equation (2) of mechanical part of the SRM together with the equation for the drives (3) represent the mathematical model of the SRM as electromechanical system which is necessary for the synthesis of energy-economic algorithms for the SRM' attitude control. It is necessary to note that the arm actuators must possess by the irreversible property, hence at $M_{\alpha_i} = 0, i = 1,2$, we have $\dot{\alpha}_i = 0, \alpha_i = const$ and the coincident equations of the system (2) are vanished. The model (3) of the idealized arm actuator enables to understand very important peculiarity of the SRM's mathematical model. At the moment $M_{\alpha i}(t) = 0$ the *i*-th joint becomes "necrosis". As the effect of the necroses we have the temporary loss in the system (2) of "equation for α_i ". Mathematically these modification are expressed in decreasing (or increasing) of the system's (2) order on $2 \times r$ units, where r is the number of simultaneously operating (or taking off) restrictions $M_{\alpha i} = 0$.

3. SOME QUESTIONS OF BASE CONTROL SYSTEM OPERATION DURING THE SRM'S FLIGHTS NEAR BY SS

Let us remind of some pre-conditions assumed for the design of the base control system. As the measured coordinates the following ones are used: in the subsystem control for translational motion - the parameters of the SRM relative motion (range, closing rate, lateral velocity) in the quasi inertia coordinate system fixed with SS' radar range which forms the current corridor of SRM's flight (this information can be achieved with the help of the usual radio systems which are used at constant-bearing guidance in the regimes of closing and mooring (Razigraev, 1977)); for the subsystem orientation - the signals of attitude sensors and angular-rate sensors in the same coordinate system. These sensors have linear outputs with saturations. It is assumed that the actuators for both subsystems are electric-jet engines or gas-jet engines, which can realize small accelerations. Furthermore Descartes' scheme of engines nozzles' disposition, which does not require reversal, turns at the regimes of acceleration and deceleration is used.

The motion control of the manipulator's links is realized by typical robotic drives which are chosen according to the requirements of mounting-assembling operations in open space (Putz, 1999).

At the synthesis of the base control algorithms of SRM's translational-motion near by the SS the method of constant-bearing guidance is used (Razigraev, 1977). In this case we may synthesize the base algorithms without taking into account the laws of orbital motion.

When the SRM enters into the action zone of the search and tracking radio (or other type) system belonging to the considered safety corridor (Kirchhoff, et al., 2000) mutual capture is realized. After that the SRM's longitudinal axis is oriented along the line of sight of the radar range which is situated on the SS in the end of the safety corridor. After that the engines (their thrust is $F_{\rm x}$) of the closing control subsystem accelerate the SRM to the minimal necessary velocity $V_{\rho i}$ which is defined by the limitation on the disposed deceleration impulse at the interval of the intersection of the considered corridor and the next one. After the acceleration the engine is switched off $(F_r = 0)$ and the rest part of motion (before the beginning of the deceleration) the SRM flies at the expense of inertia. On this segment of SRM's motion the base control system must remove the lateral velocity Y_0 and the lateral displacement Y_0 of the SRM from the line of sight and must stabilize the angular position of the SRM's main body. In particular the coincidence of the SRM's longitudinal axis with the line of sight at the disturbing moment $M^{d}_{\vartheta}(t)$ must be fulfilled. The required economic fuel's consumption for the control with the help of gas-jet engines is realized by forming of one-sided stable limit cycle. In this case the control is optimal from the point of view of the fuel's consumption because the relation $\int_{0}^{t} |M_{\vartheta}| dt = \int_{0}^{t} |M_{\vartheta}^{d}| dt$

is fulfilled (Razigraev, 1977).

Now it is worth while to set up the task about the additional economic fuel's consumption for the attitude control in reference to the aforementioned situation. This we solve by using the mobility of manipulator's links for SRM's attitude control.

4. ALGORITHMS OF COMBINED ATTITUDE CONTROL WITH USING MANIPULATOR'S MOBILITY

The attitude control of SRM using manipulator's mobility will be referred to as combined control using two types of control devices (gas-jet engines and manipulator's arm actuators) for the solving of the single task of SRM's orientation. To some extend this method is similar to the control of spacecraft's by hand-wheels and the gas-jet engines, and it was shown (Razigraev, 1977) that the method guarantees fuel consumption-optimal control. During the flights into the safety corridors SRM moves on runout $(F_r = 0)$ and the main task of control is the angular stabilization of SRM's main body that is required for the relative motion parameters measuring and the effective work of lateral motion's control subsystem $(F_v \neq 0)$. Usually during the SRM's flight with and without payload from one point in the space to another, the manipulator's links are motionless and they are situated in the optimal position at which SRM's center of mass coincides with the center of translational forces' applying. Gas-jet engines and base algorithms realize the orientation. Giving up the manipulator's links immobility enables us to consider the task of the fuel consumption economy for the attitude control at the expense of purposeful moving of the manipulator's links. The idea of such control is the following. The required SRM's angular stabilization is realized by the "exchange" of the motions between the SRM's main body and the manipulator's links which are activated by arm actuators (electrical drives). Consumed for this type of control energy is electrical one and it can be recovered further at the expense of the SRM's accumulator additional charging on the SS' board. By virtue of the displacement restrictions of the manipulator's links the event of the "saturation" (the links turn on their maximum angular displacements) can occur and the considered type of control becomes impossible. For recovering the control workability the "discharging" regime is required. During this regime gas-jet engines realize the SRM's orientation and the manipulator's links reset at their initial positions by arm actuators.

4.1 Algorithm's synthesis of the combine control. At the forming of the combined control algorithms it is necessary to take into account the following peculiarities: 1) the domains of admissible turning of the manipulator links angles and angular velocities are restricted ($|\alpha_i(t)| \le \alpha_{i \max}$, $|\dot{\alpha}_i(t)| \le \dot{\alpha}_{i\max}$); 2) the coordinates' displacements of manipulator's links from their optimal values lead to the removal of the SRM's mass center with respect to the center of applying the lateral force F_y , therefore the disturbance moment can occur in the orientation system; 3) gas-jet engines have relay characteristics, arm actuators' moments are restricted; 4) the conditions of technical controllability (Kirchhoff, *et al.*, 2000) must be fulfilled.

Let us denote the base algorithms of the SRM's orientation control by gas-jet engines and control of the manipulator's position as $u_{\vartheta,\vartheta} = f_{\vartheta}(\vartheta, \vartheta, t)$ and $u_{\alpha,i} = f_{\alpha i}(\alpha_i, t)$ respectively and the algorithm of the SRM's orientation control by using mobility of manipulator's links as $u_{\vartheta,\alpha_i} = f_{\vartheta,\alpha_i}(\vartheta,\dot{\vartheta},\alpha_i,t)$. At the synthesis consumption-economical of algorithm $u_{\vartheta,\alpha_i} = f_{\vartheta,\alpha}(\vartheta, \dot{\vartheta}, \alpha_i, t)$ the simplified model (2) is used. On the inputs of the three last equations of the system (2) the control moments $(0, M_{n,\alpha}, M_{n,\alpha})^T$ is applied instead of base control moments $M_{\vartheta}(u_{\vartheta,\vartheta})$ and $M_{\alpha_i}(u_{\alpha_i})$. The disturbance $M_{\alpha_i}^d$ assumed to be assigned (usually $M_{\vartheta}^d = const$). In this paper we further suppose that only one control moment $M_{\eta \alpha 1}$ is used, created by arm actuator of manipulator's shoulder. The system (2) can be represented in the form

$$A(q)\ddot{q} = F_q + F_q^d , \qquad (4)$$

where $q = (\vartheta, \alpha_1, X_0, Y_0)^T$, $F_q = (0, M_{\vartheta \alpha 1}, 0, F_y)^T$ is the vector of control actions, $F_q^d = (M_{\vartheta}^d, 0, 0, 0)^T$ is the vector of a disturbances, $A(q) = [a_{ij}(\vartheta, \alpha_1, \alpha_2, \lambda)]$ - is the quadratic (4x4) symmetric matrix, the part of its coefficients depend on the coordinates $\vartheta, \alpha_1, \alpha_2$ and SRM's and payload's parameters λ . As $\alpha_2 = const$ it is considered as the parameter, so it is included into the range of λ and the index 1 at α_1 will be omitted. In the regime of angular stabilization we have $\vartheta \approx 0$. So we shall consider that the elements of the matrix A(q) depend on only the coordinate α . Matrix equation (4) can be solved with respect to the higher derivatives: $\ddot{q} = [a_{ij}(\cdot)]^{-1}(F_q + F_q^d)$. Let us select the solution for $\ddot{\vartheta}(t)$ and write it in the following form

$$\ddot{\vartheta} = (\det[a_{ij}(\cdot)])^{-1} (k_{M\alpha} M_{\vartheta\alpha} + k_{M^d} M_{\vartheta}^d + k_{Fy} F_y), \quad (5)$$

where

$$k_{M\alpha} = a_{\vartheta\alpha} (a_{X_0Y_0}^2 - a_{X_0X_0} a_{Y_0Y_0}) + a_{X_0\alpha} (a_{\vartheta X_0} a_{Y_0Y_0} - a_{\vartheta Y_0} a_{X_0Y_0}) + a_{Y_0\alpha} (a_{\vartheta Y_0} a_{X_0X_0} - a_{\vartheta X_0} a_{X_0Y_0})$$
 is

the coefficient of control moment's $M_{\vartheta \alpha i}$ efficiency

on the controlled coordinate ϑ ;

$$k_{M_{\vartheta}^{d}} = a_{\alpha\alpha}(a_{X_{0}X_{0}}a_{Y_{0}Y_{0}} - a_{X_{0}Y_{0}}^{2}) + a_{X_{0}\alpha}(a_{\alpha Y_{0}}a_{X_{0}Y_{0}} - a_{\alpha X_{0}}a_{Y_{0}Y_{0}}) + a_{Y_{0}\alpha}(a_{X_{0}\alpha}a_{X_{0}Y_{0}} - a_{Y_{0}X_{0}}a_{Y_{0}\alpha})$$

is the coefficient of the influence of the external disturbance M^d_{ϑ} on the coordinate ϑ ;

$$\begin{split} k_{F_{Y}} = & a_{\partial\alpha}(a_{\chi_{0}\chi_{0}}a_{\chi_{0}\alpha} - a_{\chi_{0}\chi_{0}}a_{\chi_{0}\alpha}) + a_{\alpha\alpha}(a_{\partial\chi_{0}}a_{\chi_{0}\chi_{0}} - a_{\partial\chi_{0}}a_{\chi_{0}\chi_{0}}) + \\ & + a_{\chi_{0}\alpha}(a_{\partial Y_{0}}a_{\alpha X_{0}} - a_{\partial X_{0}}a_{Y_{0}\alpha}) \text{ is the coefficient of the influence of the control force } F_{y} . \end{split}$$

the initialities of the control force T_y .

Let us rewrite the equation (5) in the form

$$\vartheta = k_{M\alpha}(\alpha, \lambda) M_{\vartheta\alpha}(u(t)) + M_{\Sigma}^{d}(\alpha, \lambda, t), \qquad (6)$$

where $\bar{k}_{M\alpha}(\alpha,\lambda) = (\det[a_{ij}(\cdot)])^{-1}k_{M\alpha}(\alpha,\lambda)$ - is the reduced coefficient of the control moment $M_{\partial\alpha}$ effectiveness, which is suit for the condition (it is easy to obtain using the results from (Rutkovsky and Sukhanov,2000)) $\bar{k}_{M\alpha}(\alpha,\lambda) > 0 \forall (\alpha) \in (0,\pm\pi);$ $\overline{M}_{\Sigma}^{d} = \bar{k}_{M_{\theta}^{d}}(\alpha,\lambda)M_{\theta}^{d} + \bar{k}_{Fy}(\alpha,\lambda)F_{y}$ is the resulting reduced moment of the disturbances; $\bar{k}_{M_{\theta}^{d}}(\alpha,\lambda) = (\det[a_{ij}(\cdot)])^{-1}k_{M_{\theta}^{d}}(\alpha,\lambda),$ $\bar{k}_{Fy}(\alpha,\lambda) = (\det[a_{ij}(\cdot)])^{-1}k_{Fy}(\alpha,\lambda).$

The moment $M_{\vartheta,\alpha}$ can be chosen as linear function of the SRM's angular displacement and its derivative, which are measured. So

$$M_{\vartheta\alpha} = -\tilde{k}_0 k_A (k_{\dot{\vartheta}} \dot{\vartheta} + \vartheta) \tag{7}$$

where $k_A = (k_m k_b)^{-1}$ is the static coefficient of arm actuator; \tilde{k}_0 is the control algorithm $u_{\vartheta\alpha}$ parameter. It can be adjustable if it is necessary.

If the control (7) is realizable the task of the synthesis of the control algorithm's linear part will amount to the fulfillment of the stability conditions and high indexes of quality of transient processes in the system described by the equation (6). But the coefficients of this equation depend on the coordinate α and they are not constant. Taking into account (7) let us rewrite the equation (6) in the following form:

$$\ddot{\vartheta} + k_A k_{\dot{\vartheta}} (\tilde{k}_0 \bar{k}_{M\alpha} (\alpha, \lambda)) \dot{\vartheta} + k_A (\tilde{k}_0 \bar{k}_{M\alpha} (\alpha, \lambda)) \vartheta = \overline{M}_2^d (\alpha, \lambda, t) \quad (8)$$

Considering the parameters λ to be known and $\alpha(t)$

measured it is advisable to have the coefficient $\tilde{k}_0(t)$ as adjustable one in according to the law

$$\widetilde{k}_{0}(t)\overline{k}_{M\alpha}(\alpha,\lambda)) = K = const$$
(9)

In this case the coefficients of the equation (8) are constant and positive and this guarantees the system's stability. The desired SRM's dynamic is realized using well-known methods. In particular the realization of the required static accuracy $|\vartheta(t)| \le \vartheta_{\min}$ in the system (8), (9) is satisfied by the fulfillment of the condition $k_A \ge (K \vartheta_{\min})^{-1} [M_{\Sigma}^d(\alpha, \lambda, t)]_{\max}$. The optimal relation between the index of the speed of acting and the index of oscillation is guaranteed by choosing of the relative damping coefficient in the equation (8) $\xi = 0.7$. Hence we have $k_{\dot{\vartheta}} = 1.4\sqrt{k_A}$. Now it is necessary to find $u_{\vartheta,\alpha}(t) = f_{\vartheta,\alpha}(\vartheta, \dot{\vartheta}, \alpha, t)$ in order to $M_{\vartheta\alpha}(u_{\vartheta,\alpha}(t))$ suit for the equation (7). For solving this task let us transform the equations (3) to the following form

$$J_{m}k_{m}\dot{M}_{\alpha}(t) + M_{\alpha}(t) = \frac{J_{m}\dot{t}_{g}}{k_{b}}\dot{u}(t) - M_{L}(t) \quad (10)$$

Substituting in the equation (10) the expressions for $M_{\alpha} = M_{\vartheta \alpha}$ and taking into account as simplification (Yurevich *et al.*, 1984) the correlation $M_L \approx -J_L \ddot{\alpha}$, where $J_L = a_{\alpha\alpha}$ is the reduced to the considered manipulator's joint moment of inertia, we shall obtain the control law

$$u_{\partial\alpha}(t) = -\tilde{k}_0 i_g^{-1} [k_{\dot{\vartheta}} \dot{\vartheta} + (1 + \frac{k_{\dot{\vartheta}}}{k_m J_m}) \vartheta + \frac{1}{k_m J_m} \int \vartheta dt + \frac{k_b J_L}{\tilde{k}_0 J_m} \dot{\alpha}] \quad (11)$$

This control law guarantees in the admissible domain of the α variation $\alpha \in [\alpha^1, \alpha^2]$ stable system's motion and required quality of control. The final correction of the control algorithm, taking into account the nonlinear model (1) and typical nonlinear characteristics of the control system's elements can be executed with the help of a mathematical simulation.

The combined control system of SRM's orientation is used on each linear segment (corridor) of SRM's flight after the completion of the process of a reorientation and acceleration to the velocity V_{ρ_i} and it works in this way. In general case remainder nonzero starting conditions $\vartheta_0, \dot{\vartheta}_0$ and forced motion which is the result of low disturbance moment are extinguished by moment $M_{\alpha}(u_{\beta,\alpha i})$ according to the suggested algorithm. The angles of the manipulator's links α_1, α_2 will have the displacements. This process is completed either by steady-state small oscillations in the domain $|\vartheta(t)| \leq \vartheta_{\min}$ or reaching of the angles α_1, α_2 their restrictions. In the last case it is necessary to reset the manipulator's links in their optimal position (the regime of discharge) in order to have the possibility to use further suggested method of combined control.

4.2. Algorithm's synthesis of the discharge regime.

When the manipulator links angles reach their restrictions the regime of discharge is switched on. The arm actuators applying the moments $M_{\alpha i}(u_{\alpha i})$ must reset the links at the their optimal position $\alpha_i(t) \rightarrow \alpha_i^*$ for the minimum short time. At the same time attitude control using the moment $M_{\vartheta}(u_{\vartheta,\vartheta})$ must retain the SRM's main body in the required position and guar-

antees $|\vartheta(t)| \leq \vartheta_{\min}$. Here the task of the attitude control is the task of multi-connected control by nonlinear system with restrictions on the control moments. For describing the discharge regime in our case ($\alpha_2 = const$) it is necessary in the equations (4) to accept as the control vector the following one $F_q = (M_{\vartheta\vartheta}, M_{\alpha}, 0, F_y)^T$. The equation (6) will be rewritten in the form

$$\ddot{\partial} = \bar{k}_{M\partial}(\alpha, \lambda) M_{\partial\partial} + [\overline{M}_{\Sigma}^{d}(\alpha, \lambda, t) + \bar{k}_{M\alpha}(\alpha, \lambda) M_{\alpha}], \quad (12)$$

where $\bar{k}_{M\vartheta}(\alpha,\lambda)$ is the effectiveness' coefficient of the control moment $M_{\vartheta}(u_{\vartheta,\vartheta})$, which is calculated by the same method as the coefficient $\bar{k}_{M\vartheta}(\alpha,\lambda)$. The last two terms are considered as the disturbances. At the synthesis of control algorithms by coordinates ϑ and α_1 the contradictoriness of the requirements to the work of each subsystem must be taken into account. For high speed of acting of control subsystem by coordinate α the maximum admitted rate $\dot{\alpha}_{max}$ (with taking into account the restriction on $M_{\alpha}(u_{\alpha})$ must be used. But here rather big disturbance $k_{M\alpha}(\alpha, \lambda)M_{\alpha}$ in the subsystem by coordinate ϑ at the restriction of $M_{\vartheta}(u_{\vartheta,\vartheta})$ can lead to the violation of the SRM's main body orientation accuracy. Hence the rate $\dot{\alpha}_{max}$ must be restricted by corresponding value. Optimal value of the controllers' parameters which guarantee the desired dynamic of the discharge regimes can be chosen by phase plane method on the base of the eq (12). Developed in (Razygraev, 1977) base nonlinear algorithm of SRM's orientation $u_{\vartheta,\vartheta} = f(\vartheta, \dot{\vartheta})$ forms in the steady-state and without disturbances the autooscillations which are represented on the phase plane by one-sided stable limit cycle Γ_0 . In the regime of manipulator's discharge as the result of the disturbance $M_{\alpha} = k_{M\alpha}(\alpha, \lambda)M_{\alpha}$ and other disturbances limit cycle Γ_0 is vanished. But the phase trajectories must not get out from the domain of admissible displacements of ϑ and $\dot{\vartheta}$ $(|\vartheta = \vartheta_{Allow}|\dot{\vartheta} = \dot{\vartheta}_{Allow}$. This domain is depicted in Fig. 2 by dotted lines. Choosing of the base

ing of the base algorithm's parameters can fulfill the aforementioned requirement. After accomplishment of the discharge regime the control moment

 $M_{\vartheta\alpha}(u_{\vartheta,\alpha})$ is applied again.



Fig. 2. Limit cycles for the "discharge" regime of the manipulator

5. DIGITAL SIMULATION

The validity of the suggested SRM's combined control and algorithms for the orientation in reference to the nonlinear model (1) was confirmed by a digital simulation for a test object in the regime of its flight into the corridor. Mass-inertia characteristics and geometric dimensions of the manipulator were chosen in accordance to the conditions of optimal configuration's attainability in the mechanical system: main body-manipulator-payload. They are obtained in (Rutkovsky et al., 1999) for the mass and inertia moment of main body ($m_0 = 300 kg$, $I_0 = 120 kgm$,); $m_L = 80kg$, $m_1 = 18kg$, $m_2 = 12kg$, $m_{\Sigma} = 410kg$, $r_1 = 0.8m$, $r_2 = 0.7m$. The control system coefficient's values were obtained as the result of the synthesis of the combined control taking into account the conditions of stability and the requirements on the control accuracy. They are the following: $M_{\vartheta}=2.0Nm$, $M_{\alpha}=1.0Nm$, $k_{\dot{\vartheta}}=10vs/rad$, $\varepsilon_{\vartheta}=0.02rad$, $\gamma_{\vartheta}=0.001rad$; $F_x=0, F_y=2.0N, M_{a}^d=0.02Nm, \varepsilon_Y=0.1 m, \gamma_Y=0.01m,$ $k_{\dot{y}} = 1.0vs/m$, where $k_{\dot{y}}, k_{\dot{y}}$ - response times in the base algorithms, \mathcal{E}_{ϑ} , \mathcal{E}_{Y} , γ_{ϑ} , γ_{y} - values of dead zones and hysteresis of the base algorithms nonlinear part. As the starting conditions the next ones were accepted: $\vartheta_0 = 0.05$, $\dot{\vartheta}_0 = 0$, $\dot{Y}_0 = 0.01 m/s$; $\alpha_2^* = -0.7 rad$, $\alpha_1(0) = \alpha_1 = 0.43 rad$. Typical nonlinear characteristics of the attitude sensors, the rate sensors and other control system elements (dead zones, saturations) were

taken into consideration. Transient processes for the following control law

$$u_{\vartheta,\alpha}(t) = -\widetilde{k}_0 i_g^{-1} [k_{\vartheta} \dot{\vartheta} + (1 + \frac{k_{\vartheta}}{k_m J_m})\vartheta + \frac{1}{k_m J_m} \int \vartheta dt + \frac{k_b J_L}{\widetilde{k}_0 J_m} \dot{\alpha}_1]$$

are shown in Fig 3.



Here we can see that the control by the moment $M_{\partial\alpha}(u_{\partial,\alpha})$ at $\alpha(t) < \alpha^1$ is possible to avoid the operation of gas-jet engines of the orientation system and to realize some economy of the fuel's consumption. The regime of the manipulator's discharge that occurs at t = 50 s ($\alpha(t) = \alpha^1$) lasted 9s. During these 9s three impulses of the control moment $M_{\partial}(u_{\partial,\partial})$ created by gas-jet engines were needed for the SRM's orientation. After the accomplishment of the discharge's regime at t=58s ($\alpha = \alpha *$) the orientation system goes on the functioning in the regime of control by the moment $M_{\partial\alpha}(u_{\partial\alpha})$.

6. CONCLUSION

The developed combined control by SRM using the mobility of the manipulator's links permits to decrease to some extend the fuel's consumption for control in the regime of flying into the safety corridors. This type of control is especially important for such small spacecrafts as SRM because it is very difficult to place the heavy and rather unwieldy electromechanical control devices (flywheels, gyrostabilizer etc.) in the spacecrafts at small dimensions. The suggested approach to the algorithms synthesis of combined control multi-connected processes can be useful for solving other task, which occur for the other regimes of SRM's operation.

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