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SWITCHING FEEDBACK CONTROL OF NON-HOLONOMIC SYSTEMS WITH MULTI-GENERATORS

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Abstract: In this paper, we propose a switching state feedback control algorithm for a class of non-holonomic symmetric affine systems with multi-generators. The controllability Lie algebra of a multi-generator system is structurally different from that of single-generator systems, such as conventional chained form systems. A multi-generator dynamics is partially considered a single-generator system and each subsystem can be stabilized by any existing controller proposed for chained systems. We propose a switching control algorithm, in that each generator is chosen in sequence and corresponding sub-controllers are applied, where each sub-controller is designed by existing methods for chained systems. The efficiency of the proposed strategy is evaluated via numerical simulations. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Recently, symmetric affine systems (or driftless systems) have been recognized a fundamental platform of so-called non-holonomic systems. Roughly speaking, there are two reasons behind: one is that non-holonomic kinematic constraints can be categorized into this class, and the other is that they violate Brockett's necessary condition(Brockett 1983), i.e., they cannot be asymptotically stabilized by any continuous state feedback even if they are controllable in the sense of nonlinear controllability theory, such as local accessibility.

Among the subclasses of symmetric affine systems, a lot of intensive works have been done for chained form(Murray *et al.* 1994), power form(Pomet 1992), or time-state control form(M.Sampei *et al.* 1995). Though there exist wide variety of controllers proposed for these forms, the clue for stabilization has been essentially established. In fact, the controllability Lie algebra of chained forms and their equivalents have particularly simple structure, so as to be generated by iteration of Lie brackets with a certain vector-field, called *generator*.

On the contrary, systems with two or more generators have been hardly studied, and we have not find any winning trick yet. In this paper, we investigate a switching and discontinuous state feedback control algorithm for such multi-generator systems. The key concept of the proposed algorithm is simple enough, in that each generator is chosen by turns and corresponding sub-controllers are applied. Each sub-controller design is based on existing Astolfi's and Sampei's design method proposed for chained systems. At the last section, the efficiency of the proposed strategy is evaluated via numerical simulations.

2. PRELIMINARIES

2.1 *Symmetric Affine Systems*

Consider *symmetric affine systems*(driftless systems) defined as

$$\dot{x} = g_1(x)u_1 + \cdots + g_m(x)u_m, \quad x \in \mathbb{R}^n. \quad (1)$$

, where the state space is $\mathbb{R}^n$, the input $u := (u_1, \cdots, u_m)^T$ belongs to $\mathbb{R}^m$. $g_i(x) : \mathbb{R}^n \to \mathbb{R}^n$ are smooth vector-fields defined on $\mathbb{R}^n$. The control objective is to bring the state $x(t)$ starting from an arbitrary initial state $x(0)$ sufficiently close to the origin 0.

Let us define a distribution $\mathcal{G}$ spanned by the input vector-fields

$$\mathcal{G}(x) := \mathrm{span}\{g_1(x), \cdots, g_m(x)\}, \quad {}^{\forall}x \in \mathbb{R}^n.$$

A vector field g is said to belong $\mathcal{G}$, namely $g \in \mathcal{G}$, if $g(x) \in \mathcal{G}(x)$ for all $x \in \mathbb{R}^n$. In this manner, $\mathcal{G}$ is also recognized as a set of (infinitely many) vector-fields. From now on, we assume that $\mathcal{G}$ is always nonsingular, or $\dim \mathcal{G} \equiv m$ for simplicity.

2.2 *Controllability and Generator*

For a pair of vector-fields $f, g : \mathbb{R}^n \to \mathbb{R}^n$, we define a *Lie bracket* of f and g as

$$[f,g] := \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g. \tag{2}$$

The set of all smooth vector-fields $C^\infty(\mathbb{R}^n) \otimes \mathbb{R}^n$ forms a *Lie algebra* with Lie bracketing as its product operation. Iteration of Lie bracketing is simply denoted as

$$ad_f^j\, g := [f, ad_f^{j-1}\, g], \quad ad_f^0\, g := g \tag{3}$$

for any integer $j > 0$.

Consider the smallest Lie sub-algebra $\bar{\mathcal{G}}$ which includes $\mathcal{G}$, i.e., a distribution satisfying the following closure condition

$$\bar{\mathcal{G}} \supseteq \mathcal{G}, \quad {}^\forall f, g \in \bar{\mathcal{G}} \Rightarrow [f,g] \in \bar{\mathcal{G}}. \tag{4}$$

We call $\bar{\mathcal{G}}$ the *controllability Lie algebra* of the system 1. It is well known that a symmetric affine system (1) is *controllable* if and only if $\dim \bar{\mathcal{G}} = n$ (Murray *et al.* 1994), where the system is said to be controllable if there exists a finite control sequence $u(t), t \in [0,T]$ which connects any pair of initial state x_0 and desired state in finite time.

Needless to say, controllability is the most essential requirement in treating symmetric affine systems; it is vain to try to achieve the control objective if the system is *not* controllable. Thus we naturally assume that $\dim \bar{\mathcal{G}} = n$, and we should pay attention to *how the basis of $\bar{\mathcal{G}}$ is structured*.

Now it is time to introduce a notion of *generator* as follows. For a certain input index $\alpha \in \{1, \cdots, m\}$, let us consider a set of Lie brackets of the form

$$ad_{g_\alpha}^k\, g_j \quad j = 1, \cdots, m \ \ (j \neq \alpha), \quad k = 0, 1, \cdots, \tag{5}$$

namely, each element can be written as an iterative Lie bracket by g_α. We call these Lie brackets "α-series", and q_α is called a *generator* of these brackets. Note that the range of k starts from 0, thus $g_1, \cdots, g_m$ themselves (i.e., Lie bracket of order zero) are also counted among α-series. If *all*the bases of $\bar{\mathcal{G}}$ are generated by g_α, then the system is said to have a *single generator.* Similarly, if they can be generated by g_α and g_β, then the system is said to have *two generators*, and so on.

3. SYMMETRIC AFFINE SYSTEMS WITH MULTI-GENERATORS

3.1 *System model*

In the rest of this paper, we consider the following class of symmetric affine systems:

$$\Sigma : \quad \begin{array}{llll} \dot{\boldsymbol{q}} & = \boldsymbol{u} & & \\ \dot{\theta}_{ij} & = q_j u_i - q_i u_j & & \\ \dot{\phi}_{ij1} & = \theta_{ij} u_i & \dot{\psi}_{ij1} = & \theta_{ij} u_j \\ \dot{\phi}_{ijk} & = \phi_{ij(k-1)} u_i & \dot{\psi}_{ij\ell} = & \psi_{ij(\ell-1)} u_j \\ (k = 2, \cdots, r_{ij}) & & (\ell = 2, \cdots, s_{ij}) & \end{array} \tag{6}$$

where i, j are integers taken from $1, \cdots, m$ which satisfy $i > j$. For each pair of such i, j, integers $r_{ij}, s_{ij} \geq 0$ are defined to specify the dimension of $\boldsymbol{\phi}, \boldsymbol{\psi}$-coordinates.

The control inputs are $\boldsymbol{u} \in \mathbb{R}^m$. $\boldsymbol{q} \in \mathbb{R}^m$ is a special part of state variables, which is just a direct integration of $\boldsymbol{u}$. We call it the *base coordinates*. The space $Q := \mathbb{R}^m$ which contains $\boldsymbol{q}$ is called the *base space*.

θ_{ij} is a state variable corresponding to a pair of two control inputs, u_i and u_j. All θ_{ij}'s are combined to a vector $\boldsymbol{\theta} \in \mathbb{R}^w$, where $w := \binom{m}{2}$.

ϕ_{ijk}'s $(k = 2, \cdots, r_{ij})$ together with q_i, q_j, θ_{ij} can be considered a set of state variables of a "chained form". Let us denote

$$\boldsymbol{\phi}_{ij} := (\phi_{ij1}, \cdots, \phi_{ij(r_{ij})})^T$$

and combine all the vectors $\{\boldsymbol{\phi}_{ij} | i, j = 1, \cdots, m, i > j\}$ into a single vector $\boldsymbol{\phi}$. $\boldsymbol{\psi}$ is also defined in the same manner. We call $\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\psi}$ the *fiber coordinates*, and the state sub-space which they belong to

$$G := \mathbb{R}^w \times \prod_{i>j} \mathbb{R}^{r_{ij}} \times \prod_{i>j} \mathbb{R}^{s_{ij}}$$

is called *fiber space*. Finally, combine all the state variables into the state vector

$$\boldsymbol{x} := (\ \boldsymbol{q}^T,\ \boldsymbol{\theta}^T,\ \boldsymbol{\phi}^T)^T \in \mathbb{R}^n.$$

Not that the dimension of the total system is

$$n = m + \binom{m}{2} + \sum_{i>j} r_{ij} + \sum_{i>j} s_{ij}.$$

3.2 *Controllability structure*

Let us take a look into the structure of controllability Lie algebra of the system (6). Preceding the analysis, suppose that eq. (6) is expressed as a vector-field expression (1). Then the bases of the involutive closure $\bar{\mathcal{G}}$ can be collected as follows:

$$\begin{array}{rcl} g_1, \cdots, g_m & : & m \text{ vectors}, \\ [g_i, g_j] \quad (i > j) & : & w \text{ vectors} \\ ad\, {g_i}^k g_j \quad (i > j, k = 2, \cdots, r_{ij}) & : & \sum_{i>j} r_{ij} \text{ vectors} \\ ad\, {g_j}^\ell g_i \quad (i > j, \ell = 2, \cdots, r_{ij}) & : & \sum_{i>j} s_{ij} \text{ vectors} \end{array}$$

The controllability condition $\dim \bar{\mathcal{G}} = n$ is thus satisfied. Moreover, we can see that the system (6) has m generators, i.e., $q_1, \cdots, q_m$.

3.3 *Subclasses*

The system model (6) contains most of well-known subclasses of symmetric affine systems, such as chained systems, first-order systems, and second-order systems.

Example 1 (Chained systems).
Chained systems(with single chain) can be expressed by eq. (6) if we set $m = 2$ and $s_{11} = 0$. There is no $\boldsymbol{\psi}$-variables in this case. Since $s_{11} = 0$, each basis of $\bar{\mathcal{G}}$ is 1-series, i.e., q_1 is the only generator. Note that either q_1 or q_2 can be a generator if $r_{ij} = 0$ (so-called Brockett integrator).

Example 2 (First-order systems).
First-order systems(Murray *et al.* 1994) can be expressed by eq. (6) if we set $r_{11} = 0$ and $s_{11} = 0$. There are no ϕ and ψ-variables in this case. The controllability Lie algebra $\bar{\mathcal{G}}$ can be spanned by $g_1, \cdots, g_m$ and Lie brackets of order 1.

For this class of systems, the authors have achieved a switched-feedback control algorithm(Iwatani *et al.* 2002) based on time-state control form. The main idea in that paper is to focus on a generator among base coordinates by turns.

Example 3 (2-input systems).
A class of systems obtained by letting $m = 2$ in eq.(6) plays an important role in this paper. To avoid notational complexity, we omit the subscript ij from the notation of variables, since $w = \binom{2}{2} = 1$.

$$
\Sigma_2 : \quad
\begin{array}{ll}
\dot{\boldsymbol{q}} = \boldsymbol{u} & \\
\dot{\theta} = q_2 u_1 - q_1 u_2 & \\
\dot{\phi}_1 = \theta u_1 & \dot{\psi}_1 = \theta u_2 \\
\dot{\phi}_2 = \phi_1 u_1 & \dot{\psi}_2 = \psi_1 u_2 \\
\vdots & \vdots \\
\dot{\phi}_r = \phi_{r-1} u_1 & \dot{\psi}_s = \psi_{s-1} u_2
\end{array}
\tag{7}
$$

Dimension of the system is

$$
n = m + \binom{m}{2} + \sum_{i>j} r_{ij} + \sum_{i>j} s_{ij} = 2 + 1 + r + s.
$$

The simplest case occurs when $r = s = 1$ and $n = 5$. Such systems can be found in dextrous manipulation problem(ball-and-plate problem)(Sampei *et al.* 1999), offset-hitch trailer problem(Venditteli *et al.* 1998) and snake-like mobile robot(Ishikawa 2001).

Let us see an intuitive interpretation of controllability structure of this class of systems(Fig.1). At first, control inputs u_1, u_2 are integrated to yield q_1, q_2. Then the motions of q_1, q_2 are coupled to affect θ's displacement; it is roughly proportional to curvature of the trajectory on $q_1 - q_2$. Afterwards, motions of $\phi_1, \phi_2 \cdots$ are produced by an integrator chain starting from θ along u_1, while $\psi_1, \cdots$ are also affected by a similar integrator chain along u_2.

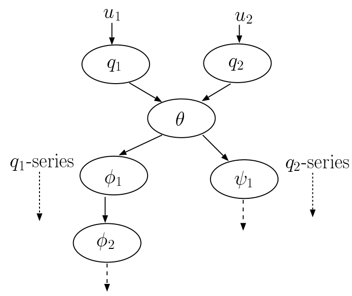

Fig. 1. An image of integrator chains for $m = 2$

4. CONTROL ALGORITHM

In this section, we present a switching control algorithm for multi-generator systems as the main result of this paper. Due to the lack of space, we restrict us to the two-input($m = 2$) case which is defined in the Example 3.

Moreover, we assume that the control method for $m = 2, n = 3$ system (Brockett integrator,
(Brockett 1983)) has been already applied preceding the main control:

Step 0.

> Execute any valid control method proposed for the Brockett integrator, in order to make $\boldsymbol{q}, \theta$ converge to the origin. In the rest of the paper, we assume that $\boldsymbol{q}(0) = \theta(0) = 0$ as initial condition.

For this two-input system (7), we propose a two-fold control algorithm : the first step focuses on q_1 as a generator, the second step focuses on q_2. The two steps will be iterated until all the state converge sufficiently close to the origin.

Step 1

> Focus on a generator q_1 in order to perform feedback stabilization for $\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\phi}$. — The rest part $\boldsymbol{\psi}$ is not fed back and its behavior follows a zero dynamics.

Consider a subsystem of (7) corresponding to $\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\phi}$

$$
\begin{array}{l}
\dot{q} = u \\
\dot{\theta} = q_2 u_1 - q_1 u_2 \\
\dot{\phi}_1 = \theta u_1 \\
\dot{\phi}_i = \phi_{i-1} u_1, \quad (i = 3, \cdots, r)
\end{array}
\tag{8}
$$

For this subsystem, we apply the following coordinate transformation

$$
\begin{aligned}
\xi_1 &= q_1, \quad \xi_2 = q_2 \\
\xi_3 &= \frac{1}{2}(\theta + q_1 q_2) \\
\xi_{i+3} &= \frac{1}{(i+2)!} \sum_{j=0}^{i+1} (i-j+1)!\, q_1^j\, \phi_{i-j} \\
&\qquad (i = 1, \cdots, r),
\end{aligned}
$$

(where $\phi_0 = \theta, \phi_{-1} = q_2$), yielding the 2-input $(r+3)$-state chained form:

$$\dot{\xi}_1 = u_1, \quad \dot{\xi}_2 = u_2 \qquad (9)$$
$$\dot{\xi}_i = \xi_{i-1} u_1, \quad (i = 3, \cdots, r+3).$$

Once the subsystem is expressed in this form, one can apply any existing control method proposed for chained form systems. In this paper, we mix up the following two approaches in order to simplify the error analysis in the next section.

[Step 1-a]

This sub-step is based on *time-state control form method* proposed by (M.Sampei *et al.* 1995), to achieve mild divergence of the neglected values.

Suppose a positive scaler $c_1 > 0$ and let $u_1 := c_1$, then we have $\dot{\xi}_1 = c_1$ and

$$\dot{\Xi} = c_1 \begin{bmatrix} 0 & 0 \\ & \vdots \\ I_{r+1} & 0 \end{bmatrix} \Xi + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2 \qquad (10)$$

$$\dot{\xi}_{r+3} = c_1 \xi_{r+2} \qquad (11)$$

where $\Xi := (\xi_2, \cdots, \xi_{r+2})^T$, thus the dynamics of Ξ is written as a controllable linear system. Then apply a linear state feedback $u_2 = F_a \Xi$ to asymptotically stabilize (10).

Note that ξ_{r+3} is *not* fed back, thus subset $\{\xi | \Xi = 0, \xi_{r+3} = \text{const.}\}$ becomes equilibria. A proper switching condition to move to Step 1-b will be presented in the next subsection(15). •

[Step 1-b]

This sub-step is based on Astolfi's discontinuous feedback controller(Astolfi 1996) to achieve rapid convergence. Suppose a negative scaler $\lambda_0 < 0$ and let $u_1 := -\lambda \xi_1$. Then perform the following coordinate transformation

$$\zeta_1 = \xi_1$$
$$\zeta_i = \frac{\xi_i}{(i-2)!\, \xi_1^{(i-2)}}, \quad i = 2, \cdots, r+3,$$

which is discontinuous when $\xi_1 = 0$. Then

$$\dot{\zeta}_1 = \lambda_0 \zeta_1 \qquad (12)$$
$$\dot{Z} = \lambda_0 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 2 & -2 & 0 \\ & \vdots & & \\ 0 & 0 & (r+3) & -(r+3) \end{bmatrix} Z + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_2$$
$$Z := (\zeta_2, \cdots, \zeta_{r+3})^T,$$

thus the behavior of Z is written as a controllable linear dynamics. Then apply a linear state feedback $u_2 = F_b Z$ which asymptotically stabilizes (10). If $|\zeta_1|, \|Z\|$ get sufficiently close to 0, then it is allowed to proceed to the Step 2. •

Step 2

Focus on a generator q_2 in order to perform feedback control for $\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\psi}$. — The rest part $\boldsymbol{\phi}$ is not fed back and its behavior follows a zero dynamics.

This step is virtually same as the Step1 except that some of the variables are swapped. Applying the following coordinate transformation

$$\xi_1 = q_2, \quad \xi_2 = q_1$$
$$\xi_3 = \frac{1}{2}(-\theta + q_1 q_2)$$
$$\xi_{i+3} = -\frac{1}{(i+2)!} \sum_{j=0}^{i+1} (i-j+1)!\, q_2^j\, \psi_{i-j}$$
$$(i = 1, \cdots, s),$$

(where $\psi_0 = \theta, \psi_{-1} = q_1$), we have a two-input $(s+3)$-state chained form system as in eq.(9). The procedure after this is as quite same as in Step1-a and Step 1-b. •

Termination

Repeat Step 1 and Step 2 until $\|\boldsymbol{x}\|$ gets sufficiently small. •

4.1 *Error convergence analysis of $\boldsymbol{\phi}, \boldsymbol{\psi}$*

In the previous subsection, we gave up controlling $\boldsymbol{\psi}$ in Step 1 and $\boldsymbol{\phi}$ in Step 2, so the behavior of these "neglected states" follow a zero dynamics. Since the zero dynamics are of higher-order, they are generally slow compared to the *main* (linear) dynamics, and will never diverge as the main dynamics converges.

Now let us analyze whether the neglected states decays or not, when the Steps 1 and 2 are repeated reciprocally.

To begin with, we investigate the amount of terminal error of $\boldsymbol{\psi}$ in Step 1-b under the discontinuous controller. For notational simplicity, we suppose that the initial time of Step 1-b is $t = 0$ and the initial values are $\zeta_1(0), Z(0)$.

[Step1-b]

Under the linear state feedback $u_2 = F_b Z$, the closed loop system of (10) takes the form of

$$\dot{Z} = A_b Z, \qquad (13)$$

where A_b is the designed asymptotically stable matrix. According to the coordinate transformation (9), we have

$$\theta = 2\xi_3 - \xi_1 \xi_2 = \zeta_1 (2\zeta_3 - \zeta_2) = \zeta_1 C Z$$

where $C = [-1,\ 2,\ 0, \cdots,\ 0\ ]$. Using this relation, the behavior of ψ_1 in Step 1-b is

$$\begin{aligned}\psi_1(t) &= \int_0^t \theta u_2 d\tau \\ &= \int_0^t e^{\lambda_0 \tau}\zeta_1(0)\cdot Ce^{A_b\tau}Z(0)\cdot F_b e^{A_b \tau}Z(0)d\tau.\end{aligned}$$

Assume A_b is a simple matrix for simplicity, so that it can be diagonalized using a nonsingular matrix P as

$$P^{-1}A_bP = \Lambda := \text{diag}(\lambda_1, \cdots, \lambda_{r+2}).$$

Now let (a_{ij}) denote a matrix whose (i,j)-element is a_{ij}, and define $P^TC^TF_bP =: (d_{ij})$. Then we have an explicit expression of the neglected error response

$$\psi_1(t) = \zeta_1(0)Z(0)^TE(t)Z(0)$$

where

$$E(t) = (P^{-1})^T \left(\frac{d_{ij}\{e^{(\lambda_0+\lambda_i+\lambda_j)t} - 1\}}{\lambda_0+\lambda_i+\lambda_j}\right)P^{-1}. \tag{14}$$

Obviously, $E(t)$ converges to a constant value

$$\lim_{t\to\infty} E(t) = (P^{-1})^T\left(\frac{d_{ij}}{\lambda_0+\lambda_i+\lambda_j}\right)P^{-1}$$

as $t \to \infty$. The rest of $\boldsymbol{\psi}$'s elements $\psi_2(t), \cdots, \psi_s(t)$ can be computed via straightforward integration following the same manner. •

According to the formula above, terminal error of ψ at the end of Step 1 is determined by by $Z(0)$ and $\zeta_1(0)$ at the initial time of Step 1-b. Thus, in the preceding Step 1-a, we can know an answer for question "how much error of ψ would be left if the step is changed to Step 1-b *at this moment*?". This leads us to the following switching criterion.

[Step1-a]

At the beginning of Step 1-a, suppose that $\boldsymbol{q} = 0, \boldsymbol{\theta} = 0, \boldsymbol{\psi} = 0$ is satisfied. Let $\boldsymbol{\phi}_0$ denote the initial error of $\boldsymbol{\phi}$. Under the control $u_1 = c_1, u_2 = F_a\Xi$ designed for Step 1-a, ξ_1 increases monotonically and $\|Z(\xi)\|$ will converge to a certain (small) constant. Thus the expected terminal error (in the succeeding Step 1-b) of $\boldsymbol{\psi}$, say $\|\boldsymbol{\psi}_\infty(\xi)\|$, will also converge to a certain constant.

Then switch to the step 1-b if when $\|\boldsymbol{\psi}_\infty(\xi)\|$ gets sufficiently small, e.g., if

$$\|\boldsymbol{\psi}_\infty(\xi)\| < k\|\boldsymbol{\phi}_0\| \tag{15}$$

is satisfied for some constant $0 < k < 1$. •

Similarly, we can think of the following switching condition for Step 2-a.

[Step2-a]

At the beginning of Step 1-a, suppose that $\boldsymbol{q} = 0, \boldsymbol{\theta} = 0, \boldsymbol{\phi} = 0$ is satisfied. Let $\boldsymbol{\psi}_0$ denote the initial error of $\boldsymbol{\psi}$. Expected terminal error of $\boldsymbol{\phi}$ (in the succeeding Step 2-b), say $\|\boldsymbol{\phi}_\infty(\xi)\|$, will also converge to a certain constant.

Then switch to the step 2-b when $\|\boldsymbol{\phi}_\infty(\xi)\|$ gets sufficiently small, e.g., if

$$\|\boldsymbol{\phi}_\infty(\xi)\| < k\|\boldsymbol{\psi}_0\| \tag{16}$$

is satisfied for some constant $0 < k < 1$. •

This error propagation mechanism is illustrated in Fig.2. Step 1 receives an initial error $\boldsymbol{\phi}_0$ and lefts a terminal error $\boldsymbol{\psi}_\infty$ to Step 2, and Step 2 receives an initial error $\boldsymbol{\psi}_0$ and lefts a terminal error $\boldsymbol{\phi}_\infty$ to Step 1. In order to decrease the error in this propagation loop, k must be less than 1.

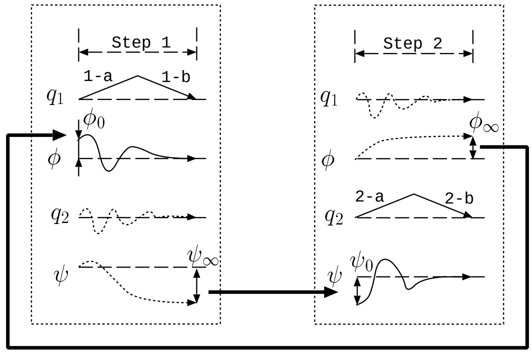

Fig. 2. Error propagation mechanism

5. SIMULATION

Let us see efficiency of the proposed control algorithm by performing a numerical simulation. The target system is 7-dimensional systems obtained by setting $m = 2, r = s = 2$ in (7).

$$\Sigma_2^7: \quad \begin{aligned} &\dot{q} = u \\ &\dot{\theta} = q_2u_1 - q_1u_2 \\ &\dot{\phi}_1 = \theta u_1 \qquad \dot{\psi}_1 = \theta u_2 \\ &\dot{\phi}_2 = \phi_1 u_1 \qquad \dot{\psi}_2 = \psi_1 u_2 \end{aligned} \tag{17}$$

The initial state is $\boldsymbol{x}(0) = (0,0,0,1,-1,1,-1)^T$. F_a, F_b are determined via LQ optimal regulator design, and $k = 0.1$.

Fig.4 shows time response of the state variables and Fig.3 illustrates the trajectory of base variables on $q_1 - q_2$ plane.

At first, q_1 plays a role of generator in Step 1, and $\boldsymbol{\psi}$'s error is left at the end of the step. Then the generator is switched to q_2 in Step 2, and $\boldsymbol{\phi}$'s error is left at the end of the step, which is less than the previous terminal error of $\boldsymbol{\psi}$ times k. The algorithm is terminated after performing Step 1 and Step 2 once again.

6. CONCLUSION

In this paper, we proposed a switching state feedback control algorithm for a class of symmetric affine

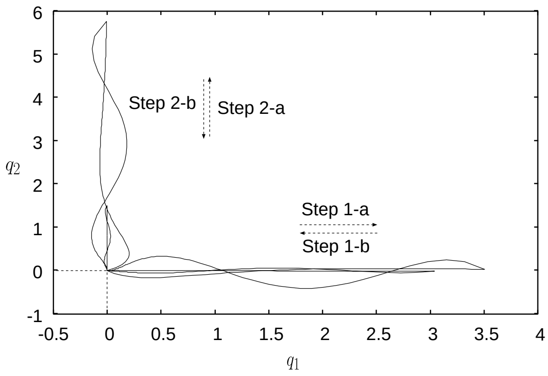

Fig. 3. on q_1-q_2 plane

systems with multi-generators. In the proposed algorithm, each generator is chosen in sequence and corresponding sub-controllers are applied, where each sub-controller design is based on existing Astolfi's and Sampei's design method proposed for chained systems. The error propagation mechanism due to the repetition of generator switching was also analyzed.

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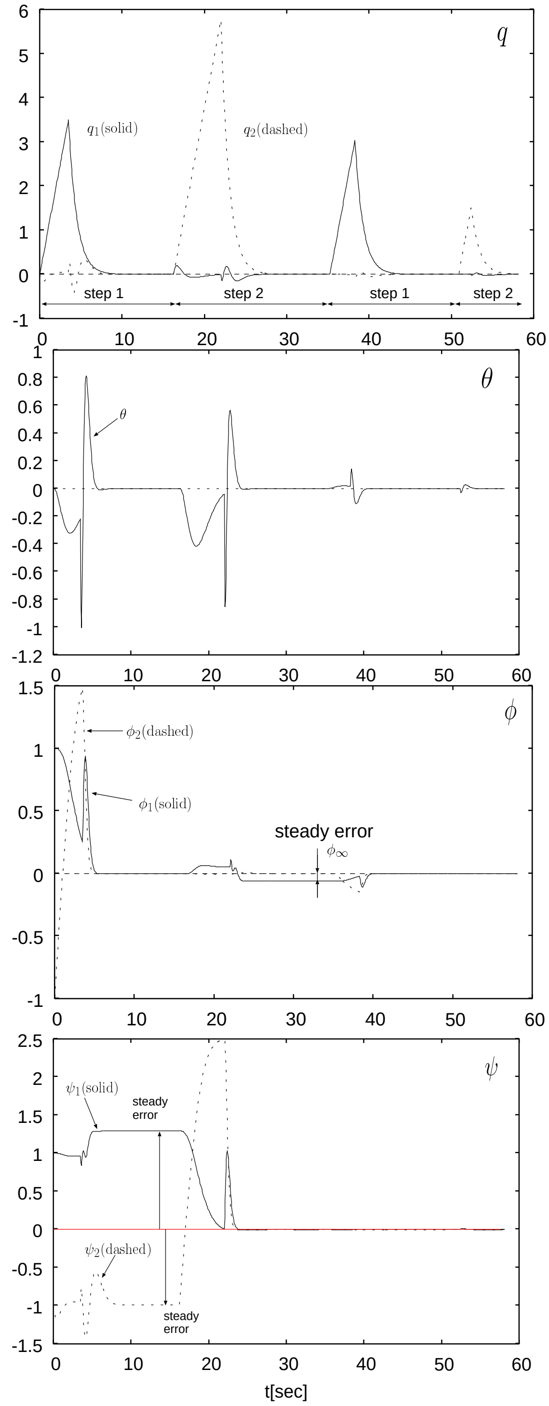

Fig. 4. State variables