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DESIGN OF RESTRUCTURABLE ACTIVE FAULT-TOLERANT CONTROL SYSTEMS

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Abstract: In this paper, a new design scheme for restructurable fault-tolerant control systems (RFTCS) is proposed. Restructurable controllers are designed on-line based on the information about system states and parameters, post-fault system model, decisions for diagnosis and activation of restructurable controllers from a fault detection and diagnosis (FDD) scheme. The feedback part of the restructurable controller is designed automatically using linear quadratic regulator (LQR) technique, while the feedforward part is designed based on either an explicit model-following structure or a command-tracking scheme. Several design strategies have been proposed and evaluated to demonstrate the variable-structure characteristics of the proposed RFTCS. The proposed scheme is evaluated using an aircraft example. *Copyright © 2002 IFAC*

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1. INTRODUCTION

Fault-tolerant control systems (FTCS) are control systems that possess the ability to accommodate failures *automatically*. Such systems are capable of maintaining overall system stability and acceptable performance in the event of component failures. FTCS are also known as reconfigurable (Jiang, 1994; Patton, 1997; Zhang and Jiang, 2001a; 2001c), or restructurable (Eterno, *et al.*, 1985; Huang and Stengel, 1990; Looze, *et al.*, 1985; Ochi and Kanai, 1991) control systems.

As the names implied, *reconfigurable* control implies that some of the control system parameters can be modified to account for the fault-induced changes in the system, such as failed actuators, sensors, or damaged components in the system. *Restructurable* control subsumes reconfigurable control, implying that not only parameters but also the structure of the control system can be changed so as to accommodate such changes (Eterno, *et al.*, 1985). Therefore, there is a fine distinction between reconfigurable and restructurable control systems. One important feature associated with the restructurable control systems is its variable-structure characteristics of controller so that the controller is able to deal with system structural/severe changes as results of failures. In fact, this makes it more challenging to design restructurable control systems than reconfigurable control systems.

Although the primary objective of FTCS is to achieve restructurable control (Eterno, *et al.*, 1985), to date, there is little work that has really focused on the issue of designing truly *restructurable* control systems, although the terminologies of restructurable control systems have been used in the literature. On the other hand, reconfigurable control systems have drawn much more attention as evidenced by the number of publications in this field (Bodson and Groszkiewicz, 1997; Jiang, 1994; Patton, 1997; Zhang and Jiang, 2001a; 2001c). However, in practical engineering systems, when a severe fault occurs, not only the parameters, but also the structure of the system may change. In this case, to achieve desired control performance during normal and fault conditions, respectively, and to design new controllers with allowable performance degradation after a fault, restructurable control systems are more suitable. The objective of this paper is to develop a new design scheme for such *restructurable* control systems in conjunction with a fault detection and diagnosis (FDD) scheme for actuator failures. Fig. 1 depicts the structure of the proposed integrated FDD and restructurable c

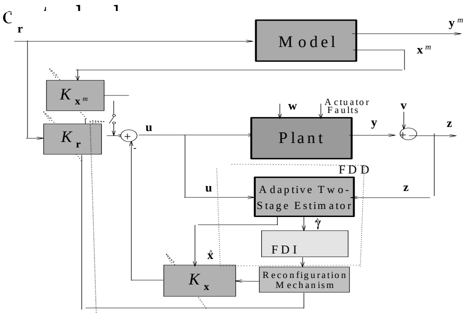

Fig. 1 *Integrated FDD and restructurable control systems.*

In the above structure, to achieve steady-state command tracking, design of feedback control only is not sufficient. It is necessary that a feedforward controller needs to be synthesized simultaneously. Therefore, there are feedback and feedforward controllers, $K_{\mathbf{x}}$, and $K_{\mathbf{r}}$, during normal operation of the system. In this case, the feedback controller is for stabilizing system and achieving desired dynamic performance and the feedforward controller is for tracking command input. Once a fault occurred, three new controller gains, $K_{\mathbf{x}}^{'}$, $K_{\mathbf{r}}^{'}$ and $K_{\mathbf{x}^m}^{'}$, need to be synthesized simultaneously in order to follow the dynamics of a prescribed reference model, which takes into consideration of degraded performance due to the fault, through a model-following scheme. $K_{\mathbf{x}^m}^{'}$ is an extra controller for tracking the dynamics of the reference model. It should be noted that, in the context of restructurable control systems, not only are there different types of controller, but also the orders of each controller may be different from those of the nominal controller. After the post-fault system has been recovered through the model-following control structure with the three controller gains, and the reconstructed system has reached a new steady-state condition, it may be desirable to redesign a new controller based on more precise post-fault system model with only two controller gains, $K''_{\mathbf{x}}$ and $K''_{\mathbf{r}}$, to achieve same/similar control performance with simpler control structure.

In practice, due to the non-deterministic nature of faults and unavailability of full state variables, on-line and real-time estimation for states and fault parameters and a FDD scheme need to be designed jointly with the above restructurable controllers. For such purpose, a two-stage adaptive Kalman filter (Zhang and Jiang, 1999; Wu, *et al.*, 2000) is used for simultaneous state and fault parameter estimation in the event of actuator faults, statistical decisions for fault diagnosis and activation of the controller reconstruction.

2. MODELING OF ACTUATOR FAULTS

Consider a fault-free system described by the following linear stochastic dynamic model:

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}} \\ \mathbf{y}_k &= H_r\mathbf{x}_k + D\mathbf{u}_k \\ \mathbf{z}_k &= H\mathbf{x}_k + D\mathbf{u}_k + \mathbf{v}_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in \Re^n$ is the system state, $\mathbf{u}_k \in \Re^l$ the input, and $\mathbf{y}_k \in \Re^l$ the output. $\mathbf{z}_k \in \Re^m$ corresponds to the measurements. $\mathbf{w}_k^{\mathbf{x}} \in \Re^n$ is a zero-mean white Gaussian noise sequence with covariance $Q_k^{\mathbf{x}} \in \Re^{n\times n}$ representing the modelling errors. $\mathbf{v}_k \in \Re^m$ is a zero-mean white Gaussian measurement noise sequence with covariance $R_k \in \Re^{m\times m}$. The initial state $\mathbf{x}_0$ is a Gaussian vector with mean $\bar{\mathbf{x}}_0$ and covariance $\bar{P}_0$. $H_r \in \Re^{l\times n}$ is a matrix which relates to the subset of outputs that track the command inputs. $H \in \Re^{m\times n}$ is the measurement matrix.

To model actuator faults, control effectiveness factors are introduced. The dynamic part of the system in the presence of actuator faults becomes:

$$\mathbf{x}_{k+1} = F\mathbf{x}_k + G^f\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}} \tag{2}$$

where the post-fault input matrix G^f relates to the nominal input matrix G and the control effectiveness factors $\gamma_k^i, i = 1, ..., l$, in the following way

$$G^f = G(I - \Gamma_k), \ \Gamma_k = \text{diag}[\gamma_k^1 \ \gamma_k^2 \ ... \ \gamma_k^l] \tag{3}$$

where $\gamma_k^i = 0, i = 1, ..., l$, denotes a healthy ith actuator and $\gamma_k^i = 1$ corresponds to total failure of the ith actuator. Naturally, $0 < \gamma_k^i < 1$ represents partial loss in control effectiveness.

To determine G^f, the control effectiveness factors $\gamma_k^i, i = 1, ..., l$, need to be estimated on-line. However, the above fault model (2) is not suitable for direct estimation of the control effectiveness factors. An alternative representation of (2) is as follows

$$\mathbf{x}_{k+1} = F\mathbf{x}_k + G\mathbf{u}_k + \Pi_k(\mathbf{u}_k)\boldsymbol{\gamma}_k + \mathbf{w}_k^{\mathbf{x}} \tag{4}$$

where $\boldsymbol{\gamma}_k = [\gamma_k^1 \ \gamma_k^2 \ ... \ \gamma_k^l]^T$ and $\Pi_k(\mathbf{u}_k)$ is defined by $\Pi_k(\mathbf{u}_k) = GU_k$, with $U_k = \text{diag}[-u_k^1 \ -u_k^2 \ ... \ -u_k^l]$.

Due to the random nature of actuator faults and in the absence of the knowledge on their true values, the effectiveness factors can be modeled as a random bias vector:

$$\boldsymbol{\gamma}_{k+1} = \boldsymbol{\gamma}_k + \mathbf{w}_k^{\gamma} \qquad \begin{cases} \boldsymbol{\gamma}_k = 0, \ k < k_F \ \text{fault-free} \\ \boldsymbol{\gamma}_k \neq 0, \ k \geq k_F \ \text{with fault} \end{cases} \tag{5}$$

where k_F denotes an unknown time when a fault (reduction of the control effectiveness) occurred. $\mathbf{w}_k^{\gamma}$ is a zero-mean white Gaussian noise sequence with covariance Q_k^{γ}.

Consequently, the combined state and control effectiveness model with the available measurement has the following form

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + \Pi_k(\mathbf{u}_k)\boldsymbol{\gamma}_k + \mathbf{w}_k^{\mathbf{x}} \\ \boldsymbol{\gamma}_{k+1} &= \boldsymbol{\gamma}_k + \mathbf{w}_k^{\gamma} \\ \mathbf{y}_k &= H_r\mathbf{x}_k + D\mathbf{u}_k \\ \mathbf{z}_k &= H\mathbf{x}_k + D\mathbf{u}_k + \mathbf{v}_k \end{aligned} \tag{6}$$

3. DESIGN OF RESTRUCTURABLE CONTROLLER

3.1 *Design Objective of Restructurable Control*

For the sake of simple restructurable controller design, the above system model (6), under the conditions of normal and fault operations, can be represented alternatively by following models:

$$\begin{aligned} &\begin{cases} \mathbf{x}_{k+1} = F\mathbf{x}_k + G\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}}, & k < k_F \ \text{fault-free} \\ \mathbf{x}_{k+1} = F\mathbf{x}_k + G^f\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}}, & k \geq k_F \ \text{with fault} \end{cases} \\ &\mathbf{y}_k = H_r\mathbf{x}_k + D\mathbf{u}_k \\ &\mathbf{z}_k = H\mathbf{x}_k + D\mathbf{u}_k + \mathbf{v}_k \end{aligned} \tag{7}$$

During the normal operation, system matrices are represented by $\{F, G, H_r, D\}$. Once an actuator fault occurs at an unknown time with unknown changes in the system, the input matrix G becomes G^f, and $G\mathbf{u}_k + \Pi_k(\mathbf{u}_k)\boldsymbol{\gamma}_k$ in (6) is replaced by $G^f\mathbf{u}_k$. If a total actuator failure occurs, the corresponding column in G to the actuator becomes zero and the dimension of G^f will then be reduced by one which corresponds to a lost of the control channel.

For the system (7), suppose one has designed a controller $\{K_{\mathbf{x}}^n, K_{\mathbf{r}}^n\}$ for the normal operation of the system with the control signal obtained by:

$$\mathbf{u}_k = -K_{\mathbf{x}}^n\mathbf{x}_k + K_{\mathbf{r}}^n\mathbf{r}_k, \quad k < k_F \tag{8}$$

Note that there are two control gains corresponding to the feedback part $K_{\mathbf{x}}^n \in \Re^{l\times n}$ and the feedforward part $K_{\mathbf{r}}^n \in \Re^{l\times l}$ of the overall controller, respectively.

To design a FTCS that considers performance degradation, a model-following design scheme has been proposed recently in (Zhang and Jiang, 2001b). Such a design approach makes it possible to design reconfigurable controllers for achieving different performance levels and specifications under normal and fault conditions, through the selection of appropriate reference models for the two modes of system operation.

Suppose that the following two reference models, one describing the desired behavior of the system during normal operation (referred as to desired reference model) and the other describing the desired behavior of the system in the event of actuator fault with consideration of performance degradation (referred as to degraded reference model), are chosen as

$$\begin{cases} \mathbf{x}_{k+1}^m = F_n^m \mathbf{x}_k^m + G_n^m \mathbf{r}_k \\ \mathbf{y}_k^m = H_n^m \mathbf{x}_k^m + D_n^m \mathbf{r}_k \end{cases} , \quad k < k_F \\ \begin{cases} \mathbf{x}_{k+1}^m = F_f^m \mathbf{x}_k^m + G_f^m \mathbf{r}_k \\ \mathbf{y}_k^m = H_f^m \mathbf{x}_k^m + D_f^m \mathbf{r}_k \end{cases} , \quad k \geq k_F \tag{9}$$

where $\mathbf{x}_k^m \in \Re^{n^m}$ is the state, $\mathbf{r}_k \in \Re^l$ the command input, and $\mathbf{y}_k^m \in \Re^{l^m}$ the output.

Based on the above system (7), the reference models (9) and the control structure in Fig. 1, the design objective of the restructurable control system is to synthesize a new controller $\{K_{\mathbf{x}}^f, K_{\mathbf{r}}^f, K_{\mathbf{x}^m}^f\}$ (restructured controller) in response to the changes in the system due to actuator fault, such that the stability can be maintained and the post-fault system can track the desired outputs of the degraded reference model, with following control signal:

$$\mathbf{u}_k = -K_{\mathbf{x}}^f \mathbf{x}_k + K_{\mathbf{r}}^f \mathbf{r}_k + K_{\mathbf{x}^m}^f \mathbf{x}_k^m, \quad k \geq k_R \tag{10}$$

where k_R represents the controller reconfiguration time, and $K_{\mathbf{x}}^f \in \Re^{l^f \times n}, K_{\mathbf{r}}^f \in \Re^{l^f \times l^f}$, $K_{\mathbf{x}^m}^f \in \Re^{l^f \times n^m}$. $l^f \leq l$ denotes the numbers of available control channels in the event of an actuator fault.

Note that in addition to the original two controller gains, a new controller gain relating to the reference model needs to be designed. Therefore, the structure of the new controller is different from the nominal controller in that numbers of controller gains and/or dimension of each controller gain may be changed, hence the name of *restructurable* controller.

It should be pointed out also that, using the control structure shown in Fig. 1, strategies for implementing the above restructurable controller, with two or three controller gains, can be arranged in several ways. Performance evaluation of the proposed restructurable control system will be carried out by the four strategies described in Table 1, to demonstrate the variable structure characteristics of the proposed scheme.

Table 1 Strategies of restructurable control system design

Controllers	Nonimal $k < k_F$	Restructured C1 $k \geq k_{R_1} (\geq k_F)$	Restructured C2 $k \geq k_{R_2} (> k_{R_1})$
Strategy 1	command-tracking $\{K_{\mathbf{x}}^n, K_{\mathbf{r}}^n\}$	model-following $\{K_{\mathbf{x}}^{f'}, K_{\mathbf{r}}^{f'}, K_{\mathbf{x}^m}^{f'}\}$	output-tracking $\{K_{\mathbf{x}}^{f''}, K_{\mathbf{r}}^{f''}\}$
Strategy 2	command-tracking $\{K_{\mathbf{x}}^n, K_{\mathbf{r}}^n\}$	model-following $\{K_{\mathbf{x}}^{f'}, K_{\mathbf{r}}^{f'}, K_{\mathbf{x}^m}^{f'}\}$	command-tracking $\{K_{\mathbf{x}}^{f''}, K_{\mathbf{r}}^{f''}\}$
Strategy 3	model-following $\{K_{\mathbf{x}}^n, K_{\mathbf{r}}^n, K_{\mathbf{x}^m}^n\}$	model-following $\{K_{\mathbf{x}}^{f'}, K_{\mathbf{r}}^{f'}, K_{\mathbf{x}^m}^{f'}\}$	output-tracking $\{K_{\mathbf{x}}^{f''}, K_{\mathbf{r}}^{f''}\}$
Strategy 4	model-following $\{K_{\mathbf{x}}^n, K_{\mathbf{r}}^n, K_{\mathbf{x}^m}^n\}$	model-following $\{K_{\mathbf{x}}^{f'}, K_{\mathbf{r}}^{f'}, K_{\mathbf{x}^m}^{f'}\}$	command-tracking $\{K_{\mathbf{x}}^{f''}, K_{\mathbf{r}}^{f''}\}$

3.2 *Restructurable Controller Design for Model Following*

3.2.1. *Feedforward Control Gain Design Methodology*

As mentioned previously, the function of the feedforward control is to make the selected outputs of the system to track the outputs of the desired or the degraded reference model during normal or fault operation respectively, i.e. to find a control sequence $\mathbf{u}_k$ that forces the command tracking error $\mathbf{e}_k$ to zero at the steady-state

$$\mathbf{e}_k = \mathbf{y}_k - \mathbf{y}_k^m = \begin{bmatrix} H_r & D \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{u}_k \end{bmatrix} - \begin{bmatrix} H^m & D^m \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^m \\ \mathbf{r}_k \end{bmatrix} \tag{11}$$

Once this condition is satisfied, the resulting ideal system state and control trajectories are denoted as $\mathbf{x}_k^*$ and $\mathbf{u}_k^*$, which satisfy

$$\mathbf{y}_k^* = \begin{bmatrix} H_r & D \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^* \\ \mathbf{u}_k^* \end{bmatrix} = \begin{bmatrix} H^m & D^m \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^m \\ \mathbf{r}_k \end{bmatrix} \tag{12}$$

and the system dynamics

$$\mathbf{x}_{k+1}^* = \begin{cases} F\mathbf{x}_k^* + G\mathbf{u}_k^*, & \text{fault-free} \\ F\mathbf{x}_k^* + \hat{G}_k^f \mathbf{u}_k^*, & \text{with fault} \end{cases} \tag{13}$$
$$\mathbf{y}_k^* = H_r \mathbf{x}^*$$

where $\hat{G}_k^f$ is an estimate of G^f at time k.

The solutions of $\mathbf{x}_k^*$ and $\mathbf{u}_k^*$ to achieve perfect tracking for step input can be represented as:

$$\mathbf{x}_k^* = S_{11}\mathbf{x}_k^m + S_{12}\mathbf{r}_k \tag{14}$$
$$\mathbf{u}_k^* = S_{21}\mathbf{x}_k^m + S_{22}\mathbf{r}_k \tag{15}$$

where $S_{ij}, i, j = 1, 2$, are calculated by

$$S_{11} = \Phi_{11} S_{11}(F^m - I) + \Phi_{12} H^m \tag{16}$$
$$S_{12} = \Phi_{11} S_{11} G^m + \Phi_{12} D^m \tag{17}$$
$$S_{21} = \Phi_{21} S_{11}(F^m - I) + \Phi_{22} H^m \tag{18}$$
$$S_{22} = \Phi_{21} S_{11} G^m + \Phi_{22} D^m \tag{19}$$

and $\Phi_{ij}, i, j = 1, 2$, are given by

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \begin{cases} \begin{bmatrix} F - I & G \\ H_r & D \end{bmatrix}^{-1}, & \text{fault-free} \\ \begin{bmatrix} F - I & \hat{G}_k^f \\ H_r & D \end{bmatrix}^{-1}, & \text{with fault} \end{cases} \tag{20}$$

where I is an identity matrix, and $D = D^m = 0$.

To incorporate feedback into the design, define

$$\tilde{\mathbf{x}}_k = \mathbf{x}_k - \mathbf{x}_k^*, \ \tilde{\mathbf{u}}_k = \mathbf{u}_k - \mathbf{u}_k^*, \ \tilde{\mathbf{y}}_k = \mathbf{y}_k - \mathbf{y}_k^* \tag{21}$$

then, we have

$$\tilde{\mathbf{x}}_{k+1} = \begin{cases} F\tilde{\mathbf{x}}_k + G\tilde{\mathbf{u}}_k, & \text{fault-free} \\ F\tilde{\mathbf{x}}_k + \hat{G}_k^f \tilde{\mathbf{u}}_k, & \text{with fault} \end{cases} \tag{22}$$

$$\tilde{\mathbf{y}}_k = H_r \tilde{\mathbf{x}}_k \tag{23}$$

For a state feedback control signals given by

$$\tilde{\mathbf{u}}_k = -K_{\mathbf{x}}\tilde{\mathbf{x}}_k = -K_{\mathbf{x}}(\mathbf{x}_k - \mathbf{x}_k^*) \tag{24}$$

From the definition of $\tilde{\mathbf{u}}_k$ in (21), it follows:

$$\mathbf{u}_k = \mathbf{u}_k^* + \tilde{\mathbf{u}}_k = \mathbf{u}_k^* - K_{\mathbf{x}}(\mathbf{x}_k - \mathbf{x}_k^*) \tag{25}$$

Substituting (14) and (15) into (25), the total control signal can be shown as:

$$\mathbf{u}_k = -K_{\mathbf{x}}\mathbf{x}_k + \underbrace{(S_{22} + K_{\mathbf{x}} S_{12})}_{K_{\mathbf{r}}} \mathbf{r}_k + \underbrace{(S_{21} + K_{\mathbf{x}} S_{11})}_{K_{\mathbf{x}^m}} \mathbf{x}_k^m \tag{26}$$

It should be noted that (26) is suitable to both normal and fault conditions. In the presence of an actuator fault, the control input matrix G will be replaced by $\hat{G}_k^f$, S_{ij}, Φ_{ij} and the control gain matrices $\{K_{\mathbf{x}}, K_{\mathbf{r}}, K_{\mathbf{x}^m}\}$ need to be re-calculated on-line, where $K_{\mathbf{r}}$ and $K_{\mathbf{x}^m}$ are calculated from $K_{\mathbf{x}}$ and $K_{\mathbf{x}}$ is obtained using linear quadratic regulator technique.

3.3 *Restructurable Controller Design for Command Tracking*

To track command inputs (instead of tracking dynamics of a reference model), a simpler control structure with only two controller gains, as shown in (8), is desirable. Such a command-tracking (CT) scheme can be derived via simplifying the above model-following design scheme.

Consider the above pre-fault and post-fault systems (7) and the reference model (9) if only the command input needs to be tracked, the above reference model can be chosen as an identity model, i.e., the output of the model is equal to its input:

$$\mathbf{y}_k^m = \mathbf{r}_k \tag{27}$$

then, the identity reference model can be described by

$$F^m = I, G^m = 0, H^m = 0, D^m = I \tag{28}$$

Following the similar derivation, (14) and (15) become

$$\mathbf{x}_k^* = \Phi_{12}\mathbf{r}_k \tag{29}$$

$$\mathbf{u}_k^* = \Phi_{22}\mathbf{r}_k \tag{30}$$

with $\Phi_{ij}, i, j = 1, 2$, are given by

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = \begin{cases} \begin{bmatrix} F - I & G \\ H_r & 0 \end{bmatrix}^{-1} & \text{fault-free} \\ \begin{bmatrix} F - I & \hat{G}_k^f \\ H_r & 0 \end{bmatrix}^{-1} & \text{with fault} \end{cases} \tag{31}$$

and the control law is given by

$$\mathbf{u}_k = -K_{\mathbf{x}}\mathbf{x}_k + (\Phi_{22} + K_{\mathbf{x}}\Phi_{12})\mathbf{r}_k \tag{32}$$

Note that this control law consists of only a feedback part, $K_{\mathbf{x}}$, and a feedforward part, $K_{\mathbf{r}} = \Phi_{22} + K_{\mathbf{x}}\Phi_{12}$.

4. ISSUES IN IMPLEMENTING OVERALL RFTCS

To design a truly functional RFTCS, a precise post-fault system model, $\{F, G^f, H_r, D\}$, is needed. Due to random nature of faults, usually, this model is not available *a priori*. It requires an on-line FDD scheme to provide information about the fault and the post-fault system model. Several issues closely related to implementation of overall restructurable control system need to be addressed: (1) the algorithm for estimating the system states and the control effectiveness factors; (2) the FDD scheme; and (3) the mechanisms for activating restructurable controllers.

4.1 *Estimation of States and Control Effectiveness Factors*

To design restructurable controllers in real-time, it is necessary to determine the control input matrix G_k^f in (7) on-line. This matrix can be obtained based on the estimated control effectiveness factors $\hat{\boldsymbol{\gamma}}_k = [\hat{\gamma}_k^1 \ \hat{\gamma}_k^2 \ ... \ \hat{\gamma}_k^l]^T$ by $\hat{G}_k^f = G(I - \hat{\Gamma}_k), \hat{\Gamma}_k = \mathrm{diag}[\hat{\gamma}_k^1 \ \hat{\gamma}_k^2 \ ... \ \hat{\gamma}_k^l]$. This leads to a combined state and parameter estimation problem. To provide estimated state variables, $\hat{\mathbf{x}}_{k|k}$ (to replace the state vector $\mathbf{x}_k$ in (26) and (32)), the control effectiveness factors, $\hat{\boldsymbol{\gamma}}_k$, and the post-fault system model, a two-stage adaptive Kalman filter (Wu, *et al.*, 2000; Zhang and Jiang, 1999) has been used. The structure of the filter is depicted in Fig. 2.

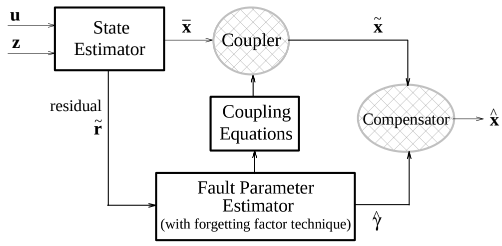

Fig. 2. *Structure of a two-stage adaptive Kalman filter.*

4.2 *Fault Detection and Isolation (FDI) Scheme*

For on-line reconfiguration purpose, a FDI scheme is needed to provide information for fault detection time and fault location soon after the fault occurrence. The magnitude of the actuator faults is further obtained from estimates of the control effectiveness factors. To provide fast and reliable FDI, a statistical hypothesis test (Zhang and Jiang, 1999) has been used.

4.3 *Mechanisms for Activating Restructurable Controllers*

As discussed in Sections 3.2 and 3.3, design of the restructurable controllers (26) and (32) are dependent on the post-fault model. Due to the time-varying nature of the estimated control matrix, $\hat{G}_k^f$, therefore, the time to activate the design process of the restructurable controllers is important for overall performance of the system. On the other hand, once the post-fault system has been recovered through restructured controller 1 in Table 1 and reach to its new steady-state, the issue becomes one of when to restructure and activate the controller 2 based on updated post-fault system model? These issues should also be considered for the overall restructurable control system design.

Since it is important to design a restructurable control law based on the estimates of converged control effectiveness factors, it is found that, to achieve good control performance, the activation of the reconstruction process should only take place when the errors in consecutive control effectiveness factor estimates satisfy the following smooth condition:

$$\left|\hat{\gamma}_k^i - \hat{\gamma}_{k-1}^i\right| \leq \ \delta_i, \ i = 1, ..., l; \ k \geq k_D \geq k_F \tag{33}$$

The time instant when the smooth condition is satisfied is defined as the first reconfiguration time k_{R_1}. The threshold δ_i is a design parameter.

To activate the second restructurable controller, additional conditions relating to the tracking accuracy of system outputs in two consecutive samples can be used:

$$\begin{cases} |\epsilon_k| \leq \varepsilon_1 \\ |\epsilon_k - \epsilon_{k-1}| \leq \varepsilon_2 \end{cases}, \quad k \geq k_{R_1}$$

where

$$\epsilon_k = \frac{\left\{\sum_{i=1}^l \left(y_k^{degraded}(i) - y_k(i)\right)^2\right\}^{1/2}}{\left\{\sum_{i=1}^l \left(y_k^{degraded}(i)\right)^2\right\}^{1/2}}, \ k \geq k_{R_1} \tag{34}$$

and $y_k^{degraded}$ denotes the outputs of the degraded reference model at time k, and y_k denotes the outputs of the restructured system. ε_1 and ε_2 are thresholds for activating the second restructurable controller.

5. PERFORMANCE EVALUATION

To demonstrate the effectiveness of the proposed scheme, a fourth-order lateral F-8 aircraft dynamic model (Sobel and Shapiro, 1986) is used for evaluation.

5.1 *System and Reference Models*

The linearized aircraft model can be described as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + B\mathbf{u}(t) \\ \mathbf{y}(t) &= C\mathbf{x}(t) \end{aligned} \tag{35}$$

where state and input vectors are defined as $\mathbf{x} = [p\ r\ \beta\ \phi]^T$ and $\mathbf{u} = [\delta_a\ \delta_r]^T$, with p representing the roll rate, r the yaw rate, β the sideslip angle, ϕ the bank angle, δ_a the aileron deflection, and δ_r the rudder deflection.

To track the sideslip and bank angles during the normal condition, the output matrix H_r is chosen as $H_r^n = C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. In the event of a total failure occurred in one actuator, only one system output can be fully controlled. H_r is chosen as $H_r^f = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$.

Taking into account of noises in the system and converting the system into discrete representation, (35) becomes:

$$\begin{aligned} \mathbf{x}_{k+1} &= F\mathbf{x}_k + G\mathbf{u}_k + \mathbf{w}_k^{\mathbf{x}} \\ \mathbf{y}_k &= H_r\mathbf{x}_k \\ \mathbf{z}_k &= C\mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{36}$$

where the sampling period $T = 0.1$ second is used.

Following the design consideration, system matrices for the degraded reference models as well as for open-loop system are given in Table 2.

Table 2 System matrices of system and reference models

Models	A	B
Open-loop system	$\begin{bmatrix} -3.598 & 0.1968 & -35.18 & 0 \\ -0.0377 & -0.3576 & 5.884 & 0 \\ 0.0688 & -0.9957 & -0.2163 & 0.0733 \\ 0.9947 & -0.1027 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 14.65 & 6.538 \\ 0.2179 & -3.087 \\ -0.0054 & 0.0516 \\ 0 & 0 \end{bmatrix}$
Desired ref. model	$\begin{bmatrix} -10.0 & 0 & -10.0 & 0 \\ 0 & -0.7 & 4.5 & 0 \\ 0 & -0.5 & -0.7 & 0 \\ 1 & 0 & 0 & -0.5 \end{bmatrix}$	$\begin{bmatrix} 20.0 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
Degraded ref. model	$\begin{bmatrix} -10.0 & 0 & -10.0 & 0 \\ 0 & -0.6 & 3.5 & 0 \\ 0 & -0.5 & -0.6 & 0 \\ 1 & 0 & 0 & -0.4 \end{bmatrix}$	$\begin{bmatrix} 2.8 \\ -0.39125 \\ 0 \\ 0 \end{bmatrix}$

5.2 *Simulation Results and Performance Evaluation*

To evaluate the performance of the proposed design strategies, a total failure (100% loss of the control effectiveness) in the aileron occurred at $k_F = 5$ sec has been simulated. A constant input vector, $\mathbf{r} = [\,5\ 5\,]^T$, has been used as command input for the system and the reference models.

5.2.1. *Results for Strategy 1 and Strategy 2*

The designed controller gains for normal and fault conditions are illustrated in Table 3. Two or three controller gains are obtained in on-line and real-time manner using either the command-tracking or the model-following scheme based on the identified post-fault model from the FDD algorithm. For the normal system, a command-tracking scheme has been used to synthesize feedback and feedforward controllers. After the actuator failure has been detected at $k_D = 5.2$ sec and the smooth condition (33) has been satisfied at time $k_{R_1} = 5.4$ sec, new controllers with different structures/orders are synthesized. The three restructured controller gains then replace the two nominal controller gains. After the closed-loop system has reached steady-state, another new controller with two restructured control gains is calculated based on more accurate post-fault model. The l_2 norms of the control gain matrices can be used as a measure of the control effort. The values of l_2 norm associated with different controllers are also shown in Table 3.

Table 3 Restructurable control gain matrices

Methods of tracking	Type	Control Gains	$\|K\|_2$
Nominal (CT)	$K_{\mathbf{x}}$	$\begin{bmatrix} 0.3746 & 0.4707 & -1.9824 & 0.5288 \\ 0.1036 & -0.9263 & 0.0242 & 0.0895 \end{bmatrix}$	2.3343
	$K_{\mathbf{x}^m}$	–	–
	$K_{\mathbf{r}}$	$\begin{bmatrix} -0.5484 & 0.5709 \\ 2.1610 & 0.0137 \end{bmatrix}$	2.3015
Degraded ref. ($k_{R_1} = 5.4$s)	$K_{\mathbf{x}}$	$\begin{bmatrix} -0.0563 & -2.1255 & 3.3507 & -0.4583 \end{bmatrix}$	3.9948
	$K_{\mathbf{x}^m}$	$\begin{bmatrix} -0.1546 & -0.1680 & 0.3179 & -0.2620 \end{bmatrix}$	0.4710
	$K_{\mathbf{r}}$	$[-0.1966]$	0.1966
Degraded ref. ($k_{R_2} = 10.5$s)	$K_{\mathbf{x}}$	$\begin{bmatrix} -0.0562 & -2.1255 & 3.3507 & -0.4583 \end{bmatrix}$	3.9948
	$K_{\mathbf{x}^m}$	–	–
	$K_{\mathbf{r}}$	$[-0.6409]$	0.6409
CT ($k_{R_2} = 10.5$s)	$K_{\mathbf{x}}$	$\begin{bmatrix} -0.0562 & -2.1255 & 3.3507 & -0.4583 \end{bmatrix}$	3.9948
	$K_{\mathbf{x}^m}$	–	–
	$K_{\mathbf{r}}$	$[-0.6409]$	0.6409

The responses of restructured system and those without controller reconstruction are shown in Fig. 3. The corresponding control signals are demonstrated in Fig. 4. Because of the total failure in the first control channel, the number of available control channels is reduced from two to one. Therefore, given H_r^f, only bank angle can track the command input after the fault occurrence.

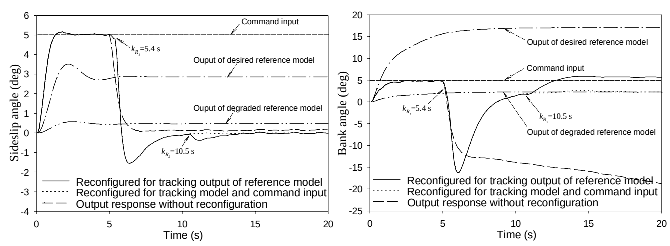

Fig. 3 *System output responses.*

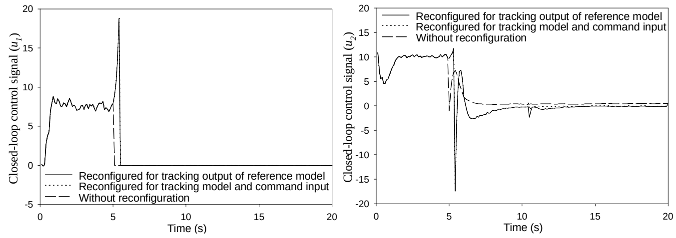

Fig. 4 *Corresponding closed-loop control signals.*

It is interesting to note that before the reconstruction process is activated at $k_{R_1} = 5.4\,\text{sec}$, the output response of the closed-loop system tends to diverge. After the restructurable controller has been activated, the bank angle can follow the output of the degraded reference model gradually. Once the tracking error becomes smaller than a prescribed threshold at $k_{R_2} = 10.5\,\text{sec}$, second reconstruction takes the place of the first reconstruction by synthesizing a new controller based on more accurate post-fault system model and with a simpler control structure of only two controller gains. For the Strategy 1, the design objective is to continuously track the output of the reference model as an output-tracking. For the Strategy 2, the objective is to track the original command input. It is obvious that both objectives have been achieved by designed restructurable controllers.

5.2.2. *Results for Strategy 3 and 4*

In the Strategies 3 and 4, the nominal controllers have been designed based on the model-following structure, the first restructured controller is synthesized using three controller gains but with reduced order in each controller. In the second restructured controller, new structure with two controller gains has been used. The restructurable controllers have been designed for tracking the output of the reference model and the original command input by the Strategy 3 and Strategy 4, respectively. Conclusions similar to those in Strategies 1 and 2 can be drawn. The design objectives have been achieved satisfactorily, as shown in Figs. 5 and 6. However, during the normal operation and the period of first reconstruction process, the bank angle and the associated control signal are smaller, with significantly smoother transients than those in the Strategies 1 and 2, due to the design objective to track dynamics of reference models in which degraded performance requirements have been incorporated.

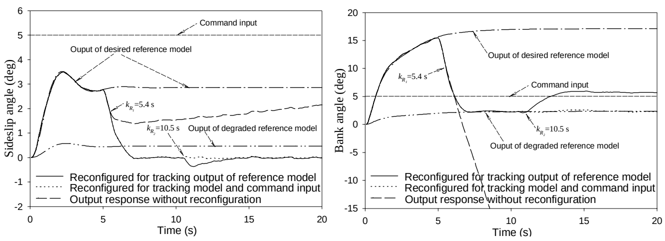

Fig. 5 *System outputs with and without reconfiguration.*

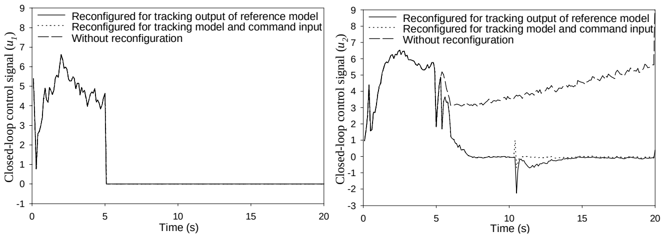

Fig. 6 *Closed-loop control signals.*

Simulation results using the above four strategies have shown the effectiveness of the proposed RFTCS. It should be pointed out, however, that selection of an appropriate strategy for a practical engineering application depends on particular performance requirements and specifications, constraints on complexity, cost and real-time implementation of the controller, among others. However, in general, to achieve the same or similar performance, the simpler the controllers, the better the design.

6. CONCLUSIONS

In this paper, a new scheme for restructurable fault-tolerant control systems (RFTCS) has been proposed based on model-following and command tracking techniques. By using model-following design strategy, performance degradation in the RFTCS has been taken into account. The restructurable controllers were designed online and automatically, based on linear quadratic regulator technique for feedback control, and model-following and command-tracking approaches for feedforward control. Several design strategies have been proposed and evaluated to demonstrate the variable-structure characteristics of the restructurable controllers. Simulation results have demonstrated the effectiveness of the proposed scheme using an aircraft model.

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