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15th Triennial World Congress, Barcelona, Spain

ADAPTIVE PRE-EMPTIVE CONTROL OF VACUUM DEWATERING IN PAPER MANUFACTURING [1]

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Abstract: Paper manufacturing involves the sequential removal of water from pulp by means of gravity, vacuum dewatering, mechanical pressing and thermal drying. This research proposes a new control topology for moisture content using surrogate measurements to infer in-process moisture content, and the pressure settings in the vacuum dewatering section as actuators. The new topology has the potential to overcome the performance, robustness and energy efficiency limitations that the current process control topology has, due to a long dead-time and excessive use of the dryers. A pre-emptive control law was previously proposed for the new topology to decouple the upstream disturbances from propagating downstream in the process. In this paper, we develop an adaptive version of this pre-emptive controller to alleviate the need to know the dewatering coefficients precisely. The adaptive control problem has the interesting aspects that the unknown parameters are nonlinearly embedded in the error dynamics, and there is a time delay before the error in the estimated parameters can be observed.

Keywords: Feedforward, time delay, adaptive control, nonlinear parameterization, paper industry.

1. INTRODUCTION

Paper manufacturing (Fig.1) involves the sequential removal of water from pulp with 99.5% moisture by means of gravity, vacuum dewatering, mechanical press, and thermal drying in a paper machine, to form the final product with moisture content of $4-8\%$. Uniform and accurate moisture content of the final product is a key quality measure. Current process control strategies make use of an online moisture content scanner at the end of the paper machine to adjust the steam pressure in the dryer sections. Although a variety of control laws have been proposed (e.g. P-I, adaptive, stochastic control, self-tuning LQ, neuro-fuzzy, model algorithmic control etc.), because of the long time delay between the actuators and the sensor, this control topology limits control performance. Moreover, the control strategy relies heavily on the drying section, which is the most energy intensive of all the water removal processes.

An alternate control strategy is proposed in which the pressure settings of the 5 - 10 suction boxes within the vacuum dewatering section are used as process control actuators. Since direct in-process moisture content measurements are not available at the wet end of the process, the moisture content is inferred from air flow rate through the wet sheet (air flow decreases as moisture content is increased). The existence of such a correlation is supported by (Brundrett and Baines, 1966; Washburn and Buchanan, 1964; Ramaswamy *et*

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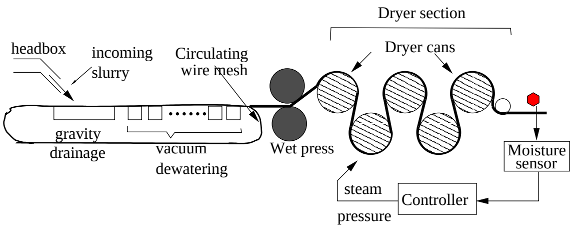

Fig. 1. Schematic of a water removal processes in a paper machine and the current process control strategy

al., 1999; Cui *et al.*, 1999; Polat *et al.*, 1992; Polat *et al.*, 1987; Tietz and Schlunder, 1993). The fundamental quantitative relationships between air flow, moisture content and other factors are currently being experimentally investigated.

Based on the concept of surrogate in-process moisture content measurements, a pre-emptive distributed control approach was proposed in (Li *et al.*, 2001). In this approach, there is a desired target moisture content for each dewatering (or suction) box. The surrogate moisture content measurements are used as local feedback for that dewatering box and are provided for the downstream dewatering box to pre-emptively eliminate the moisture content error due to the deviation of the incoming moisture content from nominal values.

Although the pre-emptive control law in (Li *et al.*, 2001) does have some robustness properties due to the measurements and actuation being co-located, for pre-emptive error elimination to be successful, the dewatering coefficients K_i of each dewatering box i must be known. This is not realistic in application because K_i depends on pulp composition which is a major uncertainty in the process. In this paper, an adaptive pre-emptive control law is proposed to relax the need for knowing K_i precisely. The interesting aspects of this adaptive control problem are that a) the unknown parameters K_i's are embedded nonlinearly in the error dynamics; b) because estimation errors of the K_i's affect the error dynamics in the downstream dewatering boxes, there is a significant transport delay before the effect of K_i estimation error and the time when the effects are observed. To overcome these difficulties, we utilize 1) a similar adaptive control strategy for nonlinearly parameterized systems proposed in (Ai-Poh *et al.*, 1999; Annaswamy *et al.*, 1998), 2) a first order system approximation of the time delay to derive the adaptation law, and 3) a tuning function to take care of any approximation errors. The proposed control law causes the moisture content error to converge to its nominal value within a predefined desired precision.

The paper is organized as follows. The development of the control design model is given in Section 2. Section 3 presents the original preemptive control algorithm. The robustness of the preemptive control scheme to model parameter uncertainty is analyzed in section 4. The adaptive control algorithm is presented in section 5. Section 6 presents the simulation results that illustrate the usefulness of the approach. Section 7 contains some concluding remarks.

2. SYSTEM MODEL

The one-lump vacuum dewatering box model developed in (Li *et al.*, 2001; Bjegovic, 2001) is given by:

$$\frac{d}{dt}w_i(t) = -\Omega_i(t)w_i(t) + v \cdot c_{i-1,out}(t-\tau_i), \qquad (1)$$

where $w_i(t)$ is the total moisture content in the domain of the $i = 1, \cdots N$-th dewatering box, v is the transport speed, $c_{i-1,out}(t)$ is the exit moisture content of the $i-$th box (taken to be the exiting moisture content at the end of the gravity section when $i = 1$), τ_i is the transport delay between the $i-1$st and the $i-$th boxes,

$$\Omega_i(t) := \frac{K_i\sqrt{P_i(t)}}{1-f_i(t)}, \qquad (2)$$

$$\text{with} \qquad f_i(t) := e^{-\frac{K_i\sqrt{P_i(t)}}{v}B}. \qquad (3)$$

K_i denotes the dewatering coefficient assumed to be constant in time and in the domain of the i-th dewatering box, B is the length of the dewatering slot length, and $P_i(t)$ is the pressure difference applied on a paper sheet, which is the manipulated variable. The exit moisture content $c_{i,out}(t)$ is related to $w_i(t)$ by

$$\begin{aligned} c_{i,out}(t) &= e^{-\frac{K_i\sqrt{P_i(t)}}{v}B}c_{i-1,out}(t) \\ &= \frac{K_i\sqrt{P_i(t)}}{v}\frac{f_i(t)}{1-f_i(t)}w_i(t). \end{aligned} \qquad (4)$$

The complete derivation of the vacuum dewatering box model can be found in (Bjegovic, 2001).

3. PREEMPTIVE CONTROL USING SURROGATE MEASUREMENTS

3.1 Control problem formulation

The control objective is to control the pressure settings P_i so that for each $i = 1, \ldots, N$, $w_i(t) \to w_i^*$ which is the target moisture content for each vacuum dewatering box i. w_i^* are designed so that water removal takes place gradually without saturating the control or causing catastrophic sealing of the sheets. We assume that w_i^*, $i = 1 \ldots N$ have been designed so that

$$0 = \dot{w}_i^* = -\Omega_i^* w_i^* + v \cdot c_{i-1,out}^* \qquad (5)$$

where $\Omega_i^* := \frac{K_i\sqrt{P_i^*}}{1-f_i^*}$, $f_i^* := e^{-K_i\sqrt{P_i^*}\frac{B}{v}}$, in which P_i^* is the nominal operating pressure, and

$$c_{i-1,out}^* = \frac{K_{i-1}\sqrt{P_{i-1}^*}}{v}\frac{f_{i-1}^*}{1-f_{i-1}^*}w_{i-1}^*$$

is the target exit moisture content, $c_{0,out}^*$ is taken to be the nominal incoming slurry moisture content, and P_i^* is the nominal pressure setting.

3.2 Preemptive control law

Let $e_i(t) := w_i(t) - w_i^*$ be the moisture content error for the $i-$th dewatering box. It can be shown that

$$\dot{e}_i(t) = -\Omega_i(t)e_i(t) + (\Omega_i^* - \Omega_i(t))\, w_i^* + v\cdot \tilde{c}_{i-1,out}(t-\tau_i), \quad (6)$$

where $\tilde{c}_{i-1,out}(t-\tau_i) := c_{i-1,out}(t-\tau_i) - c^*_{i-1,out}$. Moreover, $c_{i-1,out}(t-\tau_i)$ is available from Eq.(4), because $w_{i-1}(t-\tau_i)$ and $P_{i-1}(t-\tau_i)$ are known.

The basic pre-emptive control strategy is to choose $\Omega_i(t)$ as

$$\Omega_i(t) = \Omega_i^* + \frac{v}{w_i^*}\left[\lambda_{ff,i}\cdot \tilde{c}_{i-1,out}(t-\tau_i) + \frac{\lambda_{fb,i}}{v}e_i(t)\right]$$

or equivalently,

$$\Omega_i(t) = \lambda_{fb,i}e_i(t) + \frac{v}{w_i^*}\left[\lambda_{ff,i}\cdot c_{i-1,out}(t-\tau_i) + (1-\lambda_{ff,i})c^*_{i-1,out}\right] \quad (7)$$

where $1 \geq \lambda_{ff,i} \geq 0$ and $\lambda_{fb,i} \geq 0$ are the preemptive feedforward and feedback control gains respectively. For $i = 1$, since measurement of the incoming moisture content is not available, $\lambda_{ff,1} = 0$. The resulting error dynamics for $i > 1$ are then given by:

$$\dot{e}_i(t) = -\left(\Omega_i(t) + \lambda_{fb,i}\right)e_i(t) + (1-\lambda_{ff,i})v\cdot \tilde{c}_{i-1,out}(t-\tau_i). \quad (8)$$

Remark: Given choices for Ω_i in (7), the actual pressure settings P_i are obtained by inverting (2), which is invertible for $\Omega_i > v/B$ (until pressure saturation limits occur).

From (8), it is obvious that the control law Eq.(7) increases the convergence rate using local feedback and compensates for any discrepancy in the incoming moisture content to the present box. In particular, if $\lambda_{ff,i}$ is chosen to be 1, and $\lambda_{fb,i} \geq 0$, then the effect of any upstream disturbances will be completely decoupled from the i-th box and e_i converges to 0 exponentially.

The localization of the feedback information to the present dewatering box and its neighbor can be conceptualized as a Smart Vacuum Dewatering Box which controls its own total moisture content, and provides an estimate of the exiting moisture content to its neighbors. The complete control scheme is made up of the interconnection of such Smart Vacuum Dewatering boxes.

4. EFFECT OF THE PARAMETRIC UNCERTAINTY

We analyze the effect of uncertainty in the dewatering transport coefficients K_i. Assume that the surrogate moisture content measurements are correct, so $\hat{w}_i(t) = w_i(t)$, and the feedforward gain is: $\lambda_{ff,i} = 1$. Let $\hat{K}_i$ be the estimate of K_i. We will use the notation $\hat{\Omega}_i(t)$ to denote the expression for $\Omega_i(t)$ in Eq.(2) with K_i substituted by $\hat{K}_i$ (including inside f_i), and $\hat{c}_{i-1,out}(t)$ to denote the $c_{i-1,out}(t)$ in Eq.(4) with K_{i-1} substituted by $\hat{K}_{i-1}$. Then, the computation of the control effort P_i will be based on:

$$w_i^*\hat{\Omega}_i(t) = v\hat{c}_{i-1,out}(t-\tau_i) + \lambda_{fb,i}e_i(t). \quad (9)$$

Here,

$$\hat{c}_{i-1,out}(t) = \hat{\alpha}_{i-1}(t)\cdot w_{i-1}(t),$$

with

$$\hat{\alpha}_{i-1}(t) := \frac{\hat{K}_{i-1}\sqrt{P_{i-1}(t)}}{v}\frac{\hat{f}_{i-1}(t)}{1-\hat{f}_{i-1}(t)}. \quad (10)$$

The values of $\hat{\Omega}_i(t)$ and $\hat{f}_i(t)$ are given by Eq.(2) and Eq.(3), with K_i replaced by $\hat{K}_i$. In the actual situation, the pressure $P_i(t)$ would be computed based on $\hat{\Omega}_i(t)$ in (9), but applied to $\Omega_i(t)$ in (2) and then in turn to the error dynamics given by (5) and (6). Since $\hat{\Omega}_i(t)$ and $\Omega_i(t)$ are positive, there exists $s_i(t) > 0$ such that

$$s_i(t) := \frac{\Omega_i(t)}{\hat{\Omega}_i(t)} = \frac{K_i}{\hat{K}_i}\frac{1-\hat{f}_i(t)}{1-f_i(t)}. \quad (11)$$

Similarly, we can define

$$g_i(t) := \frac{\alpha_i(t)}{\hat{\alpha}_i(t)} = s_i(t)\frac{f_i(t)}{\hat{f}_i(t)} = s_i(t)e^{-(K_i-\hat{K}_i)\sqrt{P_i(t)}\frac{B}{v}} \quad (12)$$

so that

$$\hat{c}_{i-1,out}(t) = \hat{\alpha}_{i-1}(t)w_{i-1}(t) = \frac{1}{g_{i-1}(t)}c_{i-1,out}(t).$$

The error dynamics for $i = 2,\ldots,N$ then become:

$$\dot{e}_i(t) = -\left[\Omega_i(t) + s_i(t)\lambda_{fb}\right]e_i(t) + \left(1 - \frac{s_i(t)}{s_{i-1}(t)}e^{(K_{i-1}-\hat{K}_{i-1})\sqrt{P_{i-1}(t-\tau_i)}\frac{B}{v}}\right)\cdot v\cdot c_{i-1,out}(t-\tau_i) \quad (13)$$

Under normal operating conditions, we expect that the exponents in $f_i(t)$ and $\hat{f}_i(t)$ are in the range $[0.025, 0.4]$, so $s_i(t)$ is given approximately by:

$$s_i(t) \approx \frac{K_i}{\hat{K}_i}\frac{[1-(1+\hat{K}_i\sqrt{P_i(t)}B/v)]}{[1-(1+K_i\sqrt{P_i(t)}B/v)]} = 1. \quad (14)$$

From Eq.(15), the error bound can be obtained as:

$$\|e_i(t\to\infty)\| \leq \frac{\|c_{i-1,out}(\cdot)\|_\infty}{\underline{\Omega_i} + \underline{s_i}(t)\lambda_{fb,i}} \cdot \|1 - \frac{s_i(t)}{s_{i-1}(t)}e^{(K_{i-1}-\hat{K}_{i-1})\sqrt{P_{i-1}}\frac{B}{v}}\|_\infty \quad (15)$$

where $\underline{s_i}(\cdot)$ is the lower bound of $s_i(t) > 0$. From this equation we can conclude that the feedback gain $\lambda_{fb,i}$ can be beneficial in the case of uncertain process parameters. Also, the uncertainty in K_i only has a benign effect on the feedback action by changing the effective feedback gain from $\lambda_{fb,i}$ to $s_i(t)\lambda_{fb,i}$.

5. ADAPTIVE CONTROL

The poor knowledge of the process parameters K_i directly affects the performance of the preemptive control algorithm, as shown in Eqs.(13) and (15). In the vacuum dewatering process, these coefficients are expected to vary due to changes in paper basis weight and pulp composition. These variations of the process parameters are very slow, compared to the dynamics of the process in the vacuum dewatering section, so we can assume that each K_i is unknown, but constant. In this section, we develop an adaptive control approach in order to cope with the uncertainty in the process parameters K_i.

The proposed adaptive control algorithm provides error convergence within a predefined desired precision ε, in the case of unknown parameters K_i. The following assumptions are made:
1) measurements of the state variable $w_i(t)$ are correct,
2) the parametric uncertainty is bounded such that each constant but unknown parameter K_i belongs to a known compact set $[K_{i,min}, K_{i,max}]$,
3) the complete pre-emptive feedforward control is used, i.e. $\lambda_{ff,i} = 1$, and
4) the saturation limits for the pressure settings are sufficiently large.

From Eq.(13), the two interesting aspects of this adaptive control problem are that a) the unknown parameters K_i are nonlinearly embedded in the error dynamics; and b) the error $e_i(t)$ are affected by the estimation error $\tilde{K}_{i-1}(t-\tau_i)$ in the past, i.e. there is a time delay.

The adaptive control law is modified from the preemptive control Eq.(9), with an addition of the tuning gain $\theta_i(t)$:

$$w_i^* \hat{\Omega}_i(t) = v\hat{c}_{i-1,out}(t) + \lambda_{fb,i} e_i(t) + \theta_i(t). \quad (16)$$

The purpose of introducing a tuning gain which is first introduced in (Ai-Poh *et al.*, 1999; Annaswamy *et al.*, 1998), is to cope with error in the approximations to be introduced later. In Eq.(16), the calculation of $\hat{c}_{i-1,out}(t)$ is based on the current estimate $\hat{K}_{i-1}(t)$. The control input $P_i(t)$ is then computed based on $\hat{\Omega}_i(t)$ in (9), and estimate $\hat{K}_i(t)$. It is then applied to $\Omega_i(t)$ given by Eq.(2). When this control action is applied to the model given by Eq.(1), and if we use the relation between the actual and estimated moisture content given by Eq.(12), the error dynamics become:

$$\dot{e}_i(t) = -\left[\Omega_i(t) + s_i(t)\lambda_{fb}\right] e_i(t) + [s_{i-1}(t)\tilde{f}_{i-1}(t-\tau_i) - s_i(t)] v \cdot c_{i-1,out}(t-\tau_i) - s_i(t)\theta_i(t) \quad (17)$$

where

$$\tilde{f}_{i-1}(t-\tau_i) = e^{(\hat{K}_{i-1}(t-\tau_i) - K_{i-1})\sqrt{P_{i-1}(t-\tau_i)}\frac{B}{v}}.$$

Following (Annaswamy *et al.*, 1998), define the desired control precision $\varepsilon > 0$ of the measured total moisture content w_i, so that when $|e_i| = |w_i - w_i^*| < \varepsilon$, this error is taken to be acceptable. Introduce the synthetic error (Fig.2),

$$e_{\varepsilon,i}(t) = e_i(t) - \varepsilon \cdot S\left(\frac{e_i(t)}{\varepsilon}\right). \quad (18)$$

Here, $S(x)$ is the saturation function defined as:

$$S(x) = \begin{cases} 1, & x \geq 1 \\ x, & |x| < 1 \\ -1, & x \leq -1 \end{cases} \quad (19)$$

Note that $e_{\varepsilon,i} = 0$ if the measured error $|e_i| < \varepsilon$.

To cope with the time delay issue, we introduce the approximation:

$$\hat{K}_i(t-\tau_i) \approx \hat{K}_{old,i}(t), \quad (20)$$

where the effect of the time delay is approximated by the first order dynamics:

$$\hat{K}_{old,i} = \frac{\lambda_i}{s+\lambda_i}\hat{K}_i. \quad (21)$$

Here, the relation between transport time delay τ_i and constant λ_i is defined as: $\tau_i = \frac{1}{\lambda_i}$. Equation (21) will yield:

$$\dot{\hat{K}}_{i,old}(t) = -\lambda_i\left(\hat{K}_{i,old}(t) - \hat{K}_i(t)\right). \quad (22)$$

Define the Lyapunov function candidate for the $i-$th dewatering box:

$$V = \frac{1}{2}e_{\varepsilon,i}^2(t) + \frac{1}{2\gamma}\tilde{K}_i(t)^2,$$

where

$$\tilde{K}_i(t) = \hat{K}_{old,i}(t) - K_i,$$

and $\gamma > 0$ is an adaptation gain. Since the discontinuity at $|e_i| = \varepsilon$ is of the first kind, and since $e_{\varepsilon,i}(t) = 0$ if $|e_i| < \varepsilon$, the derivative of V with respect to time exists and it is given by:

$$\begin{aligned} \dot{V} = &-\left[\Omega_i(t) + s_i(t)\lambda_{fb}\right] e_i(t) e_{\varepsilon,i}(t) + \\ &\left(s_{i-1}(t)\tilde{f}_{i-1}(t-\tau_i) - s_i(t)\right) \cdot \\ &\cdot v \cdot \hat{c}_{i-1,out}(t-\tau_i) e_{\varepsilon,i}(t) \\ &- s_i(t)\theta_i(t) e_{\varepsilon,i}(t) + \frac{1}{\gamma}\tilde{K}_{i-1}(t) \cdot \dot{\hat{K}}_{old,i-1}(t). \end{aligned} \quad (23)$$

Now, approximate the term with the nonlinear parameterization by a linear parameterization term, and utilize the approximations in Eq.(20), and that $s_i(\cdot) \approx 1$. Let $\Delta_i(\cdot)$ be the error associated with these approximations:

$$\begin{aligned} \Delta_i(t) := &\left(s_{i-1}(t)\tilde{f}_{i-1}(t-\tau_i) - s_i(t)\right) \\ &- \left(\tilde{K}_{i-1}(t)\sqrt{P_{i-1}(t-\tau_i)}\frac{B}{v}\right). \end{aligned} \quad (24)$$

The value of $\|\Delta_i(t)\|_\infty$, where $\|\cdot\|_\infty$ denotes the supremum of the magnitude of the argument, has to be estimated. The bound on Δ_i exists, since 1) the uncertain parameters belong to a known, bounded set, and 2) the error due to approximating the effect of the known

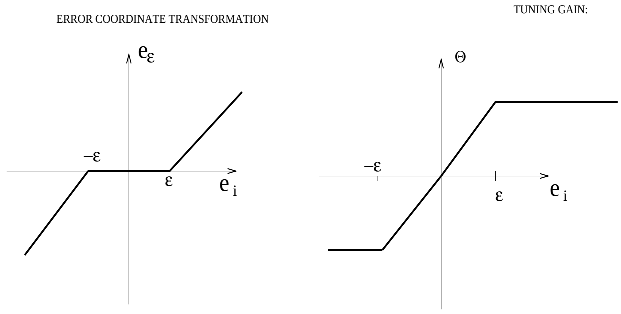

Fig. 2. Error coordinate transformation and the tuning gain used for adaptive algorithm

time delay by a first order transfer function is finite. Eq.(23) will then become:

$$\dot{V} = -\left[\Omega_i(t) + s_i(t)\lambda_{fb}\right] e_i(t)e_{\varepsilon,i}(t) + \left(\tilde{K}_{i-1}(t)\sqrt{P_{i-1}(\cdot)}\frac{B}{v} + \Delta_i(t)\right) v \cdot \hat{c}_{i-1,out}(t-\tau_i)e_{\varepsilon,i}(t) - s_i(t)\theta_i(t)e_{\varepsilon,i}(t) + \frac{1}{\gamma}\tilde{K}_{i-1}(t)\dot{\hat{K}}_{old,i-1}(t). \quad (25)$$

Parametric adaptation algorithm (PAA) is defined as follows. In order to cancel out the linearized term in Eq.(25), we need

$$\dot{\hat{K}}_{old,i-1}(t) = -\gamma \cdot v \cdot \hat{c}_{i-1,out}(t-\tau_i) \cdot \sqrt{P_{i-1}(t-\tau_i)} \cdot e_{\varepsilon,i}(t).$$

If we use the relation given by Eq.(22), PAA then consists of (22) and the feedback,

$$\hat{K}_{i-1}(t) = \hat{K}_{old,i-1}(t) - \frac{\gamma}{\lambda_{i-1}} v\hat{c}_{i-1,out}(t-\tau_i) \cdot \sqrt{P_{i-1}(t-\tau_i)}e_{\varepsilon,i}(t). \quad (26)$$

Finally, the tuning gain $\theta_i(t)$ is designed so that discrepancy due to the given approximation errors (for using a linear parameterization, and for using a first order system to model a time delay) does not adversely affect the sign of $\dot{V}$. This condition is satisfied if:

$$\Delta_i(t) \cdot v \cdot \hat{c}_{i-1,out}(t-\tau_i) - s_i(t)\theta_i(t) \leq 0. \quad (27)$$

Therefore, we can define the tuning gain to be:

$$\theta_i(t) = \frac{||\Delta_i||_\infty}{\underline{s_i}} \cdot v \cdot ||\hat{c}_{i-1,out}(t-\tau_i)||_\infty S\left(\frac{e_i(t)}{\varepsilon}\right), \quad (28)$$

where $\underline{s_i}$ denotes the lower bound on $s_i(\cdot)$, with respect to the worst-case parametric uncertainty.

Theorem 1. Consider the system with the bounded parametric uncertainty and the transport time delay τ_i between control sections, modeled by Eq.(1). For the complete feed-forward based control given by Eq.(16), process parameter updating equation (26), and with the tuning gain defined as in (28),

$$\lim_{t\to\infty} e_{\varepsilon,i}(t) = 0,$$

meaning that the measured total moisture content stays within a desired precision ε about its nominal value.

PROOF. We will consider the two distinct cases:
1) First, consider the case where $|e_i| \geq \varepsilon$. The PAA is designed to cancel out the linearized term in Eq.(25). When PAA is applied, Eq.(25) can be rewritten as:

$$\dot{V} = -\left[\Omega_i(t) + s_i(t)\lambda_{fb}\right] e_i(t) \cdot e_{\varepsilon,i}(t) + \Delta_i(t) \cdot v \cdot \hat{c}_{i-1,out}(t)e_{\varepsilon,i}(t) - s_i(t)\theta_i(t)e_{\varepsilon,i}(t). \quad (29)$$

With respect to the sign of the error $e_{\varepsilon,i}(t)$, the tuning gain is defined such that Eq.(27) is satisfied, and therefore:

$$\dot{V} \leq -\left[\Omega_i(t) + s_i(t)\lambda_{fb}\right] e_i(t)e_{\varepsilon,i}(t).$$

Since $sign(e_i) = sign(e_{\varepsilon,i})$, it implies that $\dot{V} \leq 0$.

2) Consider now the case when $e_i(t) < \varepsilon$. It implies that $e_{\varepsilon,i}(t) = 0$, and therefore $\dot{V} = 0$ since $\dot{\hat{K}}_{old,i-1}(t) = 0$ in this case.

This proves that $\dot{V} \leq 0$, and therefore V is a proper Lyapunov function, which leads to global boundedness of both $e_{\varepsilon,i}(t)$ and $\tilde{K}_i(t)$. This further implies that $\dot{e}_{\varepsilon,i}(t)$, given by Eq.(17) is bounded, which from Barbalat's lemma, proves that the error dynamics are globally asymptotically convergent: $\lim_{t\to\infty} |e_{\varepsilon,i}| = 0$. Therefore, the error in the total moisture content converges to a region ε around the origin. This completes the proof. □

6. SIMULATIONS

The proposed control system is simulated for the paper machine with N=6 dewatering boxes of slot length $B = 0.05$m and with machine speed of 20m/s. The transport coefficient K_i decreases from $K_1 = 0.8s^{-1}Pa^{-1/2}$ to $K_6 = 0.3s^{-1}Pa^{-1/2}$, and the desired total moisture contents, which are designed based on increasing nominal pressures from 8000Pa to 56000Pa, decreases from $w_1^* = 0.68$ to $w_6^* = 0.22$. Saturation limits of ±10% of the nominal pressures are imposed on input pressures. The nominal incoming moisture content $c_{out,0}^*$ is 15kg/kg. In all situations, $\lambda_{ff,i} = 1$, $\lambda_{fb,i} = v/B = 400sec^{-1}$. The transport delay between dewatering boxes are $\tau_i = 0.050$ sec.

We consider the situation when there is a $20rad/s$ sinusoidal disturbance at the incoming moisture content, with the magnitude of $1kg/kg$. The nominal preemptive controller with perfect knowledge of K_i (Fig. 3) enables the moisture content w_i to converge to the desired values towards the last vacuum box. At the other boxes, the effect of the disturbances has been gradually attenuated. Convergence does not occur at each box because of the saturation limits.

To test the adaptive control algorithm, 25% deviation in the nominal process parameters K_i's are also assumed. The actuator saturation limits are relaxed to 50% for this case. Fig.4 shows the desired and actual total moisture content with and without the adaptation after the 4-th vacuum box. Time trajectory of moisture content with adaptation converges to the nominal

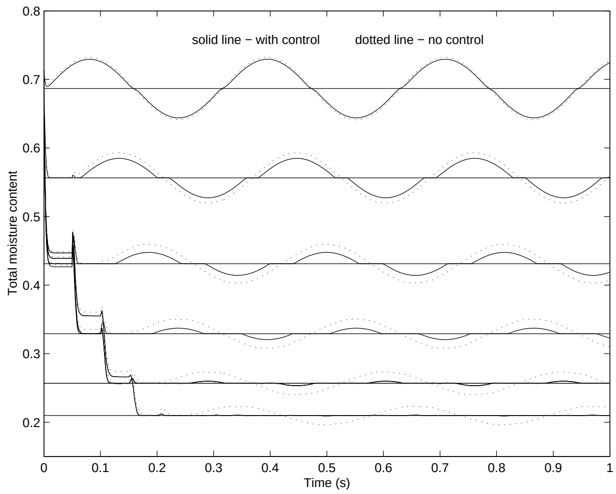

Fig. 3. Desired and actual total moisture content (w_i, $i = 1, \ldots, 6$) with sinusoidal moisture content disturbance from headbox, with and without the nominal preemptive control. K_i are assumed well known.

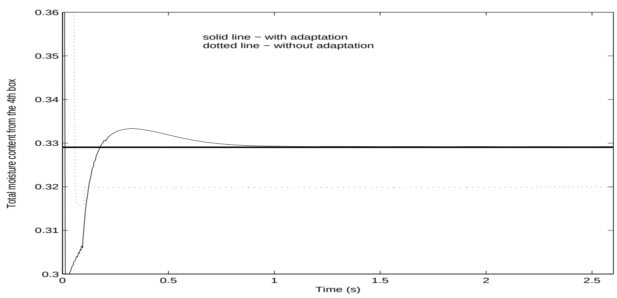

Fig. 4. Nominal and achieved total moisture content when 25% error in the process model - parameter K is introduced, with and without adaptation

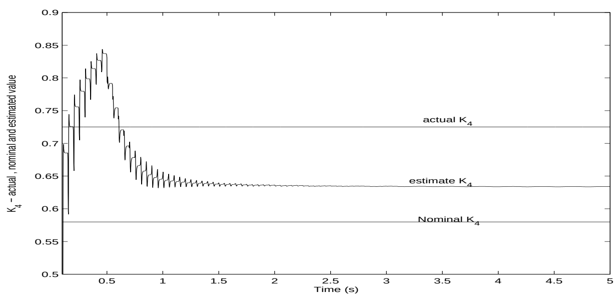

Fig. 5. Nominal, actual and estimated K_4

value within the defined precision $\varepsilon = 10^{-4} kg/kg$, although the estimate of the process parameter does not converge to its actual value (Fig.5). This is because for the $i-$th box, uncertainties in both K_{i-1} and K_i affect e_i, but only K_i is adapted based on e_i. It can also be shown that if the time delay is not taken into consideration, the adaptive controller would be unstable.

7. CONCLUSION

In this paper, we presented an adaptive version of the pre-emptive control law presented in (Li *et al.*, 2001) for the control of vacuum dewatering in paper manufacturing. The two key aspects of this problem are the nonlinear parameterization and the time delay. Both issues are dealt with analytically. The proposed control law has been validated in simulations, even in the presence of input saturation. Since the availability and accuracy of the moisture content estimates from air-flow measurement is critical to the success of the proposed control approach, our current research aims to experimentally develop such a model.

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