15th Triennial World Congress, Barcelona, Spain

ABSOLUTE STABILITY OF DYNAMICAL SYSTEMS (SURVEY)

Mark R. Liberzon*

* *Moscow State Aviation Technological University,*
27 Petrovka Street, Moscow 103031, Russia, mark@fund.ru

Abstract: This paper presents results of research on the theory of absolute stability and their applications from the theory's birth in 1944 to our days. This is a continuation and wide extention of historical essay presented at the NOLCOS-2001 in St-Petersburg, Russia, July 4-6, 2001. Monographs, surveys, more than one thousand papers on absolute stability are discussed.

Keywords: Absolute stability, Lur'e problem, Dynamical system

1. THE BEGINNING

In 1944 A.I. Lur'e and V.N. Postnikov published a paper (Lur'e and Postnikov, 1944), formulating a new problem on the stability of controlled systems. The origin of the theory of absolute stability is due to that paper.

It is necessary to mention the names of two more researchers who played a significant role in the formation of the theory of absolute stability. B.V. Bulgakov had been solving another problem (Bulgakov, 1942, 1946) – he was interested in conditions of absence of auto-oscillations. However, for the first time he considered differential equations with such non-linear characteristic that it is only known that it lies in some given angle. This method of defining the non-linear characteristic was used by the authors of the above paper (Lur'e and Postnikov, 1944) and was later widely accepted in the theory of absolute stability. Moreover, Bulgakov's problem on accumulation of oscillations served as a basis for a series of studies on absolute stability of non-stationary systems with the use of variation methods. The paper of M.A. Aizerman (Aizerman, 1949) gives an elaboration for stating the problem of absolute stability and formulates a famous Aizerman's Hypothesis. This and his later works (Aizerman and Gantmacher, 1963; Aizerman, 1966) affected the development of the theory of absolute stability.

During subsequent years the notion of absolute stability was substantially widened, in the sense of definitions given by different authors, and in the sense of variety of considered systems. Nonetheless, the main idea of the Lur'e Problem is still the same. The problem of absolute stability of automatic control systems consisted of determining the conditions for the zero solution $y = 0$ of the system

$$dy/dt = Ay + b\varphi(\sigma),\ \ \sigma = c^*y,\ \ \varphi(0) = 0 \qquad (1)$$

to be asymptotically stable in the large for any function $\varphi(\sigma)$ such that

$$0 \le \varphi(\sigma)\sigma \le k\sigma^2,\ k>0 \text{ a number.} \qquad (2)$$

That is, the function $\varphi(\sigma)$ lies in the angle [0,k]. Here, y is an n-dimensional vector of generalized coordinates; A is the square (n x n) matrix, b – a column, c* – a row, $\varphi(\sigma)$ – a real number. It is required, of course, that the function $\varphi(\sigma)$ provides the existence and uniqueness of the solution of system (1),(2).

The Lur'e Problem on absolute stability has been and still is attracting attention of numerous scientists and engineers from different countries. This is not only due to theoretical interest, directed toward the development of dynamic systems, but also to great significance of the Lur'e Problem in applied matters. The problem is related to providing dependable functioning of control systems with changes of technical characteristics, initially unknown but inevitable in the maintenance process. By far, about 1500 papers, surveys and monographs have been published, regarding the theory of absolute stability, and the research in this field is being continued.

2. HYPOTHESES

It is significant that the progress of solving the Lur'e Problem of absolute stability is related with a large number of hypotheses, which had been set, not confirmed, rejected by counterexamples, but still were influential for approaching the solution of the problem. Let us dwell on some of them, apparently, the most essential.

In 1949 M.A. Aizerman (Aizerman, 1949) formulated a problem on stability in the Hurwitz angle, which descends to the following.

Consider a linear function from the angle [0,k] instead of the non-linear $\varphi(\sigma)$ in system (1),(2):

$$\varphi(\sigma) = h\sigma, \ \ h \in [0,k]. \qquad (3)$$

Suppose that the system obtained this way is asymptotically stable regardless of the choice of the number $h \in [0,k]$; i.e., the angle [0,k] is the Hurwitz angle. Does this imply that the non-linear system (1) is absolutely stable within the class of non-linear characteristics (2)?

The Aizerman's Hypothesis: the answer is affirmative. If this hypothesis were true, then the exploration of absolute stability of non-linear systems would be limited to solving the problem of asymptotic stability of linear systems. This is why many studies were devoted to solving the Aizerman's problem. It was possible to identify the whole classes of non-linear systems that were stable in the Hurwitz angle. In general, however, the answer to the above question turned out negative, as was shown by V.A. Pliss in 1956 by constructing an example of the third order system (Pliss, 1958).

In 1957 R. Kalman (Kalman, 1957) proposed a hypothesis that stability in the Hurwitz angle takes place for systems with stronger conditions for non-linearity:

(a) the differentiated function $\varphi(\sigma)$ lies in the given angle [0,k];
(b) its derivative with respect to σ satisfies the following conditions:

$$\varepsilon_1 \le d\varphi(\sigma)/d\sigma \le k - \varepsilon_2, \ \varphi(0) = 0,$$

where ε_1 and ε_2 are arbitrarily small positive numbers.

The Kalman's Hypothesis was assumed to be true for a long time, although there was no proof. In 1966 R.E. Fitts (Fitts, 1966) constructed examples of systems of the fourth order, contradicting the Kalman's Hypothesis. Later N.E. Barabanov (Barabanov, 1982, 1988b) indicated mistakes in the Fitts' example, constructed another examples, contradicting the Kalman's Hypothesis, and proved that the Kalman's Hypothesis is faithful for systems of order no more than three.

Each of the described above hypotheses, even though eventually incorrect, stimulated scientific research. This created an important branch of the theory of absolute stability, namely, the singling out of non-linear systems whose stability can be studied with linear methods.

An original and effective V.-M. Popov's criterion (Popov, 1961, 1973), obtained in 1959-1961 as a frequency sufficient condition for absolute stability, triggered the appearance of a large number of papers directed to expansion and modification of this criterion. The hypothesis that sufficient conditions of absolute stability, obtained with the use of frequency methods, are also necessary conditions, seemed immutable to many scholars. M.A. Aizerman commented on this in the preface to the Russian translation of S.L. Lefschetz's book (Lefshetz, 1965): «... the obtained sufficient conditions of absolute stability turned out so broad, that everyone working in this field is sure that the necessary and sufficient conditions of absolute stability lie somewhere very close». This hypothesis was rejected by V.A. Yakubovich, who in (Yakubovich, 1967) constructed an example of an absolutely stable system, for which the Popov's frequency condition is not satisfied. Later the hypothesis was also rejected by E.S.Pyatnitsky, who in (Pyatnitsky, 1973a) constructed with the variation method an example of a system of order six, whose complete range of absolute stability is impossible to identify with the Popov's criterion.

The abundance of hypotheses and counter-examples induced the publication of papers, dedicated to their analysis, and stimulated periodic booms and troughs during the period of development of the theory of absolute stability. This added some dramatic tension to the history of this field.

3. LYAPUNOV FUNCTION AND FREQUENCY APPROACH

The first stage of research in the theory of absolute stability was related to the name of its founder A.I. Lur'e, who tried to solve his problem with the Lyapunov function method. The results achieved on that stage are reflected in the well-known books by A.I. Lur'e (Lur'e, 1951), A.N. Letov (Letov, 1955), V.A. Pliss (Pliss, 1958). The frequency approach in the theory of absolute stability was first used by V.-M. Popov (Popov, 1961, 1973), starting a new stage of this field. The books by J. La Salle and S.L. Lefschetz (La Salle and Lefschetz, 1961), M.A. Aizerman and F.R. Gantmacher (Aizerman and Gantmacher, 1963), S.L. Lefschetz (Lefschetz, 1965), Ya.N. Roitenberg (Roitenberg, 1971), V.-M. Popov (Popov, 1973), K.S. Narendra and J.H. Taylor (Narendra and Taylor, 1973), V. Rasvan (Rasvan, 1975) and other form an idea of the state of the problem in the 1960's – beginning of the 1970s. This period in the theory of absolute stability is characterized by a complete superiority of two methods: using the Lyapunov function and the frequency approach. Numerous studies were dedicated to finding the links between these methods. During that time V.A. Yakubovich (Yakubovich, 1962) and R. Kalman (Kalman, 1963) obtained exceptionally important results, which influenced many scientists.

Numerous papers by V.A. Yakubovich and his subordinate group of scientists describe profound elaboration of both methods mentioned above, studies of the links between them, a series of new original approaches to solving the Lur'e Problem, and obtained on this basis important and sometimes outstanding results (Yakubovich, 1975), (Gelig, Leonov, Yakubovich, 1978), (Barabanov, et al., 1996). The central result was the quadratic criterion of absolute stability (Yakubovich, 1967). Later in (Yakubovich, 1975) this criterion is discussed in detail, its advantages compared to the well-known frequency conditions of absolute stability are shown. The final formulation of the quadratic criterion of absolute stability is given in (Yakubovich, 1998).

E.S. Pyatnitsky in (Pyatnitsky, 1970) used for the first time the Pontryagin's Maximum Principle in the absolute stability theory. This paper led to new stadies of Lur'e problem with use of both Lyapunov function method and frequency approach: (Pyatnitsky, 1972, 1973b), (Molchanov and Pyatnitsky, 1986), (Kamenetsky and Pyatnitsky, 1987). The Tsipkin criterion of absolute stability for impulse systems was extended on the basis of the same idea in (Molchanov, 1979, 1983), (Barabanov, 1988a).

The paper by P.-A. Bliman (Bliman, 1999) extends the Popov criterion to systems with delays and also gives a review in this area.

4. VARIATIONAL APPROACH

A paper by E.S. Pyatnitsky (Pyatnitsky, 1970) set the grounds for applying the variation approach to the absolute stability analysis of non-stationary non-linear systems, whose non-linear characteristic has the form $\varphi(\sigma,t)$. The system (1),(2) turns to the form

$$dy/dt = Ay + b\varphi(\sigma,t),\ \ \sigma = c^*y,\ \ \varphi(0,t) = 0 \qquad (4)$$

$$0 \le \varphi(\sigma,t)\sigma \le k\sigma^2,\ \ k>0 \text{ a number} \qquad (5)$$

A topic in (Pyatnitsky,1970) is the linear system

$$dy/dt = Ay + bu(t)\sigma,\ \ \sigma = c^*y, \qquad (6)$$

which is obtained from the system (4) by choosing only functions $\varphi(\sigma,t)$ linear in σ. In this case the condition (5) passes into the inequality

$$0 \le u(t) \le k \qquad (7)$$

It is proved in (Pyatnitsky, 1970) that the absolute stability of the system (4),(5) is equivalent to the absolute stability of the system (6),(7) for which is necessary and sufficient the asymptotic stability in the Lyapunov sense of the special piece-linear system of order 2n. This special system was obtained from the solution of the Bulgakov's variational problem on accumulation of perturbations. It is easily written out if the system (6),(7) is given. The solution of Bulgakov's variation problem on accumulation of oscillations, with the help of the Pontryagin's Maximum Principal, allowed to obtain the necessary and sufficient conditions of absolute stability, sometimes written in algebraic form (Aleksandrov and Zhermolenko, 1975; Liberzon, 1979, pp. 32-36). Further development of the variation approach using the theory of inners allowed to establish the easily checked algebraic sufficient conditions of absolute stability (Liberzon, 1989a, b).

5. INNER METHOD

The inner method in the absolute stability analysis is based on definitions from the book by E.I. Jury (Jury, 1974) and variational approach. Let the system (6) (system(4)) is controllable and observable. By linear nonsingular transformation it is possible to reduce the system (6) to a differential equation of the following form:

$$x^{(n)} + \rho_n(v)\, x^{(n-1)} + \rho_{n-1}(v)\, x^{(n-2)} + \ldots + \rho_1(v)\, x = 0, \qquad (8)$$

which coefficients $\rho_i(v)$ $(i = 1,\ldots,n)$ are continuos functionals defined on the set V of arbitrary measurable limited functions v(t):

$$v \in V = \{v(t),\ |\, v(t)\,| \le 1\} \qquad (9)$$

Functions v(t) are called disturbances. Existence and uniqueness of the solution of equation (8) are insured. With the use of coefficients $\rho_i(v)$ of the equation (8) let's build the square matrix Δ shown below, elements of which are numbers or functions of time t. The order of the matrix Δ is (2n-1). Outlined square matrixes are called inners. Constant matrix of the type Δ is called inner-positive if determinants of all inners from matrix Δ and also determinant of matrix Δ are positive (Jury, 1974).

If the following conditions **A** and **B** are satisfied for any constant disturbance v from the class V:

A) Coefficients $\rho_i(v)$ (i =1,…, n) are strictly positive,

B) Matrix Δ is inner-positive,

then the equation (8),(9) (the system (6),(7), the system (4),(5)) is absolutely stable.

1	p_n	…	p_4	p_3	p_2	p_1	0	…	0	0
0	1	…	p_5	p_4	p_3	p_2	p_1	…	0	0
⋮	⋮		⋮	⋮	⋮	⋮	⋮		⋮	⋮
0	0	…	1	p_n	p_{n-1}	p_{n-2}	p_{n-3}	…	p_1	0
0	0	…	0	1	p_n	p_{n-1}	p_{n-2}	…	p_2	p_1
0	0	…	0	0	n	$(n-1)p_n$	$(n-2)p_{n-1}$	…	$2p_3$	p_2
0	0	…	0	n	$(n-1)p_n$	$(n-2)p_{n-1}$	$(n-3)p_{n-2}$	…	p_2	0
0	0	…	n	$(n-1)p_n$	$(n-2)p_{n-1}$	$(n-3)p_{n-2}$	$(n-4)p_{n-3}$	…	0	0
⋮	⋮		⋮	⋮	⋮	⋮	⋮		⋮	⋮
0	n	…	$4p_5$	$3p_4$	$2p_3$	p_2	0	…	0	0
n	$(n-1)p_n$	…	$3p_4$	$2p_3$	p_2	0	0	…	0	0

The given above algebraic sufficient conditions for absolute stability is very easy for checking out. The procedure is shown and explained in mentioned papers.

6. APPLICATIONS

The interest to the Lur'e problem of absolute stability is very high also because it is of great importance in different applications. Let us mention two applications only: in mechanics and technology.

The investigation of motion in air of an axisymmetric rotating finned vehicle (Benton, 1964) was made step by step in a series of papers with the help of the absolute stability methods (Zhermolenko and Lokshin, 1977; Skorodinsky, 1984; Liberzon, 1999). The absolute stability methods are in use for the aircraft stable flight operation during the different flight regimes and disturbances.

The problem of absolute stability of dynamic systems is the subject of research in the number of papers on such applied topics as developing of new technological processes using the influence of impulse current or electromagnetic field and stability analysis of control systems for these technological processes, stabilization of rolling mill operation with the given parameters of the output (metal tapes, wires, plates), production of bi-metal and three-metal using the diffusion welding technique (Beklemishev and Liberzon, 1993; Liberzon, 1989b).

7. CONCLUSION

A series of methods, such as the method of vector Lyapunov functions, developed by a group of scientists headed by V.M. Matrosov, the original approach to the absolute stability problem suggested by M.A. Krasnosel'sky and A.V. Pokrovsky called a principle of absence of bounded solutions, the method of generating functions by S.V. Shilman, and others allowed to consider the Lur'e problem of absolute stability from various positions and study it in different formulations and for different types of dynamic systems, including time-lag, stochastic, discrete and impulse systems, and other types of systems. Applications in industry, electrical engineering, technology, fundamental sciences, other areas make the absolute stability theory interesting for a lot of researches. It is impossible to prepare a complete survey in frames of a paper, but maybe this is not necessary. There is a decision to create at the http://www.fund.ru/stability a list of publications on the absolute stability problem and comments to some of these publications. This job is making by Mr. Aleksey Matveev who would highly appreciate any assistance, notes, comments sent to matveev@granit.ru

REFERENCES

Aizerman, M.A. (1949). About one problem concerning the stability "in large" of dynamic systems. *Uspekhi Matem. Nauk,* **4**, 186-188.

Aizerman, M.A. and Gantmacher, F.R. (1963). *Absolute stability of regulation systems*. Acad. Nauk SSSR – Press (In Russian, English transl. SF:Holden Day, 1964).

Aizerman, M.A. (1966). *Theory of Automatic Control.* Moscow, Nauka.

Aleksandrov, V.V. and Zhermolenko, V.N. (1975). A criterion for absolute stability of third-order systems. *Doklady Akad. Nayk SSSR,* **222**, pp. 309-311.

Barabanov, N.E. (1982). Existence of periodic solutions of nonlinear controlled systems and the Kalman's hypothesis. *Uspekhi Matem. Nauk,* **37**, No. 1, p. 164.

Barabanov, N.E. (1988a). On the absolute stability problem for impulse control systems. *Avtomatika i Telemekhanika,*

Barabanov, N.E. (1988b). About the Kalman's hypothesis. *Sibirsky Matem. Zhurnal,* No. 3.

Barabanov, N.E., Gelig, A.Kh., Leonov, G.A., Likhtarnikov, A.L., Matveev, A.S., Smirnova, V.B. and Fradkov, A.L. (1996). The frequency theorem (Kalman-Yakubovich lemma) in control theory. *Avtomatika i Telemekhanika,* **57**, No. 10, pp. 3-40.

Beklemishev, N.N. and Liberzon, M.R. (1993). The Use of Inner Method in Stability Analysis of Control Systems for Special Technological Processes. In: *Fundamentals of Discrete-Time Systems*, Albuquerque, New Mexico, USA, TSI Press, pp. 189-194.

Bliman, P.-A. (1999). Extension of Popov criterion to time-varying nonlinearities: LMI, frequential and grafical conditions. In: *Stability and stabilization of nonlinear systems (Ghent, 1999*), pp. 95-114*, Lecture Notes in Control and Inform. Sci.,* **246**, Springer, London.

Bulgakov, B.V. (1942). Auto oscillations of controlled systems. *Dokladi Akad. Nauk SSSR*, **37**, 283-287.

Bulgakov, B.V. (1946). Some problems of the control theory with nonlinear characteristics. *Prikl. Mat.: Mekh*, **10**, 313-332.

Fitts, R.E. (1966). Two counterexamples to Aizerman's conjecture. *IEEE Trans. On Autom. Control* **AC-11**, pp. 553-556.

Gelig, A.Kh., Leonov, G.A. and Yakubovich, V.A. (1978). *Stability of Nonlinear Systems With Nonsingular Balance Position.* Nauka, Moscow.

Harris, C.J. and Valenca, J.M.E. (1983). *The Stability of Input-Output Dynamical Syatems.* Acad. Press, Math. In Sci. And Eng., **168**.

Jury, E.I. (1974). *Inners and Stability of Dynamic Systems*. Wiley (Second Edition in 1982).

Kalman, R.E. (1957). Physical and mathematical mechanisms of instability in nonlinear automatic control systems. *Trans. ASME,* **79**, 553-566.

Kalman, R.E. (1963). Liapunov functions for the Problem of Lurie in automatic control. *Proc. Nat. Acad. Sci. USA,***49**, pp. 201-205.

Kamenetsky, V.A. and Pyatnitsky, E.S. (1987). Gradient method for constructing of Lyapunov functions in the absolute stability problems. *Avtomatika i Telemekhanika,* No. 1, pp. 3-12.

Kuntsevich, V.M. and Lychak, M.M. (1977). *Synthesis of Automatic Control Systems With Help of the Lyapunov Function.* Nauka, Moscow.

La Salle, J. and Lefschetz, S.L. (1961). *Stability by Liapunov's Direct Method.* Acad. Press, New York, London.

Lefschetz, S.L. (1965). *Stability of Nonlinear Control Systems.*Academic Press, New York – London.

Letov, A.M. (1955). *Stability of Nonlinear Controlled Systems.*Gostekhizdat (In Russian).

Liberzon, M.R. (1979). New results on absolute stability of nonstationary controlled systems (survey). *Avtomatika i Telemekhanika,,* No. 5, pp. 29-48.

Liberzon, M.R. (1986). Criterion of Absolute Stability of Nonstationary Systems. *Avtomatika i Telemekhanika*, No. 2, pp. 39-46.

Liberzon, M.R. (1989a). An Inners Criterion of Absolute Sstability of Nonstationary Control Systems. *Doklady Akademii Nauk SSSR*, **304**, No. 5, pp. 1060-1064.

Liberzon, M.R. (1989b). Iinner Criterion of Absolute Stability of Nonstationary Systems. *Avtomatika i Telemekhanika*, No. 5, pp. 40-46.

Liberzon, M.R. (1993). Inner Indication of Absolute Stability for Discrete Systems. In: *Fundamentals of Discrete-Time Systems*, Albuquerque, New Mexico, USA, TSI Press, pp. 333-338.

Liberzon, M.R. (1999). On the stability of controlled flight of an axisymmetric rotating vehicle with incomplete model information. *Proc. 1999 American Control Conference*, **5**, pp. 3520-3524, San Diego, CA, USA.

Liberzon, M.R. (2001). Lur'e problem of absolute stability – a historical essay. *Proc. of 5th IFAC Symposium "Nonlinear Control Systems NOLCOS'01",***1**, pp. 22-25, Saint-Petersburg, Russia.

Lur'e, A.I., and V.M. Postnikov (1944). On the theory of stability of controlled systems. *Prikl. Mat.i Mekh*, **8**, 283-286 .

Lur'e, A.I. (1951). *Certain Nonlinear Problems in the Automatic Regulating Theory.* Gostekhizdat (In Russian, English Transl.: London, HMSO, 1975).

Molchanov, A.P. (1979). Absolute stability of impulse systems with a number of nonlinear nonstationary elements. *Avtomatika i Telemekhanika,* No. 3, pp. 43-51.

Molchanov, A.P. (1983). Absolute stability criterion for impulse systems with nonstationary nonlinearity. *Avtomatika i Telemekhanika,* No. 5, pp. 73-81; No. 6, pp. 42-52.

Molchanov, A.P. and Pyatnitsky, E.S. (1986). Lyapunov functions which define necessary and sufficient conditions of absolute stability for nonlinear nonstationary control systems. *Avtomatika i Telemekhanika,* No. 3, pp. 63-73; No. 4, pp. 5-15; No. 5, pp. 38-49.

Narendra, K.S. and Taylor, J.H. (1973). *Frequency Domain Criteria for Absolute Stability.* Acad. Press, New York.

Naumov, B.N. (1972). *Theory of Nonlinear Automatic Systems.* Nauka, Moscow.

Pliss, V.A. (1958). *Certain Problems of the Theory of Stability of Motion in the Whole.* Leningrad University Press .

Popov, V.-M. (1961). On the absolute stability of nonlinear controlled systems. *Avtomatika i Telemekhanika,* No. 8, pp. 961-970.

Popov, V.-M. (1973). *Hiperstability of Control Systems.* Springer-Verlag, New York.

Pyatnitsky, E.S. (1968). New research on absolute stability of automatic control systems (survey). *Avtomatika i Telemekhanika,* No. 6, pp. 5-36.

Pyatnitsky, E.S. (1970). Absolute stability of nonstationary nonlinear systems. *Avtomatika i Telemekhanika,* No. 1, pp. 5-15.

Pyatnitsky, E.S. (1972). An extention of thje frequency criterion for absolute stability of control system with a single nonlinear nonstationary element. *Izvestiya Vuzov, Radiofizika,* **15**, No. 3, pp. 319-322.

Pyatnitsky, E.S. (1973a). On the existence of absolute stability systems, for which the criterion of V.-M. Popov is not satisfied. *Avtomatika i Telemekhanika,* No. 1, pp. 30-37.

Pyatnitsky, E.S. (1973b). On uniform stability in the presence of parametric perturbations. *Differentsial. Uravneniya,***9**, No. 7, pp. 1262-1274.

Resvan, V. (1983). *Absolute Stability of Control Systems With Time Delay.* Nauka, Moscow.

Roitenberg, Ya.N. (1971). *Automatic Control.* Nauka, Moscow.

Shilman, S.V. (1978). *Method of Generating Functions in the Theory of Dynamical Systems.* Nauka, Moscow.

Vidal, P. (1968). *Systems Echantillones Nonlineatres.* Cordon and Breach Paris .

Voronov, A.A. (1966). *Fundamentals of the Theory of Automatic Control.* Energia Press (Second Ed. In 1981).

Voronov, A.A. (1982). Stability theory: state of art and problems. *Avtomatika i Telemekhanika,* No. 5, pp. 5-28.

Yakubovich, V.A. (1962). Solving some matrix inequalities, which are meeting in control theory. *Doklady Acad. Nauk SSSR,* **143**, pp. 1304-1307.

Yakubovich, V.A. (1967). Frequency conditions of absolute stability of the control systems with some non-linear and linear non-stationary blocks. *Avtomatika i Telemekhanika,* No. 6, pp. 5-30.

Yakubovich, V.A. (1975). Methods of the theory of absolute stability. In: *Methods for Investigating Nonlinear Control Systems* (R.A. Nelepin, Ed.). Nauka, Moscow.

Yakubovich, V.A. (1998). Quadratic criterion of absolute stability. *Doklady Acad. Nauk,* **361**, pp. 608-611.