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STOCHASTIC POWER CONTROL FOR WIRELESS SYSTEMS: CENTRALIZED DYNAMIC SOLUTIONS AND ASPECTS OF DECENTRALIZED CONTROL [1]

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Abstract: This paper considers power control for log-normal fading channels. A rate based power set point control model and associated performance measures are introduced. The value function of the stochastic optimal control is a viscosity solution to the associated HJB equation and is approximated by smooth functions. Finally, by introducing multi-objective indices, we give a game theoretic formulation to the dynamic power optimization problem.

Keywords: Power control, log-normal fading channels, HJB equations, dynamic games, decentralization, Nash equilibrium, Pareto optimality.

1. INTRODUCTION

Power control in cellular telephone systems is an important design task for the minimization of energy requirements at the user level and in order to insure a constant or adaptable Quality of Service (QoS) in the face of cellular telephone mobility and fading channels. This is particularly crucial in CDMA (code division multiple access) systems where individual users are identified not by a particular frequency carrier and a particular frequency content, but by a wideband signal associated with a given pseudo-random number code. In such a context, the received signal of a given user at the base station views all other user signals, as well as other cell signals arriving at the base station, as interference or noise, because they both degrade the decoding process of identifying and extracting a given user's signal. Thus, it becomes crucial that individual mobiles emit power at a level which will insure adequate signal to noise ratio at the base station. More specifically, excess levels of signalling from a given mobile will act as interference on other mobile signals and contribute to an accelerated depletion of cellular phone batteries. Conversely, low levels of signalling will result in inadequate QoS. In fact, tight power control is indirectly related to the ability of the CDMA base station to accommodate as many users as possible while maintaining a required QoS.

Extensive research has been done on power control for static models which largely ignore the dynamics of channel fading as well as mobility, see (Viterbi A.M. and Viterbi A.J., 1993; Sun and Wong, 1999, 2000; and references therein). In this paper, the modelling and analysis of power control strategies employs wireless models which are time-varying and subject to fading. Specifically, we consider log-normal fading channels. Motivation and background information on power control for log-normal fading channels can be found in (Huang et al., 2001b).

The paper is organized as follows: in Section 2 we propose the optimal control formulation of the CDMA power adjustment where the performance function is intended to reflect power minimization objectives under signal to noise ratio constraints. In Section 3, we analyze the singular HJB equation associated to this stochastic control problem and suboptimal approximation of the value function, and Section 4 contains

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numerical solutions of the suboptimal control. Section 5 gives a game theoretic formulation for dynamic power optimization and introduces the notion of Pareto optimality under decentralized information.

2. AN OPTIMAL CONTROL FORMULATION

2.1 *The Channel Model*

Let $x_i(t), 1 \leq i \leq n$, denote the attenuation (expressed in dBs and scaled to the natural logarithm basis) at the instant t of the power of the i-th mobile of a network and let $\alpha_i(t) = e^{x_i(t)}$ denote the corresponding power loss. Based on the work of Charalambous and Menemenlis (1999), we model the power attenuation dynamics by

$$dx_i = -a_i(x_i + b_i)dt + \sigma_i dw_i, 1 \leq i \leq n, \quad (2.1)$$

where n denotes the number of mobiles, $\{w_i, 1 \leq i \leq n\}$ are n independent standard Wiener processes, and the initial states $x_i(0)$, $1 \leq i \leq n$ are mutually independent Gaussian random variables which are also independent of the Wiener processes. In (2.1) a_i, b_i, $\sigma_i > 0$, $1 \leq i \leq n$. The first term in (2.1) implies a long-term adjustment of x_i towards the long-term mean $-b_i$, and a_i is the speed of the adjustment. Correspondingly, the i-th power loss α_i has a long-term adjustment towards its long-term mean, which is the average large-scale path loss (Charalambous and Menemenlis, 1999).

2.2 *Rate Based Power Control*

Currently, the power control algorithms employed in the mobile telephone domain use gradient type algorithms with bounded step size. This is motivated by the fact that cautious algorithms are sought which behave adaptively in a communications environment in which the actual position of the mobile and its corresponding channel properties are unknown and varying. We model the adaptive step-wise adjustments of the (sent) power p_i of the i-th mobile by the so-called rate adjustment model (Huang et al., 2001a,b)

$$dp_i = u_i dt, \quad |u_i| \leq u_{i max}, \quad 1 \leq i \leq n, \quad (2.2)$$

where the bounded input u_i controls the size of increment dp_i. Without loss of generality, $u_{i max}$ will be set equal to one. The adaptive nature of practical rate adjustment control laws is replaced here by an optimal control calculation based on full knowledge of channel parameters a_i, b_i, and σ_i, $1 \leq i \leq n$. In the intended practical implementation of our solution these parameters would be replaced by on-line estimates. Write

$$x = [x_1, \cdots, x_n]^\tau, \quad \alpha = [\alpha_1, \cdots, \alpha_n]^\tau,$$
$$p = [p_1, \cdots, p_n]^\tau, \quad u = [u_1, \cdots, u_n]^\tau.$$

2.3 *Performance Function*

Let $\eta > 0$ be the constant system thermal noise intensity which is assumed to be the same for all n mobile users. Then, in terms of the power levels $p_i \geq 0$, and the channel power attenuations α_i, $1 \leq i \leq n$, the so-called signal-to-interference ratio (SIR) for the i-th mobile is given by

$$\Gamma_i = \frac{\alpha_i p_i}{\sum_{j \neq i}^n \alpha_j p_j + \eta}, \quad 1 \leq i \leq n. \quad (2.3)$$

A standard communications quality of service constraint is to require that

$$\Gamma_i \geq \gamma_i > 0, \quad 1 \leq i \leq n, \quad (2.4)$$

where γ_i, $1 \leq i \leq n$, is a prescribed set of individual signal to noise ratios. We note that the constraints (2.4) are equivalent to

$$\Gamma_i' = \frac{\alpha_i p_i}{\sum_{j=1}^n \alpha_j p_j + \eta} \geq \mu_i, \quad 1 \leq i \leq n, \quad (2.5)$$

where $\mu_i \triangleq \frac{\gamma_i}{1+\gamma_i} > 0$. Further, from (2.5) we see

$$0 < \sum_{i=1}^n \mu_i < 1, \quad (2.6)$$

holds if we require (2.4) to be solvable.

A straightforward way to formulate the optimization problem would be to seek control functions which yield the minimization of the integrated power $\int_0^T \sum_{i=1}^n p_i(t)dt$ subject to the constraints (2.5)-(2.6) at each instant t, $0 \leq t \leq T$.

Consider the pointwise minimization of the summed power $\sum_{i=1}^n p_i$ under (2.5)-(2.6) and $p_i \geq 0$, $1 \leq i \leq n$. Setting n inequalities in (2.5) as equalities and taking into account constraint (2.6), we get a positive vector $p^0 = (p_1^0, \cdots, p_n^0)$ given by

$$p_i^0 = \frac{\mu_i \eta}{\alpha_i(1 - \sum_{i=1}^n \mu_i)}, \quad 1 \leq i \leq n. \quad (2.7)$$

It turns out that p^0 is the unique positive vector minimizing $\sum_{i=1}^n p_i$ under constraints (2.5)-(2.6). In other words, the solution to minimizing $\sum_{i=1}^n p_i$, $p_i \geq 0$,subject to (2.5), (2.6) is the unique solution to

$$\frac{\alpha_i p_i}{\sum_{j=1}^n \alpha_j p_j + \eta} = \mu_i, \quad 1 \leq i \leq n. \quad (2.8)$$

Hence it is well motivated to replace the above pointwise constrained deterministic optimization problem with the following unconstrained deterministic penalty function optimization problem

$$\min \sum_{i=1}^n [\alpha_i p_i - \mu_i(\sum_{j=1}^n \alpha_j p_j + \eta)]^2 + \lambda \sum_{i=1}^n p_i, \quad (2.9)$$

where $\lambda > 0$. However, because the power vector is a part of the stochastic channel-power system state with dynamics (2.1), (2.2), it is impossible to instantaneously minimize (2.9) via $u(t)$ at all times t. Hence, over the interval $[0,T]$, we employ the following averaged integrated loss function:

$$E\int_0^T \{\sum_{i=1}^n [\alpha_i p_i - \mu_i(\sum_{j=1}^n \alpha_j p_j + \eta)]^2 + \lambda \sum_{i=1}^n p_i\} dt \tag{2.10}$$

subject to (2.1) and (2.2), where $\lambda > 0$.

3. ANALYSIS OF THE OPTIMAL CONTROL AND APPROXIMATION OF THE VALUE FUNCTION

We will analyze the optimal control problem in terms of the state variable (x,p); this facilitates the definition of the value function v since x_i is defined in R, while α_i is only defined in R^+, $1 \le i \le n$. Further define

$$f(x) = \begin{pmatrix} -a_1(x_1+b_1) \\ \vdots \\ -a_n(x_n+b_n) \end{pmatrix}, \quad H = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n \end{pmatrix},$$

$$z = \begin{pmatrix} x \\ p \end{pmatrix}, \quad \psi = \begin{pmatrix} f \\ u \end{pmatrix}, \quad G = \begin{pmatrix} H \\ 0_{n\times n} \end{pmatrix}.$$

We write (2.1), (2.2) in the vector form

$$dz = \psi dt + G dw, \quad 0 \le t \le T, \tag{3.1}$$

where w is an $n \times 1$ standard Wiener process determined by (2.1). We will denote the state variable by (x,p) or z, or in a mixed form. We write the integrand in (2.10) in terms of (x,p) as

$$L(z) = \sum_{i=1}^n [e^{x_i} p_i - \mu_i(\sum_{j=1}^n e^{x_j} p_j + \eta)]^2 + \lambda \sum_{i=1}^n p_i,$$

where $\lambda > 0$. The admissible control set is $\mathcal{U} = \{u(\cdot) \mid u \text{ is adapted to } \sigma(x_s, p_s, s \le t), \text{ and } u(t) \in U \triangleq [-1,1]^n, \forall 0 \le t \le T\}$. We assume p has a deterministic initial value at $s = 0$; then obviously $\sigma(x_s, p_s, s \le t) = \sigma(x_0, w_s, s \le t)$. The cost associated with (3.1) and a control u is specified to be $J(s,x,p,u) = E[\int_s^T L(x_r,p_r)dr | x_s = x, p_s = p]$, where s is the initial time; further we set $v(s,x,p) = \inf_{u\in\mathcal{U}} J(s,x,p,u)$, and simply write $J(0,x,p,u)$ as $J(x,p,u)$.

Theorem 3.1. (Huang et al., 2001b) There exists a unique optimal control $\widehat{u} \in \mathcal{U}$ such that $J(x_0,p_0,\widehat{u}) = \inf_{u\in\mathcal{U}} J(x_0,p_0,u)$, for any (x_0,p_0), and uniqueness holds in the sense: if there is $\widetilde{u} \in \mathcal{U}$ such that $J(x_0,p_0,\widetilde{u}) = J(x_0,p_0,\widehat{u})$,then $P_\Omega(\widetilde{u}_s \ne \widehat{u}_s) > 0$ only on a set of times $s \in [0,T]$ of measure 0, where Ω is the sample space. □

Definition 3.2. (Yong and Zhou, 1999) A function $\overline{v}(t,z) \in C([0,T] \times \mathrm{R}^{2n})$ is called a **viscosity subsolution** to the HJB equation

$$0 = -\frac{\partial v}{\partial t} + \sup_{u\in U}\{-\frac{\partial^\tau v}{\partial z}\psi\} - \frac{1}{2}\mathrm{tr}(\frac{\partial^2 v}{\partial z^2} GG^\tau) - L,$$
$$v\mid_{t=T} = h, \quad z \in \mathrm{R}^{2n}, \tag{3.2}$$

if $\overline{v} \mid_{t=T} \le h$, and for any $\phi(t,z) \in C^{1,2}([0,T] \times \mathrm{R}^{2n})$, whenever $\overline{v} - \phi$ takes a local maximum at $(t,z) \in [0,T) \times \mathrm{R}^{2n}$, we have

$$-\frac{\partial \phi}{\partial t} + \sup_{u\in U}\{-\frac{\partial^\tau \phi}{\partial z}\psi\} - \frac{1}{2}\mathrm{tr}(\frac{\partial^2 \phi}{\partial z^2} GG^\tau) - L \le 0, \tag{3.3}$$

at (t,z). $\overline{v} \in C([0,T] \times \mathrm{R}^{2n})$ is called a **viscosity supersolution** to equation (3.2) if $\overline{v} \mid_{t=T} \ge h$, and in (3.3) we have an opposite inequality at (t,z), whenever $\overline{v} - \phi$ takes a local minimum at $(t,z) \in [0,T) \times \mathrm{R}^{2n}$. $\overline{v}$ is called a **viscosity solution** if it is both a viscosity subsolution and a viscosity supersolution. □

We introduce the function class $\mathcal{G}$ such that each $v(t,x,p) \in \mathcal{G}$ satisfies: a) $v \in C([0,T] \times \mathrm{R}^{2n})$ and, b) there exist $C, k_1, k_2 > 0$ such that $|v| \le C\{1 + \sum_{i=1}^n e^{k_1 x_i} + \sum_{i=1}^n (|x_i|^{k_2} + |p_i|^{k_2})\}$, where the constants C, k_1, k_2 depend on each v.

Theorem 3.3. (Huang et al., 2001b) The value function v is a viscosity solution to the equation

$$0 = -\frac{\partial v}{\partial t} + \sup_{u\in U}\{-\frac{\partial^\tau v}{\partial z}\psi\} - \frac{1}{2}\mathrm{tr}(\frac{\partial^2 v}{\partial z^2} GG^\tau) - L,$$
$$v(T,x,p) = 0. \tag{3.4}$$

Moreover, there exists only one viscosity solution to (3.4) in the class $\mathcal{G}$. □

We modify (3.4) by adding a perturbing term and formally carrying out the minimization to get

$$0 = v_t^\epsilon + \frac{1}{2}\sum_{i=1}^n \sigma_i^2 v_{x_i x_i}^\epsilon + \frac{1}{2}\sum_{i=1}^n \epsilon^2 v_{p_i p_i}^\epsilon \tag{3.5}$$
$$- \sum_{i=1}^n v_{x_i}^\epsilon a_i(x_i + b_i) - \sum_{i=1}^n |v_{p_i}^\epsilon| + L(x,p),$$

where we use v^ϵ to indicate the dependence on $\epsilon > 0$. We will seek a classical solution v^ϵ in the class $\mathcal{F}$ such that any $v \in \mathcal{F}$ satisfies: a) $v \in C^{1,2}((0,T) \times \mathrm{R}^{2n}) \cap \mathrm{C}([0,\mathrm{T}] \times \mathrm{R}^{2n})$ and, b) $|v| \le C(1 + |p|^{k_1} + e^{k_2|x|})$, where C, k_1, $k_2 > 0$ depend on v, and $v(T,x,p) = 0$.

Theorem 3.4. (Huang et al., 2001b) The equation (3.5) has a unique solution v^ϵ in $\mathcal{F}$ for $\epsilon > 0$, and for $0 < \epsilon < 1$, $B \subset \mathrm{R}^{2n}$ compact, $v^\epsilon \to v$ uniformly on $[0,T] \times B$, as $\epsilon \to 0$, where v is the value function of (3.1). □

4. NUMERICAL IMPLEMENTATION OF THE SUBOPTIMAL CONTROL LAW

We consider two mobiles with i.i.d. channel dynamics

$$dx_i = -a(x_i + b)dt + \sigma dw_i, \quad i = 1,2, \ 0 \le t \le 1.$$

The performance function is $E\int_0^1 L(x_t,p_t)dt$ with

$$L=[e^{x_1}p_1-0.4(e^{x_1}p_1+e^{x_2}p_2+0.25)]^2$$
$$+[e^{x_2}p_2-0.4(e^{x_1}p_1+e^{x_2}p_2+0.25)]^2+\lambda(p_1+p_2).$$

The approximation equation (3.5) takes the form:

$$0=v_t+\frac{1}{2}\sigma^2(v_{x_1x_1}+v_{x_2x_2})+\frac{1}{2}\epsilon^2(v_{p_1p_1}+v_{p_2p_2})$$
$$-a(x_1+b)v_{x_1}-a(x_2+b)v_{x_2}-|v_{p_1}|-|v_{p_2}|+L,$$
$$v(1,x,p)=0.$$

The above equation is solved by a difference scheme (Ames, 1992) in a bounded region

$$S=\{(t,x,p),0\le t\le 1,-4\le x_i\le 3,|p_i|\le 3\}.$$

An additional boundary condition is added such that $v|_{\overline{\partial}}=0$, where $\overline{\partial}=\partial S\backslash\{(t,x,p),t=0\}$. Take step sizes $\delta t,h>0$, and denote $z=(x_1,x_2,p_1,p_2)^\tau$, $e_i=(0,\cdots,1,\cdots,0)^\tau$ where 1 is the i-th element in the row. We use the scheme

$$\begin{aligned}
0=&\tfrac{1}{\delta t}[v(t+\delta t,z)-v(t,z)]\\
&+\tfrac{\sigma^2}{2h^2}[v(t,z+e_1h)+v(t,z-e_1h)-2v(t,z)]\\
&+\tfrac{\sigma^2}{2h^2}[v(t,z+e_2h)+v(t,z-e_2h)-2v(t,z)]\\
&+\tfrac{\epsilon^2}{2h^2}[v(t,z+e_3h)+v(t,z-e_3h)-2v(t,z)]\\
&+\tfrac{\epsilon^2}{2h^2}[v(t,z+e_4h)+v(t,z-e_4h)-2v(t,z)]\\
&-\tfrac{a(x_1+b)}{h}[v(t,z+e_1h)-v(t,z)]1_{\{a(x_1+b)\le 0\}}\\
&-\tfrac{a(x_1+b)}{h}[v(t,z)-v(t,z-e_1h)]1_{\{a(x_1+b)>0\}}\\
&-\tfrac{a(x_2+b)}{h}[v(t,z+e_2h)-v(t,z)]1_{\{a(x_2+b)\le 0\}}\\
&-\tfrac{a(x_2+b)}{h}[v(t,z)-v(t,z-e_2h)]1_{\{a(x_2+b)>0\}}\\
&+\tfrac{u_1}{2h}[v(t,z+e_3h)-v(t,z-e_3h)]\\
&+\tfrac{u_2}{2h}[v(t,z+e_4h)-v(t,z-e_4h)]+L(z),\\
u_1=&-\text{sgn}[v(t,z+e_3h)-v(t,z-e_3h)],\\
u_2=&-\text{sgn}[v(t,z+e_4h)-v(t,z-e_4h)].
\end{aligned}$$

The solution to the difference equation can be obtained by an iterative procedure. The convergence to the exact solution of the iteration can be proved by the method in (Kushner and Kleinman, 1968). We take $a=4$, $b=0.3$, $\sigma^2=0.09$, $\epsilon^2=0.15$, $\delta t=h=0.1$. Case 1: $\lambda=0.01$; Case 2: $\lambda=0.001$. The value function is further interpolated to get a step size of 0.05. The control is determined by the descent direction of the value function. In the figures, q_1, q_2 are the pointwise ideal powers obtained from (2.8). Figures 2-3, Figures 4-5 show the results for Cases 1, 2, respectively. Figure 6 shows the value function surface. When at the initial time one mobile has a high power and the other has a low power, we see that an interesting equalization phenomenon takes place as shown in Figure 4. At the beginning the controller will first make the mobile with a high power reduce power and the other one increase power, and after a certain period both mobiles will increase power together. This happens because when the power difference of two mobiles is big, more penalty is caused in the performance function.

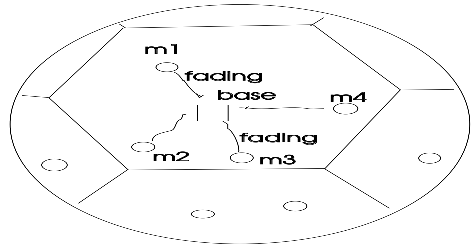

Fig.1: A typical cell

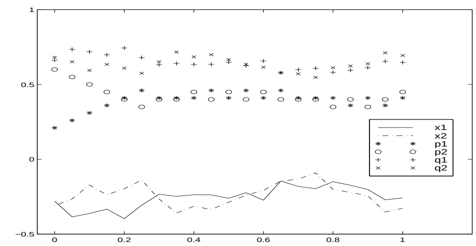

Fig.2: $p_1(0)=0.21,p_2(0)=0.6,\lambda=0.01$

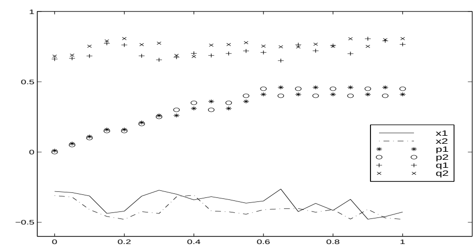

Fig.3: $p_1(0)=0.01,p_2(0)=0,\lambda=0.01$

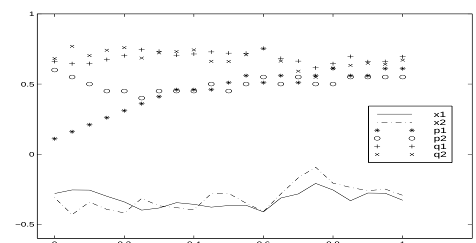

Fig.4: $p_1(0)=0.11,p_2(0)=0.6,\lambda=0.001$

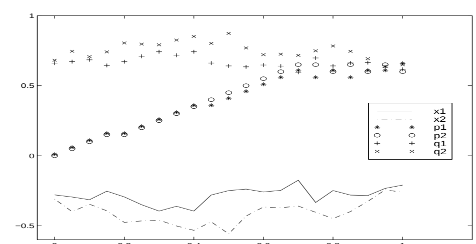

Fig.5: $p_1(0)=0.01,p_2(0)=0,\lambda=0.001$

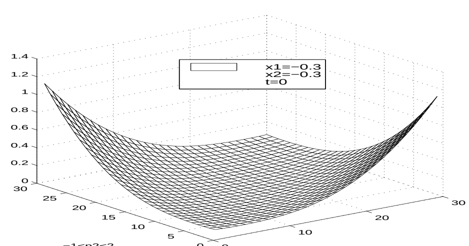

Fig.6: $x_1,x_2=-0.3,t=0,\lambda=0.001$

5. DECENTRALIZATION AND A GAME THEORETIC APPROACH

Essentially, in the stochastic optimal control framework, the performance (in terms of the SIR level and power consumption) of each mobile is influenced indirectly by its control, since the control actions of the mobiles are coordinated by a centralized cost function. Thus, to increase flexibility of individual mobiles to adjust their own performance, it is quite natural to introduce individual costs for each mobile, and this potentially helps develop decentralized optimization for networks.

Now we introduce the multi-objective optimization approach for the power control problem and give a game theoretic formulation. The reader is referred to (Orda and Shimkin, 2000; Yaiche et al., 2000; Sun and Wong, 2000) for game theoretic approaches to rate, power control, and network service allocation for static models. In practical systems, it is important to implement control strategy in a decentralized manner, i.e., each mobile user adjusts its power based on its local information concerning the network. This can significantly reduce information exchange efforts among users and base stations and thus reduce system running costs. And based on these aspects, it makes sense to place emphasis on decentralized games.

Here we also call a control as a policy or strategy. The control is taken from an admissible control set $\mathcal{U}$ to be defined appropriately. Write

$$l_i(z) = l_i(x,p) = [e^{x_i}p_i - \mu_i(\sum_{j=1}^{n} e^{x_j}p_j + \eta)]^2 + \lambda p_i,$$

and for user i, we define its objective function as

$$J_i(x,p,u) = E[\int_0^T l_i(x_t,p_t)dt | z_0 = (x,p)]. \quad (5.1)$$

Each user chooses its control to minimize its cost index. Sometime we omit (x,p) in J_i when there is no ambiguity. For illustrating a decentralized power adjustment strategy, as an example, we define the decentralized admissible control set as

$\mathcal{U}_{dec} = \{u = (u_i)_{i=1}^n | u_i \in [-1,1]$ and is adapted to $\sigma(x_i(s), p_i(s), r_i(s) \triangleq \sum_{j\neq i} e^{x_j}p_j + \eta,\ s \le t)\}$.

We write the i-th component of $\mathcal{U}_{dec}$ by $\mathcal{U}^i_{dec}$. Here r_i denotes the total interference to user i caused by all the other users and the system thermal noise. So each user utilizes its own power and attenuation history as well as the interference it receives to determine its power adjustment. This gives a decentralized control strategy. It is of interest to determine each mobile user's power by considering other information structure which is locally accessible and can efficiently reflect network information. In a practical system, bit error rate (BER) could be useful local information.

Definition 5.1. A control $\hat{u} \in \mathcal{U}$ is a **Nash equilibrium**, if for each i and any $u_i \in \mathcal{U}^i$, we have

$$J_i(\hat{u}_i, \hat{u}_{-i}) \le J_i(u_i, \hat{u}_{-i})$$

where $\mathcal{U}$ is a certain admissible control set (centralized or decentralized), $\mathcal{U}^i$ is the i-th component of $\mathcal{U}$, and $\hat{u}_{-i}$ denotes the remainder of $\hat{u}$ generated by taking out the i-th component. □

Definition 5.2. $\hat{u} \in \mathcal{U}$ is said to be **Pareto optimal** if there exists no $\overline{u} \in \mathcal{U}$ such that $J_i(\overline{u}) \le J_i(\hat{u})$ for all i with at least one strict inequality. □

Definition 5.3. $u \in \mathcal{U}_{dec}$ is called a **decentralized information Pareto optimal (DIPO) control** with respect to J, for a certain set of cost indices $J = (J_1, \cdots, J_n) : u \to \mathrm{R}^n$, if u is Pareto optimal with respect to $\mathcal{U}_{dec}$. The associated optimal cost vector is called a **DIPO solution**. □

The following gives a simple relation between the optimal control under centralized cost and centralized information, and the Pareto optimal strategy with respect to individual costs for each agent and centralized information. The additive form of the centralized cost function immediately yields the following result.

Proposition 5.4. Suppose $\hat{u}$ is the optimal control in Theorem 3.1 under the cost (2.10) and the admissible control set $\mathcal{U}$ defined in Section 3. Then $\hat{u}$ also gives a Pareto optimal strategy for the multi-objective cost indices given by (5.1) for $i = 1, \cdots, n$, with $\mathcal{U}$ given in Section 3. □

Now we use the following example to further illustrate the above notions. To reduce computational complexity, the example is extremely simple.

Example: Suppose we have two players with dynamics and cost indices given by:

$$dx = ydt + u_1 dt + dw_1,$$
$$dy = xdt + u_2 dt + dw_2,$$

$$J_1 = \limsup_{T\to\infty} E\frac{1}{T}\int_0^T (\alpha x^2 + u_1^2)dt,$$
$$J_2 = \limsup_{T\to\infty} E\frac{1}{T}\int_0^T (\alpha y^2 + u_2^2)dt, \quad (5.2)$$

where w_1, w_2 are two mutually independent standard Wiener processes, $\alpha > 0$, and the control is restricted to be linear time invariant feedback $u_1 = l_1 x$, $u_2 = l_2 y$ using only local measurements (i.e., decentralized information). □

Proposition 5.5. The associated costs $J_i, i = 1,2$ in (5.2) are finite if and only if $(l_1,l_2) \in \overline{L} = \{(l_1,l_2) : l_1 l_2 > 1,\ l_i < 0, i = 1,2\}$. For $(l_1,l_2) \in \overline{L}$, the associated costs are given by $J_1 = \frac{\alpha+l_1^2}{2}\frac{l_2}{1-l_1l_2}$, $J_2 = \frac{\alpha+l_2^2}{2}\frac{l_1}{1-l_1l_2}$. Furthermore, $\overline{l}_1 = \overline{l}_2 = -\sqrt{2+\alpha}$ gives the unique Nash equilibrium, and $l_1^* = l_2^* = -\sqrt{[3+\alpha+\sqrt{(3+\alpha)^2+4\alpha}]/2} \triangleq$

c^* gives a Pareto optimal control. In other words, the feedback control $u_1 = l_1^* x$, $u_2 = l_2^* y$ is DIPO.

Proof. In fact, for any feedback gain pair (l_1, l_2), to make the associated costs $J_i(l_1, l_2)$, $i = 1, 2$, finite, we need the matrix $A = \begin{pmatrix} l_1 & 1 \\ 1 & l_2 \end{pmatrix}$ to be strictly stable, which is equivalent to $(l_1, l_2) \in \overline{L}$. In order to obtain J_1, solving $KA + A^\tau K + M = 0$, where $M_{11} = \alpha + l_1^2$, $M_{12} = M_{21} = M_{22} = 0$, we get a positive definite symmetric solution K, and

$$J_1 = \mathrm{tr} K = \frac{\alpha + l_1^2}{2} \frac{l_2}{1 - l_1 l_2}. \qquad (5.3)$$

In the same way, we obtain

$$J_2 = \frac{\alpha + l_2^2}{2} \frac{l_1}{1 - l_1 l_2}. \qquad (5.4)$$

By taking $\frac{\partial J_1}{\partial l_1} = \frac{\partial J_2}{\partial l_2} = 0$, we find a unique solution $(\bar{l}_1, \bar{l}_2) = (-\sqrt{2+\alpha}, -\sqrt{2+\alpha})$, and it can be shown that for any $(l_1, \bar{l}_2), (\bar{l}_1, l_2) \in \overline{L}$, $J_1(l_1, \bar{l}_2) \geq J_1(\bar{l}_1, \bar{l}_2)$, $J_2(\bar{l}_1, l_2) \geq J_2(\bar{l}_1, \bar{l}_2)$. Finally, concerning the pair (l_1^*, l_2^*), if there is $(\widehat{l}_1, \widehat{l}_2)$ such that $J_i(\widehat{l}_1, \widehat{l}_2) \leq J_i(l_1^*, l_2^*), i = 1, 2$, then it follows $\underline{J}(\widehat{l}_1, \widehat{l}_2) = J_1(\widehat{l}_1, \widehat{l}_2) + J_2(\widehat{l}_1, \widehat{l}_2) \leq \underline{J}(l_1^*, l_2^*)$. On the other hand, we can show that $\underline{J}(l_1, l_2)$ attains its minimum at the interior of $\overline{L}$. By the first order necessary condition we find the minimum is attained at a unique point (c^*, c^*). So it is impossible to find $(\widehat{l}_1, \widehat{l}_2) \in \overline{L}$ such that $J_i(\widehat{l}_1, \widehat{l}_2) \leq J_i(l_1^*, l_2^*), i = 1, 2$, with at least one strict inequality. □

We modify the admissible control set by restricting the feedback gain to be bounded, i.e., $|l_i| \leq B_i > 2, i = 1, 2$. Now in the definition of J_i, take $\alpha = 1$ and replace u_i by $\beta u_i, \beta > 0, i = 1, 2$, in the integrand. β is interpreted as a pricing parameter for the input energy; then under price $\beta = \frac{1}{B^2 - 2}$, $(\bar{l}_1, \bar{l}_2) = (B_0, B_0)$ is a DIPO (Nash) equilibrium, where $B_0 = \min\{B_i, i = 1, 2\}$. (See (Sun and Wong, 2000) on reshaping equilibrium set by pricing). We note that even if only l_1 varies and l_2 is fixed, the control input process $l_2 y$ will be forced to change since the two components of the state vector are interacting. We term the feedback pair $(\bar{l}_1 x, \bar{l}_2 y)$ as a (Nash) equilibrium in a generalized sense.

For this example, finite horizon cost indices with time varying linear feedback can also be considered, and the existence of DIPO equilibria can be analyzed. But the computation should be more complex. We remark that it is an important issue to determine an appropriate decentralized information and controller structure to make efficient use of local information for each user and meanwhile keep the computation of DIPO equilibria relatively simple.

6. CONCLUSION

We have studied rate-based stochastic optimal power control for CDMA systems and its suboptimal approximation. By introducing individual costs for each agent (mobile) to replace the centralized cost in the stochastic optimal control framework, we present a game theoretic formulation for the power control problem. A comparison between the two approaches would be of interest and will be investigated in future work.

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