15th Triennial World Congress, Barcelona, Spain

INFERENTIAL PROBLEMS FOR A CLASS OF DISCRETE-TIME HYBRID SYSTEMS PART II: STATE ESTIMATION

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Abstract: This paper, the second part of our three-part contribution, is concerned with state estimation in a discrete-time hybrid system model, described in Part I (Bhowal *et al.*, 2002). The original contributions of the paper are as follow. The observable dynamics, seen through a measurement restriction imposed on the system, is characterised in terms of measurability and distinguishability of transitions between activity states. Based on the characterisation, an algorithm, to construct an Observer for estimation of the current state of the overall system composed from its component models, is developed. The whole theory is illustrated with an example of a heating system explined in (Bhowal *et al.*, 2002). This methodology for state estimation is used in a companion paper Part III (Sarkar *et al.*, 2002) for solving the problem of fault diagnosis.

Keywords: Hybrid Systems, Discrete-time Systems, Systems Modelling, State Estimation, Fault Diagnosis

1. INTRODUCTION

It is well known that limitations of measurement systems give rise to nondeterminism in terms of uncertainty in states and transitions in the observed dynamics of the systems. Inferential problems under partial observation for discrete event system are studied by many researchers (Bhowal *et al.*, 2000),(Ozveren and Willsky, 1990), (Sampath *et al.*, 1995), (Zad *et al.*, 1998). Inferential problem solving for hybrid systems has started recently (Bhowal *et al.*, 2001), (Basseville *et al.*, 1997), (McIlraith *et al.*, 2000), (Zad, 1999). In this paper, we formally introduce the concept of measurement with respect to the activity state based model formalism presented in our companion paper (Bhowal *et al.*, 2002), and characterise the consequent uncertainty in the states and transitions of the model. The concept of measurement and measurement equivalence is introduced first in the context of activity states and transitions. The existence of a finite activity transition graph for the process model is assumed. The observer for the process model under measurement restriction is constructed. Fault detection and diagnosis based on the observer is discussed in our companion paper (Sarkar *et al.*, 2002).

This paper is organized as follows. Section 2 characterizes the measurement structure and the consequent uncertainty in states and transitions. Section 3 considers the problem of estimation of activity states under the measurement restriction and constructs an observer for the same. Finally, section 4 concludes the paper.

2. MEASUREMENTS

In real life it may not be possible to measure all data variables due to inadequacy of sensors or due to physical limitations. The set V of data variables,

x	Composition	V_P				
		σ_{dx}			Δ_{xT}	b_x
		H	S	C	$\frac{\Delta T}{\Delta t}$	b_{xT}
x_1	$H_F C_L$	F	G	L	-1	$0 \leq T \leq 7$
x_2	$H_F C_H$	F	G	H	-1	$0 \leq T \leq 7$
x_3	$H_N C_H$	N	G	H	+1	$0 \leq T \leq 7$
x_4	$H_N C_L$	N	G	L	+1	$0 \leq T \leq 7$
x_5	$H_{SF} C_L$	F	B	L	-1	$0 \leq T \leq 7$
x_6	$H_{SF} C_H$	F	B	H	-1	$0 \leq T \leq 7$
x_7	$H_{SN} C_H$	N	B	H	+1	$0 \leq T \leq 7$
x_8	$H_{SN} C_L$	N	B	L	+1	$0 \leq T \leq 7$

Table 1. Activity description table of composite model of heating system

therefore, can be partitioned into two disjoint subsets, the subset V_m of *measurable* and the subset V_u of *unmeasurable* variables. We assume that $\forall v \in V_u$ there is no measurement and $\forall v \in V_m$ there is perfect measurement. Also $V_m \cap V_u = \phi$.

Let σ_m and σ_u denote a measurable and an unmeasurable part of a given data state σ respectively, at any instant of time.

Let Σ_m and Σ_u be the measurable and unmeasurable data spaces respectively.

Let Π_{V_m} (Π_{V_u}) denote the projection over the coordinates corresponding to members of V_m (V_u).

Thus $\Sigma_m = \Pi_{V_m}(\Sigma_D)$ and $\Sigma_u = \Pi_{V_u}(\Sigma_D)$.

$\Delta_{mx}(\Delta_{ux})$ is the change function (internal dynamics) of the measurable (unmeasurable) variables associated with an activity state x.

To illustrate this concept, the heating system model, explained in (Bhowal *et al.*, 2002) is considered. The ADT (set of variables and their data states, represented in a tabular form) is shown again in **Table 1**. If any of the variables is unmeasurable, in the ADT it may be seen that, with the set of measurable variables, all the activity states cannot be distinguished. This happens as the measurabe data spaces of some of the activity states become identical.

In **Table 1** if the status variable is not measurable, the pairs of activity states $(x_1, x_5), (x_2, x_6), (x_3, x_7)$, (x_4, x_8), are not distinguishable.

Also it can be seen that due to constrainta of measurement, some of the transitions become unmeasurable. In the composite model of heating system, τ_{s2} is unmeasurable. There may also be cases where, even among the measurable transitions, some can not be distinguished from others only by monitoring the occurrence of the transition. Such a case, however, does not occur in the heating system example.

2.1 *Transitions under measurement restriction*

Definition 1. **: Measurable transition**
A transition $\tau =< x, x^+ >$ is said to be measurable iff $(V_\tau \cap V_m \neq \phi) \vee (\Delta_{mx} \neq \Delta_{mx^+})$

This means that, either values or rates or both of some measurable variable are changing during such a transition.

Here V_τ is the set of target variables, which changes while the transition takes place and Δ_{mx} is the measurable dynamics of the activity state x.

Definition 2. **: Unmeasurable transition**
A transition $\tau =< x, x^+ >$ is said to be unmeasurable iff $(V_\tau \cap V_m = \phi) \wedge (\Delta_{mx} = \Delta_{mx^+})$

The set of measurable transitions is denoted as $\Im_m$ and the set of unmeasurable transitions is denoted as $\Im_u$; thus, $\Im = \Im_m \cup \Im_u$ and $\Im_m \cap \Im_u = \phi$

2.1.1. *Exit data space of a transition*

The exit data space of a transition $\tau =< x, x^+ >$ is the set of data states of the activity state x where the transition is eligible for taking place, that is $e_\tau = true$.

The limiting enabling condition $e_{\tau l}$ of a transition τ is obtained by restricting the inequalities in e_τ to respective equalities of the variables to its minimum or maximum value, depending upon increasing or decreasing dynamics of the variables. For example, for an e_τ of the form $(3 \leq T \leq 5)$, $e_{\tau l} = 3$ if $\frac{\Delta t}{\Delta t} > 0$, or $e_{\tau l} = 5$ if $\frac{\Delta t}{\Delta t} < 0$.

Let $\tau =< x, x^+, e_\tau, h_\tau, l_\tau, u_\tau >$ be a transition. The exit data space of τ is the data space

$$\rho_\tau = [e_{\tau l} + \Delta_x l_\tau, \;\; e_{\tau l} + \Delta_x u_\tau]$$

Under measurement restriction, the measurable exit data space of a measurable transition τ is the data space

$$\rho_{m\tau} = [e_{\tau l}/\Sigma_m + \Delta_{mx} l_\tau, \;\; e_{\tau l}/\Sigma_m + \Delta_{mx} u_\tau]$$

where $e_{\tau l}/\Sigma_m$ is the restriction of $e_{\tau l}$ on the measurable data space and the measurable exit data space is denoted as $\rho_{m\tau}$

2.1.2. *Exit data space computation under partial definition of e_τ*

For the variables which are not defined in the expression of e_τ, the exit data space (for the undefined variable) shall be the entire range of the variable. The is illustrated by the **Fig. 1**. In this figure the x-axis represents temperature and the y-axis represents pressure. e_τ, l_τ, u_τ and Δ_x of the transitions are given as

for $\tau_1 : e_\tau : (T \geq 2), l_\tau = 0, u_\tau = 2, \Delta_x : \frac{\Delta T}{\Delta t} = 1$
and $\rho_{\tau_1} =< (T = 2 \wedge P \geq 0), (T = 4 \wedge P \geq 0) >$

for $\tau_2 : e_\tau : (T \geq 4), l_\tau = 0, u_\tau = 2, \Delta_x : \frac{\Delta T}{\Delta t} = 1$
and $\rho_{\tau_2} =< (T = 6 \wedge P \geq 0), (T = 8 \wedge P \geq 0) >$

for $\tau_3 : e_\tau : (P \leq 10), l_\tau = 0, u_\tau = 1, \Delta_x : \frac{\Delta P}{\Delta t} = -10$

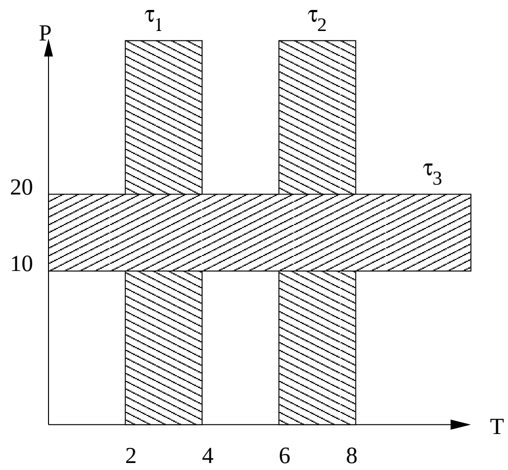

Fig. 1. Exit data space computation

and $\rho_{\tau_2} =< (T \geq 0 \wedge P = 10), (T \geq 0 \wedge P = 20) >$

The exit data spaces of the transitions are represented in **Fig. 1** as shaded areas. It can be seen that the exit data spaces of τ_1 and τ_2 are not overlapping. But the exit data space of τ_3 is overlapped with those of τ_1 and τ_2.

Definition 3. **: Distinguishability of measurable transitions**
Two measurable transitions $\tau_1 < x_1, x_1^+ >$ and $\tau_2 < x_2, x_2^+ >$ are distinguishable if any one of the following conditions is satisfied

(1) $h_{\tau_1}/\Sigma_m \neq h_{\tau_2}/\Sigma_m$
(2) $(\Delta_{mx_1} \neq \Delta_{mx_2}) \vee (\Delta_{mx_1^+} \neq \Delta_{mx_2^+})$
(3) $(\sigma_{mdx_1} \neq \sigma_{mdx_2}) \vee (\sigma_{mdx_1^+} \neq \sigma_{mdx_2^+})$
(4) $\rho_{m\tau_1} \cap \rho_{m\tau_2} = \phi$
(5) $u_{\tau_1} < l_{\tau_2}$ when $(e_{\tau_1} = e_{\tau_2}) \wedge (\Delta_{mx1} = \Delta_{mx2} = 0)$. This condition is important for discrete system. For systems with continuous dynamics, the effect of timing constrain is captured by clause 4.

Definition 4. **: Nondistinguishability of measurable transitions**
Two measurable transitions $\tau_1 < x_1, x_1^+ >$ and $\tau_2 < x_2, x_2^+ >$ are non-distinguishable iff all the following conditions are satisfied

(1) $h_{\tau_1}/\Sigma_m = h_{\tau_2}/\Sigma_m$
(2) $(\Delta_{mx_1} = \Delta_{mx_2}) \wedge (\Delta_{mx_1^+} = \Delta_{mx_2^+})$
(3) $(\sigma_{mdx_1} = \sigma_{mdx_2}) \wedge (\sigma_{mdx_1^+} = \sigma_{mdx_2^+})$
(4) $\rho_{m\tau_1} \cap \rho_{m\tau_2} \neq \phi$
(5) $(l_{\tau_2} < u_{\tau_1})$

Two non-distinguishable transitions, however, can actually be distinguished if their exit data spaces are not equal or the timing constrains are not matching. Therefore, the notions of strict non-distinguish ability is introduced.

Definition 5. **: Strict nondistinguishability of measurable transitions**
Two measurable transitions $\tau_1 < x_1, x_1^+ >$ and $\tau_2 < x_2, x_2^+ >$ are strict nondistinguishable iff all the following conditions are satisfied

(1) $h_{\tau_1}/\Sigma_m = h_{\tau_2}/\Sigma_m$
(2) $(\Delta_{mx_1} = \Delta_{mx_2}) \wedge (\Delta_{mx_1^+} = \Delta_{mx_2^+})$
(3) $(\sigma_{mdx_1} = \sigma_{mdx_2}) \wedge (\sigma_{mdx_1^+} = \sigma_{mdx_2^+})$
(4) $\rho_{m\tau_1} = \rho_{m\tau_2}$ /* perfect matching of data space */
(5) $(l_{\tau_1} = l_{\tau_2}) \wedge (u_{\tau_1} = u_{\tau_1})$ /* perfect matching of time space */

Since the strict nondistinguishability conditions are subsumed by nondistinguishability conditions **(Definition 4)**. For diagnosis, strict nondistinguishability need not be considered.

A set of nondistinguishable measurable transitions is denoted as λ. λ is also a six-tuple, i.e., $< y, y^+, e_\lambda, h_\lambda, l_\lambda, u_\lambda >$ where,

y represents a set of activity states, defined as $y = \{x | \tau < x, x^+ > \in \lambda\}$.
y^+ represents a set of activity state, defined as $y^+ = \{x^+ | \tau < x, x^+ > \in \lambda\}$.
e_λ is the enabling condition, defined as

$$e_\lambda = (\bigvee_{\tau_i \in \lambda} e_{\tau_i})/\Sigma_m$$

h_λ is the transformation function, defined as $h_\lambda = h_{\tau_i}/\Sigma_m$ (It is to be noted that by the definition of non-distinguish-ability of measurable transitions $\forall \tau_i \in \lambda, \quad h_{\tau_i}/\Sigma_m$ is identical).
l_λ is the lower time limit, defined as $l_\lambda = min(l_{\tau_i}) | \tau_i \in \lambda$
u_λ is the upper time limit, defined as $u_\lambda = max(u_{\tau_i}) | \tau_i \in \lambda$

The set of all such nondistinguishable measurable transition classes is denoted as Λ, where $\Lambda = \{\lambda_1, \lambda_2 .... \lambda_n\}$. Also,

$$\Im_m = \bigcup_j (\bigcup_i \tau_i \in \lambda_j) \;\; and \;\; \lambda_i \cap \lambda_j \;\; need \; not \; be \; \phi, \quad for \;\; i \neq j$$

A measurable transition τ_k which is distinguishable from all other measurable transitions, may also be considered as a nondistinguishable measurable transition λ_k (say), i.e. $\lambda_k = \{\tau_k\}$.

The set of all nondistinguishable measurable transitions (λ) is denoted as Λ.

3. STATE ESTIMATION UNDER MEASUREMENT RESTRICTION

Having defined the measurement structure and the consequent uncertainty in activity state, that arise, of is natural to consider the problem of activity state estimation under measurement restriction. The estimates are generated with the help of an automata called the observer, defined below.

3.1 *Construction of observer*

An observer is an activity state estimator under measurement restriction. It is represented as a digraph $O =< N, A >$, where each node $n \in N$ represents a set of activity states and each arc $a \in A$ represents a set of nondistinguishable measurable transitions. Each node represents the uncertainty about the states. Similarly, each arc represents the uncertainty of occurrence of a measurable transition.

The observer is constructed starting from the initial state of the process model. The construction is based on the following definitions.

Definition 6. **: Unmeasurable successor** $(\mathcal{U})$
The unmeasurable successor of a set n of activity states is defined as

$$\mathcal{U}(n) = \{x^+ | \forall x \in n, \tau < x, x^+ >\in \Im_u\}$$

Definition 7. **: Unmeasurable reach** $(\mathcal{U}^*)$
The unmeasurable reach of a set n of activity states is the transitive closure (Kleene closure) of unmeasurable successors of n and is denoted as $\mathcal{U}^*(n)$

$$\forall n \in N, \exists n_e \subseteq n \;\; s.t. \;\; n = \mathcal{U}^*(n_e);$$

n_e is called the *entry set* of n.

Definition 8. **: Measurable successors of a set** n **for** λ (n_λ^+)
the measurable successor set of a set n for a non-distinguishable measurable transition $\lambda = \{\tau_1, \tau_2 ...\}$, denoted as n_λ^+ is defined by the following set of entities

$$(i) \quad \lambda_n = \{\tau \mid \tau \in \lambda, \tau =< x, x^+ > \;\; and \;\; x \in n\} \qquad (1)$$

$$(ii) \;\; n_\lambda = \{x \mid x = initial(\tau), \;\; \tau \in \lambda_n\} \qquad (2)$$

$$(iii) \; n_\lambda^+ = \{x^+ \mid x^+ = \;\; final(\tau, \tau \in \lambda_n\} \qquad (3)$$

λ_n represents an arc of the observer between the nodes n and n^+, where $n_\lambda \subseteq n$ and $n_\lambda^+ \subseteq n^+$. This is shown in **Fig. 2**. Actually $n^+ = \mathcal{U}^*(n_\lambda^+)$.

More specifically, given an uncertainty node n, $n^+$is obtained using the following steps. For each set λ of measurable non-distinguishable transitions, obtain $\lambda_n \subseteq \lambda$ (having initial states of the transitions in n); next, from n_λ and λ_n, n_λ^+ is obtained. Finally, a new uncertainty node $n^+ = \mathcal{U}(n_\lambda^+)$ is obtained and $\lambda_n =< n, n^+ >= a$ is put in arc set A of the observer.

Each $a \in A$ is also a six tuple

$$a =< n, n^+, e_a, h_a, l_a, u_a >$$

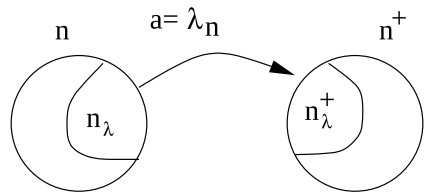

Fig. 2. Transition under measurement restriction

where

n represents the initial node, from where the arc a is defined
n^+ represents the final node of arc a.
e_a is the enabling condition, is defined as

$$e_a = (\bigvee_{\tau_i \in a} e_{\tau_i})/\Sigma_m$$

h_a is the transformation function, is defined as $h_a = h_{\tau_i}/\Sigma_m$. It is to be noted that by the definition of non-distinguish-ability of measurable transitions, $\forall \tau_i \in a, \quad h_{\tau_i}/\Sigma_m$ is identical.
l_a is the lower time limit, is defined as $l_a = min\{l_{\tau_i} | \tau_i \in a\}$
u_a is the upper time limit, is defined as $u_a = max\{u_{\tau_i} | \tau_i \in a\}$

Algorithm 1. **: Algorithm for construction of observer** O **for the process model** M

begin

C1 Let x_0 be the initial activity state of the process model M.

$$n_0 = \mathcal{U}^*(x_0)$$

let $N \leftarrow \{n_0\}; \;\; A \leftarrow \phi$
C2 .
for all $n \in N$ **do**
  for all non-distinguishable set λ of measurable transitions **do**
    compute $\lambda_n, n_\lambda, n_\lambda^+$ and n^+ [def: 8]
    if $n' = \mathcal{U}^*(n_\lambda) \notin N$,
      $N \leftarrow N \cup \{n^+\}$
      $A \leftarrow A \cup \{\lambda_n\}$
  end /*end of for loop */
end /*end of for loop */

end

Termination of the Algorithm The algorithm constructs subsets of activity states. Since the composite model has finite number of activity states, their subsets will be finite in number. Hence the algorithm will terminate.

3.2 *Example: Heating system*

The construction of observer is illustrated by the example of heating system here. It is considered that only the temperature and its rate variable (i.e. T and

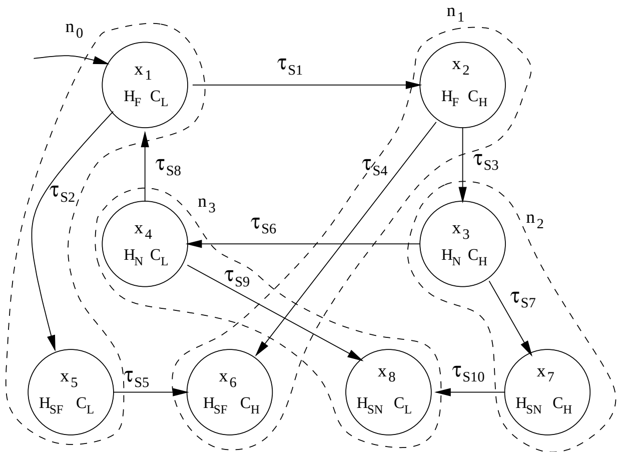

Fig. 3. Sates reachable by non-measurable transitions (τ_u)

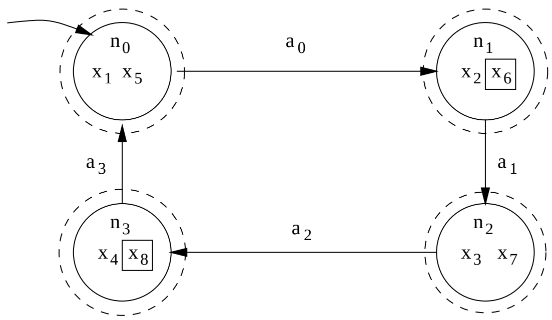

Fig. 4. Observer for heating system

$\frac{\Delta T}{\Delta t}$) is measurable . The unmeasurable transitions are shown by dotted line in the **Fig. 3**.

The set of nondistinguishable measurable transitions are $\lambda_0 = \{\tau_{S1}, \tau_{S5}\}$, $\lambda_1 = \{\tau_{S3}\}$, $\lambda_2 = \{\tau_{S6}, \tau_{S10}\}$, $\lambda_3 = \{\tau_{S8}\}$

The set of nodes are $n_0 = \{x_1, x_5\}$, $n_1 = \{x_2, x_6\}$, $n_2 = \{x_3, x_7\}$, $n_3 = \{x_4, x_8\}$

The observer constructed according to the **Algorithm 1** is shown in **Fig. 4**. In this example the arcs are correspond to the set of nondistinguishable measurable transitions that is ($a_0 = \lambda_0$, $a_1 = \lambda_1$, $a_2 = \lambda_2$, $a_3 = \lambda_3$) .

□

In the previous section, the construction of observer O, under measurement restriction was discussed. some of the important properties of the observer O are stated below.

Lemma 1. For any two nodes n_1 and n_2, if their corresponding entry sets n_{1e} and n_{2e} are different, then n_1 and n_2 are different. Thus symbolically

$$n_{1e} \neq n_{2e} \Rightarrow n_1 \neq n_2$$

Proof by construction

Lemma 2. Any transition between two nodes of the observer O is a set of measurable transitions

Proof By step C2 of observer construction algorithm

Lemma 3. All unmeasurable transitions are within a node. For a transition $\tau < x, x^+ >$, if $x, x^+ \in n$ then $\tau \in \Im_u$.

Proof By contrapositivity of Lemma 2.

Lemma 4. All the activity states in an observer node n, have identical measurable dynamics. Symbolically

$$\forall x_1, x_2 \in n, \quad \Delta_{mx_1} = \Delta_{mx_2}$$

Proof .

Either $n = \mathcal{U}^*(\{x_0\})$
/*initial uncertainty */
Or $n' = \mathcal{U}^*(n^+_\lambda)$ for some $n \in N$
/*n' is created by some λ */
Again
Case I: $x_j \in \mathcal{U}^*(\{x_i\})$
Case II: $\{x_i, x_j\}$ are final states of two members of λ

For Case I, $\Delta_{mx_i} = \Delta_{mx_j}$ by definition of unmeasurable transition
CaseII, $\Delta_{mx_i} = \Delta_{mx_j}$ by definition of non-distinguishable measurable
‘ transition

[Proved]

Lemma 5. It is possible to have two distinct nodes $n_1, n_2 \in N$, with same measurable dynamics.

Proof Two distinct nodes $n, n' \in N$ may have same measurable dynamics.

For example, consider two subsets $\lambda_1 =< n_{1\lambda_1}, \mathcal{U}^*(n^+_{1\lambda_1}) >$ and $\lambda_2 =< n_{2\lambda_1}, \mathcal{U}^*(n^+_{2\lambda_1}) >$ of a set λ of non-distinguishable measurable transitions.

Therefore $\lambda_1, \lambda_2 \subseteq \lambda$, $\Delta_{mn_{1\lambda_1}} = \Delta_{mn_{2\lambda_2}}$ and $\Delta_{m\mathcal{U}^*(n_{1\lambda_1})} = \Delta_{m\mathcal{U}^*(n_{2\lambda_2})}$

4. CONCLUSION

In this paper we have introduced the concept of measurement restriction on a hybrid system model discussed in (Bhowal *et al.*, 2002). The activity state estimation problem under measurement restriction has been discussed and an observer for the model has been developed. The properties of the observer are also discussed. The concepts of measurement restriction and construction of observer are explained with the help of an example of a heating system. Fault detection and diagnosis using these reasults is discussed in one of our companion paper (Sarkar *et al.*, 2002).

Acknowledgement: Thanks are due to the anonymous reviewers for their valuable comments.

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