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15th Triennial World Congress, Barcelona, Spain

OPTIMIZATION METHODOLOGY FOR TUNING FUZZY LOGIC CONTROLLERS

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Abstract: The scope of this paper is to present an optimization methodology for tuning fuzzy logic controllers used in suspension system for ground vehicles. The proposed optimization method combines the advantages of two categories of optimization algorithms, deterministic and stochastic. Numerical experiments show the improvement in efficiency and reliability of the search for the optimum. The fuzzy logic membership functions are optimized such that the maximum value of vertical and rotational acceleration of vehicle body at the passengers seats are minimized from the view point of ride comfort under the geometrical constraints of the car. The simulation results of the proposed fuzzy logic controller show significant improvement regarding the vehicle ride comfort. *Copyright © 2002 IFAC*

Keywords: fuzzy logic controller, optimization, hybrid algorithm, semi-active suspension systems.

1. INTRODUCTION

Advanced suspension systems play a vital role in the performance of modern vehicles. They must support the vehicle body, keep the rider's comfort within permissible allowances, retain the vehicle stability during various handling actions, control body and wheel attitude, and minimise the vertical force variation of the road-to-tire contact. Another trade-off exists between the rider's comfort and safety and the economics of producing advanced suspensions. Presently, advanced suspensions implemented in modern vehicles are often described in confusing and conflicting ways. Therefore a great effort is made nowadays to develop or perfect adaptive or active suspension systems for vehicles, see (Asami, 1991). These systems, compared to the passive ones, see (Koulocheris, *et al.*, 1997; Spentzas, *et al.*, 1995), have a superior performance, but are very expensive, technically very complicated, much less reliable, require regular service and some of them consume non negligible quantities of energy. The semi-active system achieves some performance capabilities of fully active systems with components close to passive ones in terms of cost and complexity. The idea was to employ a spring to support the isolated mass in parallel with an adjustable damper whose force-velocity relationship could be modulated. Investigation of active and semi-active suspensions of ground vehicles in transportation is recently increasing, see (Chou, *et al.*, 1998; Yoshimura, *et al.*, 1997). However the semi-active suspensions which are denoted as less expensive alternatives to the active suspensions replace the active force generators by adjustable suspension parts according to the dynamic response of the vehicles.

2. SYSTEM DESCRIPTION

2.1 Vehicle model

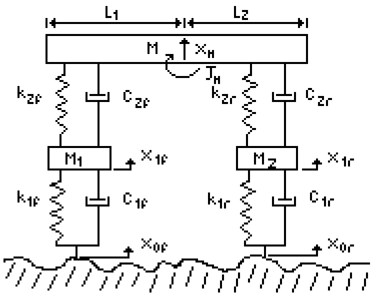

Fig.1. Half car model.

We considered a half car model Fig. 1. The elastic coefficient for tyre is k_{1f}=k_{1r}=155900 [N/m] and the damping coefficient is c_{1f}=c_{1r}=2500 [Ns/m]. The sprung mass is M=580 [kg], the mass inertia J_M=400[kg.m^2] and the unsprung mass M_1=M_2=29 [kg].
The system of differential equations of motion for both linear models can be brought in the form:

$$\begin{aligned}
\dot{x}_{1f} &= u_{1f} \\
\dot{x}_{1r} &= u_{1r} \\
\dot{x}_M &= u_M \\
\dot{\theta}_M &= \omega_M \\
\dot{u}_{1f} &= [k_{1f}(x_{0f} - x_{1f}) + k_{2f}(x_M + \theta_M L_1 - x_{1f}) + \ldots \\
&\ldots + c_{1f}(u_{0f} - u_{1f}) + c_{2f}(u_M + \omega_M L_1 - u_{1f})] / M_1 - g \\
\dot{u}_{1r} &= [k_{1r}(x_{0r} - x_{1r}) + k_{2r}(x_M - \theta_M L_2 - x_{1r}) + \ldots \\
&\ldots + c_{1r}(u_{0r} - u_{1r}) + c_{2r}(u_M - \omega_M L_2 - u_{1r})] / M_2 - g \\
\dot{u}_M &= [k_{2f}(x_{1f} - x_M - \theta_M L_1) + k_{2r}(x_{1r} - x_M + \theta_M L_2) + \ldots \\
&\ldots + c_{2f}(u_{1f} - u_M - \omega_M L_1) + \\
&c_{2r}(u_{2r} - u_M + \omega_M L_2)] / M - g \\
\dot{\omega}_M &= L_1 k_{2f}(x_{1f} - x_M - \theta_M L_1) - L_2 k_{2r}(x_{1r} - x_M + \theta_M L_2) \\
&\ldots + L_1 c_{2f}(u_{1f} - u_M - \omega_M L_1) - \\
&L_2 c_{2r}(u_{2r} - u_M + \omega_M L_2)] / J_M
\end{aligned} \tag{1}$$

2.2 Road excitations

Excitations by road irregularities can be considered, either deterministic or random. In the following, we focused our interest to two of them corresponding to the transition of obstacles: step function and harmonic excitation.

The step function excitation that we considered is:

$$x_{0f}(t) = \begin{cases} 0 \ldots\ldots\ldots t < 0.5 \\ vt \ldots\ldots\ldots 0.5 \le t \le 0.5 + \dfrac{0.06}{v} \\ 0.06 \ldots\ldots t > 0.5 + \dfrac{0.06}{v} \end{cases} \tag{2}$$

The harmonic excitation due to road irregularities is:

$$x_{0f}(t) = \begin{cases} 0 \ldots\ldots\ldots\ldots\ldots\ldots t < 0.5 \\ A[1 - \cos(2\pi f(t - 0.5))] \ldots\ldots\ldots 0.5 \le t \le 0.5 + \dfrac{1}{f} \\ 0 \ldots\ldots\ldots\ldots\ldots\ldots t > 0.5 + \dfrac{1}{f} \end{cases} \tag{3}$$

which is a cosine shaped bump of height 0.04m (peak to peak), with a wavelength proportional to the forward velocity of the car.

2.3 Solution of Equations

The system of differential equations of motion of the model was solved numerically on a PC-Pentium IV computer. The Matlab 6.1 program was used to perform the numerical simulations. Since it is necessary to solve the equations at each step of the optimization process, it is important to select an appropriate and fast solution method. Our preference was for the Runge - Kutta fourth order method with a variable time step. Convergence to the solution did not present any problem.

3. FUZZY LOGIC CONTROL

A semi-active suspension to be proposed here is realised by changing the damping coefficients of the front and rear dampers. Each fuzzy controller makes use of six inputs, three from the front and three from the rear suspension sensors:

- unsprung mass acceleration a_{1i}
- sprung mass acceleration a_{2i}
- suspension travel x_{12}=(x_1-x_2).
- relative velocity of sprung-unsprung mass

In Fig. 2-8 are shown the initial membership functions for the eight inputs and the output. The data that define these fuzzy sets compose the initial vector for the optimization procedure. The abbreviations at the input membership functions stand respectively for: large negative (L-), small negative (S-), very small (VS), small positive (S+) and large positive (L+). The use of two-sided Gaussian membership functions instead of standard trapezoidal ones achieved a smoother response and eliminated a problem of slight unsteadiness in the region of "small" excitations.

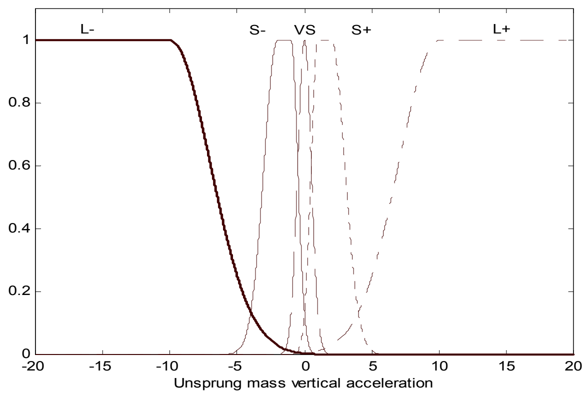

Fig.2 Input fuzzy sets-vertical acceleration.

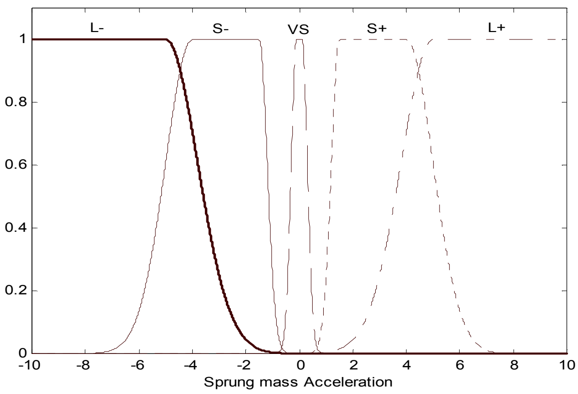

Fig. 3 Input fuzzy sets-sprung mass acceleration.

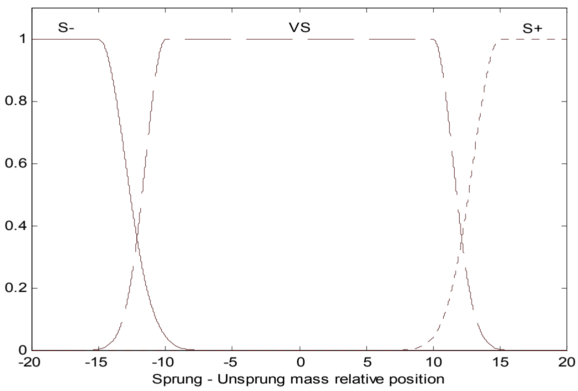

Fig. 4 Input fuzzy sets-relative position.

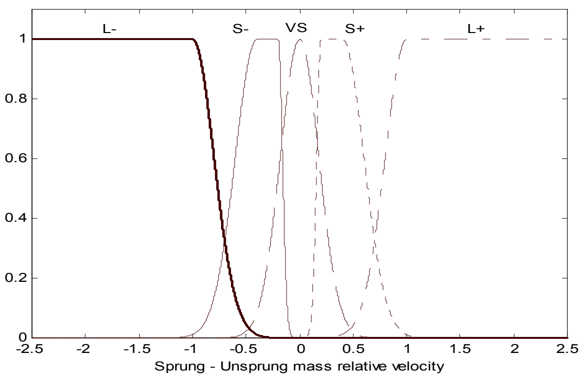

Fig.5 Input fuzzy sets-relative velocity.

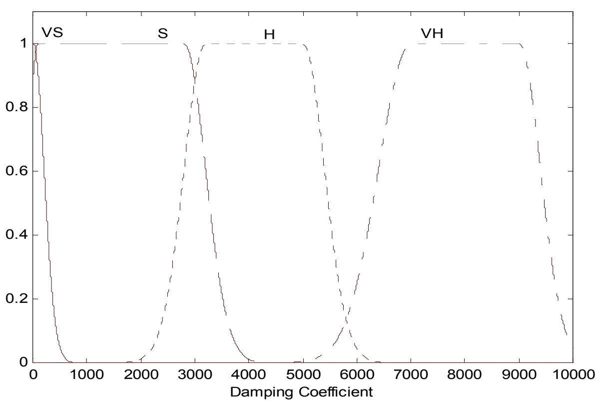

Fig.6 Output fuzzy sets-damping coefficient.

Each non-symmetric side of the Gaussian membership functions is a parameter for the optimization procedure. The main objective in designing and optimizing the fuzzy controller was increasing passenger comfort i.e. minimising the sprung mass linear and rotational acceleration. The fuzzy rules for the front half–car controller and the rear half-car controller, shown in Table 1, describe strategies to handle each case.

Table 1: Fuzzy Rules

A_{1f}	A_{2f}	X_{12f}	U_{12f}	A_{1r}	A_{2r}	X_{12r}	U_{12r}	C_{2f}	C
	S+ or L+		S- or L-					VS	
	S- or L-		S+ or L+					VS	
	S+ or L+		S+ or L+					VH	
	S- or L-		S- or L-					VH	
VS	VS	VS	VS					S	
					S+ or L+		S- or L-		V
					S+ or L+		S+ or L+		V
					S- or L-		S+ or L+		V
					S- or L-		S- or L-		V
				VS	VS	VS	VS		S
	S+		S-		S-		S+	H	H
	S-		S+		S+		S-	H	H

4. OPTIMIZATION METHOD

The proposed optimization method (hybrid) applied to the tuning of the fuzzy controller is based on a combination of a stochastic and a deterministic algorithm. The objective of this combination is the interrelation of the diverse characteristics of each category as well as the exploitation of their advantages the same time.

It is well known that stochastic algorithms are very efficient in avoiding local optima, see (Baeck, 1996; Michalewicz, 1996; Schwefel, 1995) but there is no indication regarding to their convergence rate. On the other hand, deterministic algorithms cannot avoid local optima but they converge very rapidly towards stationary points (Nocedal, 1999), this means points with zero gradient.

From the category of stochastic algorithms we chose the one parent-one offspring Evolution Strategy and from the category of deterministic algorithms the Steepest Descent method.

The proposed hybrid algorithm has been thoroughly tested, see (Kanarachos, *et al.,* 2001), with satisfying results. In particular, it has been proved statistically that the new proposed algorithm yields better vectors with decreased number of exact evaluations of the objective function, always compared with the Evolution Strategy.

4.1 Description of the method

Each vector produced by the Evolution Strategy is tested whether it satisfies the inequality:

$$f(\vec{x}_{new}) < h \cdot f(\vec{x}_{old}) \quad (4)$$

where $h \geq 1$. In the case that it does satisfy it, Steepest Descent takes over and yields the nearest local minimum. This way the proposed algorithm finds the best vector in the region of the random vector produced by the evolution strategy, feature that permits more frequent adaptation of the search space, via the adaptation of the standard deviation parameter, than that of the 1/5 rule, which is applied to the Evolution Strategy. In addition, convergence of the standard deviation of the evolution strategy to zero is not necessary because of the convergence of the norm of the gradient to zero through the Steepest Descent, thus we skip the most time-consuming phase of the Evolution Strategy. Once the standard deviation parameter of the Evolution Strategy becomes smaller than a prescribed value that is not close to zero, the final Steepest Descent phase takes place, in which the best vector so far becomes the initial vector for a deterministic search with increased accuracy. The stochastic nature of the Evolution Strategy, which is the feature that avoids local optima, is not affected.

4.2 *Wolfe condition*

The Wolfe condition ensures that the step length evaluation yields sufficient decrease in the objective function f by satisfying the following inequality

$$f(\vec{x}_k + a \cdot \vec{p}_k) \leq f(\vec{x}_k) + c \cdot a \cdot (\nabla f(\vec{x}_k))^T \cdot \vec{p}_k \quad (5)$$

where a is the step length in the steepest descent direction:

$$\vec{p}_k = -\frac{\nabla f(\vec{x}_k)}{\|\nabla f(\vec{x}_k)\|_2} \quad (6)$$

and c is a constant in the interval $(0,1)$. It can be proved that there exist step lengths that satisfy the Wolfe condition for every function f that is smooth and bounded below. The benefit of using the Wolfe condition is the certainty that the gradient sequence converges to zero.

4.3 Backtracking step length evaluation

This procedure evaluates the acceptable step length according to the Wolfe conditions. Starting from the initial step length $\bar{a}$, we reduce to ρa iteratively until the sufficient decrease condition (5) holds. The contraction factor ρ is taken here as a constant in the interval $(0,1)$.

Procedure Backtracking
set $\bar{a} > 0, \rho, c \in (0,1)$; set $a \leftarrow \bar{a}$;
while $f(\vec{x}_k + a \cdot \vec{p}_k) > f(\vec{x}_k) + c \cdot a \cdot (\nabla f(\vec{x}_k))^T \vec{p}_k$
    $a \leftarrow \rho a$;
end (while)
set $a_k \leftarrow a$

The vector to be optimized contains the data of the two-sided non-symmetric Gaussian membership functions. Geometrical constraints were applied, regarding the relative displacements of the different masses of the model, in order not to have design incompatibilities in the working space of the suspension:

$$g_1(\vec{x}) = \max\{|x_M + \theta_M L_i - x_{1i}| - 0.15, i = f, r\} < 0 \quad (7)$$

$$g_2(\vec{x}) = \max\{|x_{1i} - x_{0i}| - 0.04, i = f, r\} < 0 \quad (8)$$

4.4 Objective function

The objective function for fitness evaluation is formed by the summation of the ratios of the current values of the vertical and rotational acceleration to the corresponding initial values plus the penalty terms.

$$f(\vec{x}) = \frac{\ddot{x}_M}{\ddot{x}_M^{initial}} + \frac{\ddot{\theta}_M}{\ddot{\theta}_M^{initial}} + \sum_{i=1}^{2} 10^6 \cdot g_i^+(\vec{x}) \quad (9)$$

where

$$g_i^+(\vec{x}) = \begin{Bmatrix} g_i(\vec{x}), \ldots\ldots if \ldots\ldots g_i(\vec{x}) > 0 \\ 0, \ldots\ldots\ldots\ldots if \ldots\ldots g_i(\vec{x}) \leq 0 \end{Bmatrix} \quad (10)$$

are the constraint functions derived from the corresponding inequalities (7) and (8).

5. TEST FUNCTIONS

The stochastic feature of both the presented hybrid algorithm as well as the evolution strategy suggests a statistical way of testing their efficiency. A thousand different initial pseudorandom vectors were produced for each optimization algorithm and the results as well as the number of evaluations of the objective function until convergence were compared. Statistical procedure shows that both algorithms are capable of resulting in a vector that is very near the optimum, regardless of the pseudorandom initial vector. The advantage of the proposed hybrid algorithm is the ability of better fine-tuning in the area of the optimum, ability that is based in its deterministic feature, as well as the reduction of the number of the objective function exact evaluations.

The Griewangk's function has the following analytic form:

for $-600 \le x_i \le 600$, $i = 1,2,...n$

$$f_{GR}(\vec{x}) = 1 + \sum_{i=1}^{n} \frac{x_i^{\,2}}{400 \cdot n} - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) \qquad (11)$$

The Rastrigin's function has the following analytic form:

for $-5.12 \le x_i \le 5.12$, $i = 1,2,...n$

$$f_{RA}(\vec{x}) = 10n + \sum_{i=1}^{n} \left(x_i^{\,2} - 10\cos(2\pi x_i)\right) \qquad (12)$$

The Griewangk's Function as well as the Rastrigin's Function for n=2 are presented graphically in Figure 7 and Figure 8 respectively. It is obvious that the multiple local minima provide very difficult test cases for minimization.

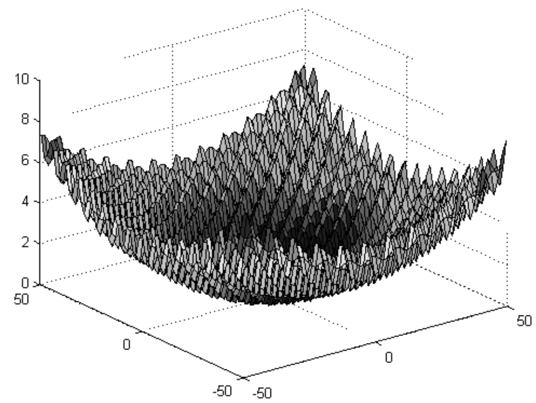

Figure 7: Graph of the Griewangk's Function.

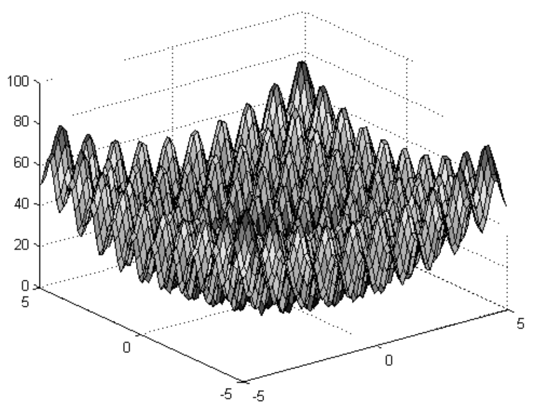

Figure 8: Graph of the Rastrigin's Function.

The statistical procedure shows that in 80% of the total number of tests for the Griewangk function and in 70% for the Rastrigin function, the hybrid algorithm resulted in a better vector than the Evolution Strategy did. The most significant statistical result is that the proposed algorithm needs 25% and 35% less evaluations of the objective function respectively in order to converge and satisfy the termination criterion, always compared with the Evolution Strategy. The efficiency of the proposed algorithm in respect to the Evolution Strategy regarding the Griewangk and Rastrigin functions are presented in Table 2.

Table 2: Statistic results

Statistics of Hybrid E.S -S.D.	Griewangk Function	Rastrigin Function
Reduction in number of evaluations	25%	35%
More efficient tests per cent	80%	70%

6. NUMERICAL RESULTS

In Fig. 9 & 10 we present numerical results to quasi-impulse excitation. The vehicle velocity is kept constant at 10m/s and the cosine frequency 8 Hz.

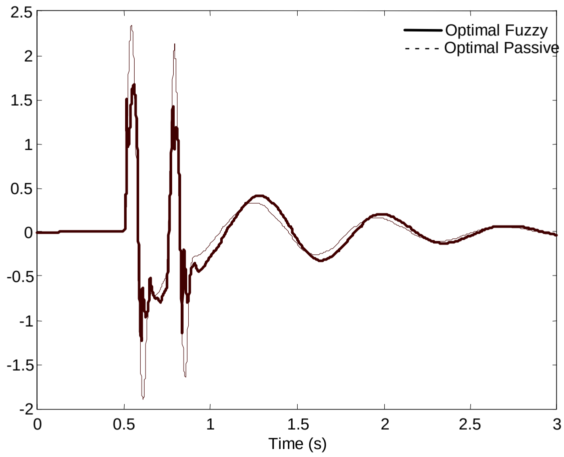

Fig.9 Sprung mass vertical acceleration.

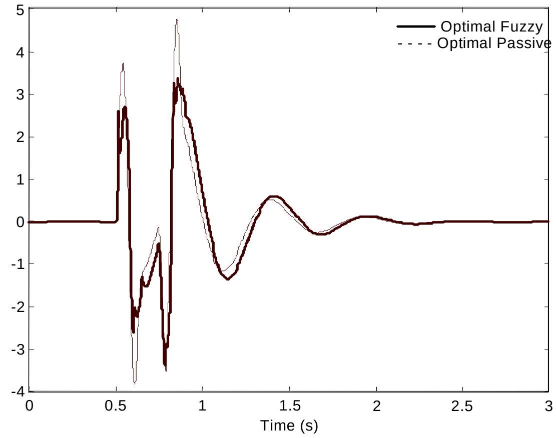

Fig.10 Sprung mass rotational acceleration.

In Fig. 11 & 12 we present numerical results to step excitation. The vehicle velocity is kept constant at 10m/s.

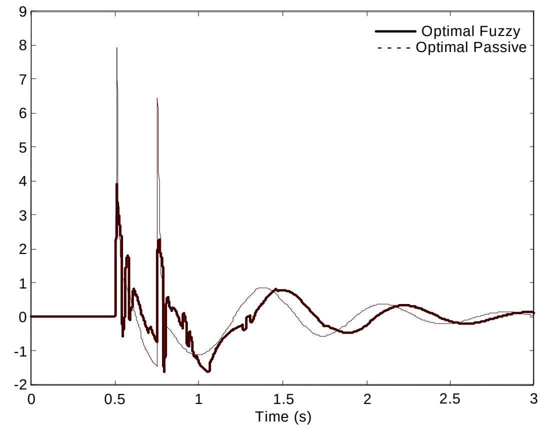

Fig.11 Sprung mass vertical acceleration.

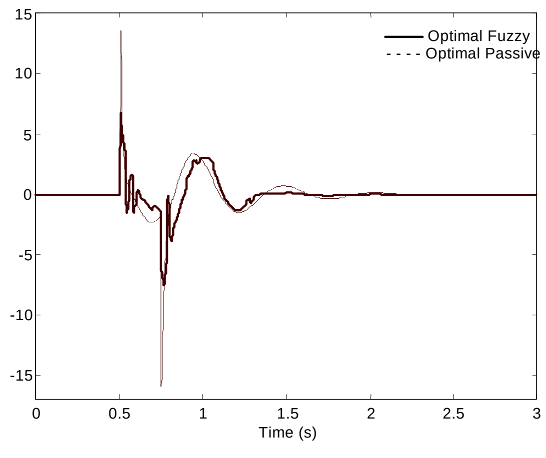

Fig.12 Sprung mass rotational acceleration.

As can be seen, the peak value of the suspended mass linear and rotational acceleration is neatly improved in the case of the semi-active half car model with fuzzy controller, compared to the optimized passive one. The semi-active model achieves equivalent or even better results in other performance criteria, such as peak pitching angle and settling time.

7. CONCLUSIONS

This paper proposed the construction of a fuzzy logic controller used in ground vehicles. The sprung and unsprung mass acceleration, the suspension travel and its time derivative were treated as the input variables and the damping coefficient of the suspension was adjusted as the output variable in the fuzzy control rules at every suspension location.

The performance of this suspension system was evaluated by considering a half car model and was proved to contribute greatly to the reduction of the peak acceleration of the suspended mass. The simulation results indicate that the proposed semi-active suspension is much improved in vertical and rotational acceleration. A future extension is planned by considering full car model with preview.

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