

OUTPUT MANEUVERING FOR A CLASS OF NONLINEAR SYSTEMS

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Abstract: The output maneuvering problem involves two tasks. The first, which is the geometric task, is to force the output to converge to a desired path parametrized by a continuous scalar variable θ . The second task is to satisfy a desired speed assignment along the path. The main concern is to satisfy the geometric task. However, the speed assignment will ensure that the output follows the path with sufficient speed. A recursive control design technique is developed for nonlinear plants in vectorial strict feedback form of any relative degree. First the geometric part of the problem is solved. Then an update law is constructed that bridges the geometric design with the speed assignment. An extra degree of freedom is provided for an operator to specify the speed $\dot{\theta}$. A computer simulation with a marine vessel is performed to illustrate the design. Copyright ©2002 IFAC

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1. INTRODUCTION

In many applications it is of primary importance to steer an object (ship, vehicle, robot arm, etc.) along a desired *path*. The speed assignment along the path may be less important. Vehicle control applications of this type are described in (Micaelli and Samson, 1993), (Hauser and Hindman, 1997) and (Pettersen and Lefeber, 2001), while an informative reference for robotic manufacturing plants is (Song *et al.*, 2000).

Rather than considering a single tracking problem as a function of time, control problems for such applications are usually approached as two separate tasks. The first task is to reach and follow a desired path as a function of a scalar variable θ , left as an extra degree of freedom for the second task. In the second task, $\dot{\theta}$ is used to satisfy an additional specification of speed assignment.

Hauser and Hindman (1995) introduced a procedure to design a *maneuver regulation* controller which ensures that the states converge to the desired path and then proceed along the path. To determine the *path variable* θ , they used a P -orthogonal projection from the current state onto the path. The already at hand tracking controller was then converted into a maneuver regulation controller, and a quadratic Lyapunov function was employed to guarantee that the states converge to the path and move along the path.

The methodology of Hauser and Hindman (1995) applies to feedback linearizable systems, where the desired path is specified for the full state. Encarnação and Pascoal (2001) further showed how the methodology of Hauser and Hindman (1995) can be extended to solve the *output maneuvering problem* by backstepping. However, for systems of relative degree higher than two, their approach has the disadvantage that each step of backstepping increases the number of time derivatives of θ .

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The contribution of this paper is a maneuvering design that is applicable to systems in vectorial strict feedback form of any relative degree. In the n recursive steps, the design solves the geometric part of the problem. It then proceeds to construct an update law that ties together the geometric design with the speed assignment. An extra degree of freedom is provided for an operator to select the speed of the maneuver variable θ .

1.1 An introductory example

To introduce the main idea of the design procedure developed in Section 2, we look at the strict feedback form system

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1^2 \\ \dot{x}_2 &= u\end{aligned}\quad (1)$$

where $y = x_1$ is the output and u is the control. We want to design a controller to

1. force the output y to converge to the path $y_d = \sin(\theta)$.
2. assign θ to $t - \phi$, that is, make the output y to oscillate with frequency 1 rad/s.

The design in Section 2 results in the control law

$$\begin{aligned}u &= -2[x_1 + x_1^2 + x_1^3 + x_1x_2 + x_2 \\ &\quad - y_d(\theta) - y'_d(\theta) - \frac{1}{2}y''_d(\theta)]\end{aligned}\quad (2)$$

where $y'_d(\theta) = \frac{dy_d}{d\theta}(\theta) = \cos(\theta)$ and $y''_d(\theta) = \frac{d^2y_d}{d\theta^2}(\theta) = -\sin(\theta)$. In the new coordinates

$$z_1 = x_1 - y_d(\theta)\quad (3)$$

$$z_2 = x_1 + x_1^2 + x_2 - y_d(\theta) - y'_d(\theta)\quad (4)$$

$$\omega_s = 1 - \dot{\theta}\quad (5)$$

the resulting closed-loop system is

$$\dot{z}_1 = -z_1 + z_2 + y'_d(\theta)\omega_s\quad (6)$$

$$\dot{z}_2 = -z_1 - z_2 + [y'_d(\theta) + y''_d(\theta)]\omega_s\quad (7)$$

For the Lyapunov function

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\omega_s^2\quad (8)$$

the derivative along the solutions of (6), (7) is

$$\dot{V} = -z_1^2 - z_2^2 + \{z_1y'_d(\theta) + z_2[y'_d(\theta) + y''_d(\theta)] + \dot{\omega}_s\}\omega_s$$

where an update law for $\dot{\omega}_s$ is yet to be assigned. To make \dot{V} negative definite, we let

$$\dot{\omega}_s = -\omega_s - z_1y'_d(\theta) - z_2[y'_d(\theta) + y''_d(\theta)]\quad (9)$$

and achieve global exponential stability (GES) of the equilibrium $(z_1, z_2, \omega_s) = 0$.

The main purpose of this example is to interpret how the design specifications have been met. From the GES property, it follows that $z_1(t) = y(t) - \sin(\theta(t)) \rightarrow 0$ and, hence, y converges to $y_d = \sin(\theta)$. To recognize

that the time assignment, $\theta(t) \rightarrow t - \phi$ has been met, we integrate $\dot{\theta}(t) = 1 - \omega_s(t)$ and get

$$\theta(t) = t + \theta(0) - \int_0^t \omega_s(\tau) d\tau$$

where $\phi = \lim_{t \rightarrow \infty} [\theta(0) - \int_0^t \omega_s(\tau) d\tau]$ exists because the convergence $\omega_s(t) \rightarrow 0$ is exponential. From (9) we see that θ , and hence $y_d(\theta)$, depends on the system state. In this way, the desired transient to the path and the speed along the path, are shaped by feedback system dynamics. This flexibility is due to the fact that $\theta \rightarrow t - \phi$ is required to be attained only asymptotically, rather than as an identity $\theta = t - \phi$.

For another interpretation, we represent the closed-loop system in the space of $(x_1, x_2, \theta, \omega_s)$ as follows

$$\dot{x}_1 = x_2 + x_1^2\quad (10)$$

$$\begin{aligned}\dot{x}_2 &= -2[x_1 + x_1^2 + x_1^3 + x_2 + x_1x_2] \\ &\quad + \sin(\theta) + 2\cos(\theta)\end{aligned}\quad (11)$$

$$\dot{\theta} = 1 - \omega_s\quad (12)$$

$$\dot{\omega}_s = -z_1\cos(\theta) - z_2[\cos(\theta) - \sin(\theta)] - \omega_s\quad (13)$$

It can be verified that this dynamical system has an invariant manifold defined by

$$\begin{aligned}x_1 &= \sin(\theta) \\ x_2 &= \cos(\theta) - \sin^2(\theta) \\ \omega_s &= 0\end{aligned}$$

on which $x_1(t)$ and $\dot{x}_1(t)$ satisfy $x_1^2(t) + \dot{x}_1^2(t) = 1$. Since this invariant manifold is globally attractive, $y(t)$ converges to the behavior of a harmonic oscillator as required by the design specifications. The phase ϕ of the oscillation is unspecified and depends on the initial condition.

A further design step can be used to assign the phase ϕ , for example to $\phi = 0$. Introducing $\omega_t = t - \theta$ such that

$$\dot{\omega}_t = \omega_s\quad (14)$$

and augmenting the Lyapunov function (8) by $\frac{1}{2}\omega_t^2$, a new update law for $\dot{\omega}_s$ is selected to be

$$\dot{\omega}_s = -\omega_s - \omega_t - z_1y'_d(\theta) - z_2[y'_d(\theta) + y''_d(\theta)]\quad (15)$$

Since the closed-loop (6), (7), (14), (15) becomes time-varying with $\theta = t - \omega_t(t)$, we resort to LaSalle-Yoshizawa (see Theorem 2.1 in Krstić *et al.* (1995)) to establish global boundedness of $(z_1, z_2, \omega_t, \omega_s)$ and convergence $(z_1, z_2, \omega_s) \rightarrow 0$. Using Theorem 1 in (Fossen *et al.*, 2001), it is straightforward to show that the equilibrium $(z_1, z_2, \omega_t, \omega_s) = 0$ is indeed UGAS. It follows that $\omega_t(t) = t - \theta(t) \rightarrow 0$ and, hence, $\phi(t) \rightarrow 0$.

1.2 Definitions and problem statement

Time derivatives of $x(t)$ are denoted \dot{x} , \ddot{x} , $x^{(3)}$, \dots , $x^{(i)}$, and derivatives with respect to θ

of $x(\theta)$ are $x', x'', x^{(3)}, \dots, x^{(i)}$. We denote $x^{(1\dots i)}$ [or $x^{(1' \dots i')}$] as a collection of derivatives of $x(t)$ [or $x(\theta)$].

\bar{x}_i denotes the vector $\bar{x}_i = [x_1^\top \ x_2^\top \ \dots \ x_i^\top]^\top$.

A **Parametrized Path** is a geometric curve $\xi \in \mathbb{R}^q$, $q \geq 1$, that is parametrized by a continuous variable θ .

The **Path Characterization Vector** is the vector

$$\xi_{cv}(\theta) \triangleq \left[(\xi'(\theta))^\top, (\xi''(\theta))^\top, \dots, (\xi^{(n')}(\theta))^\top \right]^\top$$

The Maneuvering Problem: design a controller that solves the two tasks:

1. The Geometric Task: force the state x to converge to a desired path $\xi(\theta)$,

$$\lim_{t \rightarrow \infty} [x(t) - \xi(\theta(t))] = 0 \quad (16)$$

for any continuous function $\theta(t)$.

2. The Speed Assignment Task: force the speed $\dot{\theta}$ to converge to a desired speed v_s ,

$$\lim_{t \rightarrow \infty} [\dot{\theta}(t) - v_s(\theta, t)] = 0 \quad (17)$$

The **Output Maneuvering Problem** is to solve the maneuvering problem with respect to the output y rather than the state x .

A more restrictive version of the asymptotic speed assignment (17) is to satisfy it identically, $\dot{\theta} \equiv v_s$. In particular, if $v_s = 1$, then the maneuvering problem becomes a tracking problem where the desired output $y_d(\theta(t))$ will be a prespecified time signal $\tilde{y}_d(t)$. Using the asymptotic formulation (17), on the other hand, allows for more flexibility, including the possibility to let $\dot{\theta}$ depend on the system state.

2. OUTPUT MANEUVERING

Consider the general nonlinear system in strict feedback form with vector relative degree n

$$\begin{aligned} \dot{x}_1 &= G_1(\bar{x}_1) x_2 + f_1(\bar{x}_1) \\ \dot{x}_2 &= G_2(\bar{x}_2) x_3 + f_2(\bar{x}_2) \\ &\vdots \\ \dot{x}_n &= G_n(\bar{x}_n) u + f_n(\bar{x}_n) \\ y &= h(x_1) \end{aligned} \quad (18)$$

where $x_i \in \mathbb{R}^m$, $i = 1, \dots, n$, $y \in \mathbb{R}^m$ is the output and $u \in \mathbb{R}^m$ is the control. The state dependent matrices $G_i(\bar{x}_i)$ and $\frac{\partial h}{\partial x_1}(x_1)$ are invertible for all \bar{x}_i , and G_i , f_i and h are smooth.

The control objective is to design a *maneuvering controller* that solves the *output maneuvering problem* with respect to a desired *parametrized output path*

$$\mathbf{Y}_d = \{y_d(\theta) \mid y_d \in \mathbb{R}^m, \theta \in \mathbb{R}\} \quad (19)$$

where y_d is n times differentiable with respect to θ .

2.1 Design procedure

In the recursive procedure that follows, the first two steps are separated from steps 3 through n since the first two steps involve $\frac{\partial h}{\partial x_1}$. The design procedure borrows much from adaptive tracking developed by Krstić *et al.* (1995), and the notion of a *tuning function* is used also in this design.

Step 1:

The new variables

$$v_s \triangleq v_s - \dot{\theta} \quad (20)$$

$$z_1 \triangleq y - y_d(\theta) \quad (21)$$

$$z_i \triangleq x_i - \alpha_{i-1}, \quad i = 2, \dots, n \quad (22)$$

are introduced, where v_s is a bounded \mathcal{C}^{n-1} signal, and α_{i-1} are virtual controls to be specified later. Differentiating (21) with respect to t results in

$$\begin{aligned} \dot{z}_1 &= \dot{y} - y_d' \dot{\theta} \\ &= \left(\frac{\partial h}{\partial x_1}(x_1) \right) G_1(x_1) z_2 + (\nabla h(x_1)) G_1(x_1) \alpha_1 \\ &\quad + \left(\frac{\partial h}{\partial x_1}(x_1) \right) f_1(x_1) - \nu_1 v_s + \nu_1 \omega_s \end{aligned}$$

where $\nu_1 \triangleq y_d'$. Using $p_1 > 0$, define the first Lyapunov function as

$$V_1 \triangleq p_1 z_1^\top z_1 \quad (23)$$

whose time derivative is

$$\begin{aligned} \dot{V}_1 &= 2p_1 z_1^\top \left[\left(\frac{\partial h}{\partial x_1} \right) G_1 \alpha_1 + (\nabla h) f_1 - \nu_1 v_s \right] \\ &\quad + 2p_1 z_1^\top \left(\frac{\partial h}{\partial x_1} \right) G_1 z_2 + 2p_1 z_1^\top \nu_1 \omega_s \end{aligned}$$

Then the first virtual control is picked as

$$\begin{aligned} \alpha_1 &= G_1^{-1} \left(\frac{\partial h}{\partial x_1} \right)^{-1} [-C_1 z_1 - (\nabla h) f_1 + \nu_1 v_s] \\ &= f_{\alpha_1}(x_1, y_d, y_d', v_s) \end{aligned} \quad (24)$$

where $C_1 = C_1^\top > 0$. Let $Q_1 \triangleq 2p_1 C_1$ and define the first tuning function, $\tau_1 \in \mathbb{R}$, as

$$\tau_1 \triangleq 2p_1 z_1^\top \nu_1 \quad (25)$$

Finalizing *Step 1*, the result is

$$\begin{aligned} \dot{V}_1 &= -z_1^\top Q_1 z_1 + 2p_1 z_1^\top \left(\frac{\partial h}{\partial x_1} \right) G_1 z_2 + \tau_1 \omega_s \\ \dot{z}_1 &= -C_1 z_1 + \left(\frac{\partial h}{\partial x_1} \right) G_1 z_2 + \nu_1 \omega_s \end{aligned}$$

where z_2 and ω_s are left for the next step. In aid of next step, let

$$\hat{\alpha}_1 = \sigma_1 + \nu_2 \dot{\theta} \quad (26)$$

where σ_1 is defined as the terms in $\hat{\alpha}_1$ not containing $\dot{\theta}$, and ν_2 as those multiplying $\dot{\theta}$, that is

$$\sigma_1 \triangleq \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_1}{\partial v_s} \dot{v}_s \quad (27)$$

$$\nu_2 \triangleq \frac{\partial \alpha_1}{\partial y_d} y_d' + \frac{\partial \alpha_1}{\partial y_d'} y_d'' \quad (28)$$

Steps 2 and $i=3, \dots, n$ are summarized in Tables 1 and 2.

Table 1. **Step 2**

$$\begin{aligned}
\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\
&= G_2(\bar{x}_2)z_3 + G_2(\bar{x}_2)\alpha_2 + f_2(\bar{x}_2) - \sigma_1 - \nu_2\dot{\theta} \\
V_2 &\triangleq V_1 + p_2 z_2^\top z_2 \\
\dot{V}_2 &= -z_1^\top Q_1 z_1 \\
&\quad + 2z_2^\top \left\{ G_1^\top \left(\frac{\partial h}{\partial x_1} \right)^\top p_1 z_1 + p_2 [G_2 \alpha_2 + f_2 - \sigma_1 - \nu_2 v_s] \right\} \\
&\quad + 2p_2 z_2^\top \nu_2 \omega_s + \tau_1 \omega_s + 2p_2 z_2^\top G_2 z_3 \\
\alpha_2 &= G_2^{-1} \left[-C_2 z_2 - \frac{p_1}{p_2} G_1^\top \left(\frac{\partial h}{\partial x_1} \right)^\top z_1 - f_2 + \sigma_1 + \nu_2 v_s \right] \\
&= f_{\alpha_2}(\bar{x}_2, y_d, y_d', y_d'', v_s, \dot{v}_s) \\
Q_2 &\triangleq 2p_2 C_2, \quad C_2 = C_2^\top > 0 \\
\tau_2 &\triangleq \tau_1 + 2p_2 z_2^\top \nu_2 \\
&\quad \downarrow \\
\dot{V}_2 &= -\sum_{j=1}^2 z_j^\top Q_j z_j + 2p_2 z_2^\top G_2 z_3 + \tau_2 \omega_s \\
\dot{z}_2 &= -\frac{p_1}{p_2} G_1^\top \left(\frac{\partial h}{\partial x_1} \right)^\top z_1 - C_2 z_2 + G_2 z_3 + \nu_2 \omega_s \\
\alpha_2 &= \sigma_2 + \nu_3 \dot{\theta} \\
\sigma_2 &\triangleq \frac{\partial \alpha_2}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_2}{\partial x_2} \dot{x}_2 + \frac{\partial \alpha_2}{\partial v_s} \dot{v}_s + \frac{\partial \alpha_2}{\partial \dot{v}_s} \dot{\dot{v}}_s \\
\nu_3 &\triangleq \frac{\partial \alpha_2}{\partial y_d} y_d' + \frac{\partial \alpha_2}{\partial y_d'} y_d'' + \frac{\partial \alpha_2}{\partial y_d''} y_d^{(3')}
\end{aligned}$$

Table 2. **Step i = 3, ..., n**

$$\begin{aligned}
\dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\
&= G_i(\bar{x}_i)z_{i+1} + G_i(\bar{x}_i)\alpha_i + f_i(\bar{x}_i) - \sigma_{i-1} - \nu_i \dot{\theta} \\
V_i &\triangleq V_{i-1} + p_i z_i^\top z_i \\
\dot{V}_i &= -\sum_{j=1}^{i-1} z_j^\top Q_j z_j \\
&\quad + 2z_i^\top \left\{ G_{i-1}^\top p_{i-1} z_{i-1} + p_i [G_i \alpha_i + f_i - \sigma_{i-1} - \nu_i v_s] \right\} \\
&\quad + 2p_i z_i^\top \nu_i \omega_s + \tau_{i-1} \omega_s + 2p_i z_i^\top G_i z_{i+1} \\
\alpha_i &= G_i^{-1} \left[-C_i z_i - \frac{p_{i-1}}{p_i} G_{i-1}^\top z_{i-1} - f_i + \sigma_{i-1} + \nu_i v_s \right] \\
&= f_{\alpha_i}(\bar{x}_i, y_d^{(0' \dots i')}, v_s^{(0 \dots i-1)}) \\
Q_i &\triangleq 2p_i C_i, \quad C_i = C_i^\top > 0 \\
\tau_i &\triangleq \tau_{i-1} + 2p_i z_i^\top \nu_i \\
&\quad \downarrow \\
\dot{V}_i &= -\sum_{j=1}^i z_j^\top Q_j z_j + 2p_i z_i^\top G_i z_{i+1} + \tau_i \omega_s \\
\dot{z}_i &= -\frac{p_{i-1}}{p_i} G_{i-1}^\top z_{i-1} - C_i z_i + G_i z_{i+1} + \nu_i \omega_s \\
\alpha_i &= \sigma_i + \nu_{i+1} \dot{\theta} \\
\sigma_i &= \frac{\partial \alpha_i}{\partial x_1} \dot{x}_1 + \dots + \frac{\partial \alpha_i}{\partial x_i} \dot{x}_i + \frac{\partial \alpha_i}{\partial v_s} \dot{v}_s + \dots + \frac{\partial \alpha_i}{\partial v_s^{(i-1)}} v_s^{(i)} \\
\nu_{i+1} &= \frac{\partial \alpha_i}{\partial y_d} y_d' + \frac{\partial \alpha_i}{\partial y_d'} y_d'' + \dots + \frac{\partial \alpha_i}{\partial y_d^{(i')}} y_d^{(i+1')}
\end{aligned}$$

Upon the completion of *Step n* the choice of the control law is

$$\begin{aligned}
u &= \alpha_n \\
&= G_n^{-1} \left[-C_n z_n - \frac{p_{n-1}}{p_n} G_{n-1}^\top z_{n-1} \right. \\
&\quad \left. - f_n + \sigma_{n-1} + \nu_n v_s \right] \quad (29)
\end{aligned}$$

results in

$$\dot{V}_n = -\sum_{j=1}^n z_j^\top Q_j z_j + \tau_n \omega_s \quad (30)$$

$$\dot{z}_n = -\frac{p_{n-1}}{p_n} G_{n-1}^\top z_{n-1} - C_n z_n + \nu_n \omega_s \quad (31)$$

This implies that, if $\omega_s = 0$, then $z = [z_1^\top, \dots, z_n^\top]^\top$ will converge to $z = 0$ as $t \rightarrow \infty$. Hence, $y(t) \rightarrow y_d(\theta(t))$ for any continuous $\theta(t)$, and the geometric task (16) is solved.

2.2 Speed assignment

To solve the speed assignment task (17), augment the *Step n* Lyapunov function to

$$V = V_n + \frac{1}{2\mu_1} \omega_s^2, \quad \mu_1 > 0 \quad (32)$$

The time derivative of (32) is

$$\begin{aligned}
\dot{V} &= \dot{V}_n + \frac{1}{\mu_1} \omega_s \dot{\omega}_s \\
&= -\sum_{j=1}^n z_j^\top Q_j z_j + \left[\tau_n + \frac{1}{\mu_1} \dot{\omega}_s \right] \omega_s
\end{aligned}$$

where the update law for $\dot{\omega}_s$ is yet to be constructed. To make \dot{V} negative definite, choose the $\dot{\omega}_s$ -update law

$$\dot{\omega}_s = -\lambda \omega_s - \mu_1 \tau_n \quad (33)$$

$\lambda > 0$, which gives

$$\dot{V} = -\sum_{j=1}^n z_j^\top Q_j z_j - \frac{\lambda}{\mu_1} \omega_s^2 < 0 \quad (34)$$

Consequently, as $t \rightarrow \infty$, $\omega_s(t) = v_s(\theta, t) - \dot{\theta}(t) \rightarrow 0$, and the speed assignment (17) is satisfied. From (33) it follows that $\omega_s(t)$, and therefore $y_d(\theta(t))$, is dependent on the system state $x(t)$ through the final tuning function τ_n . Hence, feedback is introduced from the system state to the desired output. This feedback gives the overall system certain good properties compared to a pure tracking design.

2.3 The main result

To obtain the closed-loop system in vector form, let $A_z(\bar{x}_{n-1})$ and $F(\bar{x}_n, v_s^{(0 \dots n-2)}(\theta, t))$ be as below

$$A_z(\bar{x}_{n-1}) = \begin{bmatrix} -C_1 & \left(\frac{\partial h}{\partial x_1} \right) G_1 & 0 & 0 & \dots & 0 \\ -\frac{p_1}{p_2} G_1^\top \left(\frac{\partial h}{\partial x_1} \right)^\top & -C_2 & G_2 & 0 & \dots & 0 \\ 0 & -\frac{p_2}{p_3} G_2^\top & -C_3 & G_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & -\frac{p_{n-1}}{p_n} G_{n-1}^\top & -C_n \end{bmatrix}$$

$$F(\bar{x}_n, v_s^{(0 \dots n-2)}(\theta, t)) = \begin{bmatrix} I & 0 & 0 & \dots & 0 \\ \frac{\partial \alpha_1}{\partial y_d}(t) & \frac{\partial \alpha_1}{\partial y_d'}(t) & 0 & \dots & 0 \\ \frac{\partial \alpha_2}{\partial y_d}(t) & \frac{\partial \alpha_2}{\partial y_d'}(t) & \frac{\partial \alpha_2}{\partial y_d''}(t) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \alpha_{n-1}}{\partial y_d}(t) & \frac{\partial \alpha_{n-1}}{\partial y_d'}(t) & \frac{\partial \alpha_{n-1}}{\partial y_d''}(t) & \dots & \frac{\partial \alpha_{n-1}}{\partial y_d^{(n-1')}}(t) \end{bmatrix}$$

Define the *path characterization vector* as

$$\xi_{cv}(\theta) \triangleq \left[(y'_d(\theta))^\top (y''_d(\theta))^\top \dots (y_d^{(n')}(\theta))^\top \right]^\top$$

which gives the relationship

$$\tau_n = 2\xi_{cv}^\top(\theta)F^\top(\bar{x}_n, v_s^{(0\dots n-2)}(\theta, t))Pz,$$

with $z = [z_1^\top, \dots, z_n^\top]^\top$ and $P \triangleq \text{diag}(p_1I, p_2I, \dots, p_nI)$. In vector form, the closed-loop equation is

$$\dot{z} = A_z(\bar{x}_{n-1})z + F(\bar{x}_n, v_s^{(0\dots n-2)})\xi_{cv}(\theta)\omega_s \quad (35)$$

$$\dot{\omega}_s = -2\mu_1\xi_{cv}^\top(\theta)F^\top(\bar{x}_n, v_s^{(0\dots n-2)})Pz - \lambda\omega_s \quad (36)$$

Under the following assumption

(A1) There exist a positive real, $r_1 > 0$, such that the path characterization vector $\xi_{cv}(\theta)$ is upper bounded, uniformly in θ , that is,

$$0 \leq |\xi_{cv}(\theta)| < r_1 < \infty, \quad \forall \theta \in \mathfrak{R}$$

we have the result:

Theorem 1. The closed-loop output maneuvering system consisting of the plant (18), the control (29) and the ω_s -update law (33), where ω_s is given by (20), has a Uniformly Globally Exponentially Stable (UGES) equilibrium $(z, \omega_s) = (0, 0)$. This means in particular that global exponential output maneuvering of the output path (19) is achieved:

$$\lim_{t \rightarrow \infty} [y(t) - y_d(\theta(t))] = 0$$

$$\lim_{t \rightarrow \infty} [\dot{\theta}(t) - v_s(\theta(t), t)] = 0$$

Remark 1. In the case of constant $v_s(\theta, t) = v_0 \neq 0$, all derivatives $v_s^{(i)} = 0$, $i \geq 1$, and the closed-loop system, (35) and (36), is time-invariant. It follows that $\lim_{t \rightarrow \infty} \theta(t) = v_0t - \phi$, where

$\phi = \lim_{t \rightarrow \infty} \left[\theta(0) - \int_0^t \omega_s(\tau) d\tau \right]$ exists because the convergence $\omega_s(t) \rightarrow 0$ is exponential.

2.4 Phase assignment

For the constant $v_s(\theta, t) = v_0 \neq 0$ case, it is possible to introduce a design step to assign the phase ϕ . Let $\omega_t = v_0t - \theta - \phi_0$ where ϕ_0 is a desired constant phase shift, for example $\phi_0 = 0$. The time derivative is

$$\dot{\omega}_t = \omega_s \quad (37)$$

and augmenting the Lyapunov function (32), to

$$V = V_n + \frac{\mu_0}{2}\omega_t^2 + \frac{1}{2\mu_1}\omega_s^2, \quad \mu_0 \geq 0 \quad (38)$$

a new update law for $\dot{\omega}_s$ is constructed as

$$\dot{\omega}_s = -\lambda\omega_s - \mu_1[\tau_n + \mu_0\omega_t] \quad (39)$$

This gives the negative semi-definite derivative of (38) along the solutions of (35), (37) and (39)

$$\dot{V} = -z^\top Qz - \frac{\lambda}{\mu_1}\omega_s^2 \leq 0 \quad (40)$$

Note that $\mu_0 = 0$ is acceptable in order to switch off the regulation of the phase. Under the assumptions

(A2) The path $y_d(\theta)$ is uniformly bounded in θ .

(A3) The map $h(x_1)$ is a local diffeomorphism.

we have the following result:

Theorem 2. The closed-loop output maneuvering system consisting of the plant (18), the control (29), the update law (39) and (37), with $v_s(\theta, t) = v_0 \neq 0$, has a Uniformly Globally Asymptotically Stable (UGAS) equilibrium $(z, \omega_t, \omega_s) = (0, 0, 0)$. In particular, this means that global asymptotic maneuvering of (19) is achieved:

$$\lim_{t \rightarrow \infty} [y(t) - y_d(\theta(t))] = 0$$

$$\lim_{t \rightarrow \infty} [\dot{\theta}(t) - v_0] = 0$$

In addition, any constant phase specification ϕ_0 is satisfied, that is

$$\lim_{t \rightarrow \infty} [\theta(t) - v_0t + \phi_0] = 0$$

Proof. First, notice that with $v_s(\theta, t) = v_0$ and $\theta(t) = v_0t - \omega_t(t) - \phi_0$, the closed-loop equations (35), (39) and (37) are time-varying. From assumption A3 and the smooth virtual controls α_i , there exist a continuous map $\Psi : (z, t) \rightarrow x$ which is uniformly bounded since $z(t)$ is exponentially converging and the time-variation only enters through either $y_d(\theta(t))$ or $\xi_{cv}(\theta(t))$, which due to assumptions A1 and A2 are uniformly bounded. Resorting to Theorem 1 in (Fossen *et al.*, 2001) where $x_1 = \text{col}(z, \omega_s)$ and $x_2 = \omega_t$, it can be verified that all assumptions are satisfied and that, therefore, the equilibrium $(z, \omega_t, \omega_s) = (0, 0, 0)$ is UGAS. It follows that as $t \rightarrow \infty$, $z_1 = y(t) - y_d(\theta(t)) \rightarrow 0$, $\omega_s(t) = v_0 - \dot{\theta}(t) \rightarrow 0$ and $\omega_t(t) = v_0t - \theta(t) - \phi_0 \rightarrow 0$. ■

3. APPLICATION TO SHIP MANEUVERING

For ship maneuvering, a dynamical model where the surge mode is decoupled from the sway and yaw modes is used. See Skjetne and Fossen (2001) and references therein. Let $\eta = [x \ y \ \psi]^\top$ be the earth-fixed position vector and $\nu = [u \ v \ r]^\top$ be the body-fixed velocity vector. The kinematics and dynamics is given by

$$\dot{\eta} = R(\psi)\nu \quad (41)$$

$$\dot{\nu} = M^{-1}N(\nu)\nu + M^{-1}GT \quad (42)$$

where $M = M^\top > 0$ and $T = [T_u, \delta_1, \delta_2]^\top$ is the control vector. The dynamical system (41) and (42) are then in the form of (18) where η is the output and T is the control. Let a desired output path be

$$\eta_d(\theta) = \left[x_d(\theta) \ y_d(\theta) \ \arctan\left(\frac{y'_d}{x'_d}\right) \right]^\top \quad (43)$$

where $x_d(\theta)$ and $y_d(\theta)$ are *three* times differentiable with respect to θ . The control objective is:

1. force the ship to converge to and follow the path.
2. let the ship speed be adjustable online.

Using the design procedure of Section 2, we ensure convergence to the path by forcing the convergence of the error state, $z_1(t) \rightarrow 0$. To solve the problem of online adjustable speed, let u_0 be a constant desired set-point for u . We then have the feedforward terms

$$v_s(\theta) = \frac{u_0}{\sqrt{(x'_d(\theta))^2 + (y'_d(\theta))^2}} \quad (44)$$

$$\dot{v}_s = \frac{-[x'_d(\theta)x''_d(\theta) + y'_d(\theta)y''_d(\theta)]\dot{\theta}}{[(x'_d(\theta))^2 + (y'_d(\theta))^2]^{3/2}} u_0 \quad (45)$$

The design procedure of Section 2 gives the following signals

$$z_1 = \eta - \eta_d(\theta)$$

$$z_2 = \nu - \alpha_1(\eta, \eta_d(\theta), \eta'_d(\theta), v_s(\theta))$$

$$\alpha_1 = R^\top(\psi) [-K_p z_1 + \eta'_d(\theta) v_s(\theta)]$$

$$\sigma_1 = \dot{R}^\top(\psi) R(\psi) \alpha_1 + R^\top(\psi) [-K_p R(\psi) \nu + \eta'_d(\theta) \dot{v}_s]$$

$$\nu_2 = R^\top(\psi) [K_p \eta'_d(\theta) + \eta''_d(\theta) v_s(\theta)]$$

The control law is given by

$$T = G^{-1} M [-K_d z_2 - P_2^{-1} R^\top(\psi) P_1 z_1 - M^{-1} N \nu + \sigma_1 + \nu_2 v_s(\theta)] \quad (46)$$

and the update law for $\dot{\omega}_s$ is

$$\dot{\omega}_s = -\lambda \omega_s - 2\mu_1 [z_1^\top P_1 \eta'_d(\theta) + z_2^\top P_2 \nu_2] \quad (47)$$

The simulation are done for a container ship of length $L = 175$ m, and the numerical values are taken from Appendix E.1.3 in (Fossen, 1994). The desired output path are given by (43) where $x_d(\theta) = \theta$ and $y_d(\theta) = 500 \sin(\frac{2\pi}{4000}\theta)$. The controller settings are as follows: $K_p = \text{diag}(0.1, 0.25, 0.25)$, $K_d = \text{diag}(0.4, 0.02, 0.02)$, $P_1 = \text{diag}(0.001, 0.01, 0.01)$, $P_2 = \text{diag}(5, 20, 20)$, $\lambda = 0.1$ and $\mu_1 = 0.005$. Initial conditions are $\eta(0) = [400, 0, \frac{\pi}{2}]^\top$, $\nu(0) = [4, 0, 0]^\top$, $\theta(0) = 0$ and $\omega_s(0) = -1$. In Figure (1) it is shown how the ship accurately converges to and follows the path.

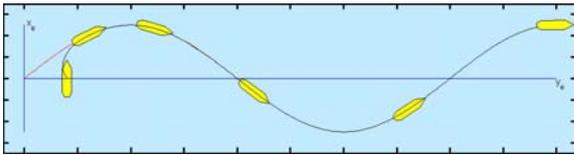


Fig. 1. Simulation of a container ship following a desired sinusoidal path on the ocean surface.

In Figure (2) it is shown how the surge speed u , by the command of an operator, is first controlled to $u_0 = 4$ m/s and then at $t = 400$ s to $u_0 = 10$ m/s.

4. CONCLUSION

As defined in section one, the maneuvering problem is separated into two tasks, the geometric task and the speed assignment task. The recursive design procedure developed in this paper shows one solution to the maneuvering problem for nonlinear systems in vectorial strict feedback form. In n recursive steps, the geometric task was solved. Finally, an update law

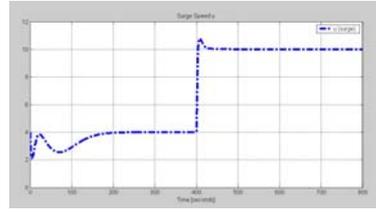


Fig. 2. Plot of the surge speed u in the container ship simulation.

was constructed to satisfy the speed assignment as an asymptotic limit rather than as an identity. This update law introduced feedback from the system state to the desired output. In the case of constant desired speed, $\dot{\theta} = v_0$, a further design step was introduced to assign a constant desired phase $\phi = \phi_0$.

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