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REDUCED ORDER ADAPTIVE OBSERVER BACKSTEPPING

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Abstract:
It is the intent of this paper to add to, and improve the existing design tools for nonlinear output feedback systems. Considered here are systems, exhibiting parametric uncertainty, and containing nonlinearities dependent on the output alone. A lower triangular state transformation is utilised, equivalent to that obtained via the traditional and recursive backstepping algorithm. Extra damping is introduced to counter the uncertainty and observer error effects. The benefits of this approach lie in the significant reduction in total controller dynamic order, as well as the simplicity and clarity provided by implementing backstepping from a purely structural perspective. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The introduction of backstepping, and subsequently, adaptive backstepping, over a decade ago, provided a great step forward, including the possibility of solving many more control problems, particularly for nonlinear systems satisfying a triangular structure condition (Krstić *et al.*, 1995).

Early adaptive state feedback results exhibited overparametrisation, a trend which was removed by the introduction of *tuning functions* (see also (Krstić *et al.*, 1995)). For output feedback backstepping designs, overparametrisation was also a characteristic early on in (Marino and Tomei, 1993) and (Kanellakopoulos *et al.*, 1991). Not unlike state feedback designs, the tuning functions method was also used for output feedback systems in (Krstić *et al.*, 1994), and (Krstić and Kokotović, 1994), to remove the overparametrisation. Two approaches were developed which used the tuning functions idea: *K-Filters* (Kreisselmeier, 1977), and *MT-Filters* (Marino and Tomei, 1993).

Both methods introduced a series of filters, often including large matrix filters, to observe and, in the case of MT-Filters, transform the system. The more unknown parameters that are present, the larger the size of the matrix filter. The controller, in its entirety, can therefore potentially be quite large in dynamic order. Using K-Filters for example, the total dynamic order of the filters alone (not including adaption and without uncertainty on the input coefficients) can be minimally $(n-1)(q+1)$, where n is the state dimension, and q is the size of the uncertainty vector. For MT-Filters, a similar order applies, however plus the order of a separate observer.

In addition to the large dynamic order of the controller, the methods tend to be quite complicated, particularly when implemented by the traditional recursive backstepping approach.

The method presented here significantly reduces the total dynamic order of the controller and simplifies the design. In fact, the large matrix filters traditionally used are not required in this design. The price paid for such a design is a more conservative controller, guaranteeing robust stability of the output and state. Asymptotic stability of the state and output requires the nonlinearities to vanish at the origin. The result is presented for systems where parametric uncertainty exists in the nonlinear output feedback terms only. It should be stated that systems considered in the literature also allow lower relative degrees and uncertainty in the input coefficients. These systems are not considered here for simplicity and clarity, and also because existing techniques, such as those contained in (Krstić *et al.*, 1995), can be applied to extend this method so as to handle this class of systems.

The design utilises a lower triangular state transformation, equivalent to that which would be obtained via a recursive backstepping algorithm. This affords a more structural approach, and it is hoped, a clearer solution to the problem.

1.1 *Notation*

Some terminology listed below is introduced in (Clements and Jiang, 1999) but is repeated here for convenience.

- A vector or matrix function f of x has *lower triangular dependence in x* if the i-th component or row is a function of $x_1,\ldots,x_i$ only. Then f is said to be **LTd in x** or $f \in LTd(x)$.
- A vector or matrix function f of x has *strictly lower triangular dependence in x* if the i-th component or row is a function of $x_1,\ldots,x_{i-1}$ only. Then f is said to be **SLTd in x** or $f \in SLTd(x)$.
- With I_n an n-th order identity matrix, denote an n^{th} order shift matrix $N_n \in \Re^{nxn}$ and input matrix $b_n \in \Re^n$ as:

$$N_n = \begin{bmatrix} \mathbf{0} & I_{n-1} \\ 0 & \mathbf{0} \end{bmatrix}, \quad b_n = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}. \tag{1}$$

Further, introduce the following definitions:

Definition 1.1. For a matrix $A \in \Re^{nxm}$, define the matrix quantity

$$|A|_p = \left(\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}(a_{ij})^2}\right)^p. \tag{2}$$

Definition 1.2. For a matrix A with rows a_i^T, $(i = 1,\ldots,n)$, and a positive integer p, define a positive diagonal matrix $\mathcal{D}_p[A]$ to be

$$\mathcal{D}_p[A] = \begin{bmatrix} ||a_1||^p & & \\ & \ddots & \\ & & ||a_n||^p \end{bmatrix}. \tag{3}$$

2. SYSTEM DESCRIPTION

Consider the class of uncertain nonlinear systems with the following description:

$$\begin{aligned} \dot{x} &= N_n x + \phi(y) + \Phi^T(y)\theta + b_n u \\ y &= c_n^T x \end{aligned} \tag{4}$$

where $x \in \Re^n$ is the state, unavailable for feedback, $u, y \in \Re$ are the input and output respectively, θ is an unknown but constant parameter vector, and ϕ and Φ^T are known vector and matrix functions of the output respectively. Assume the following:

Assumption 2.1. The matrix function $\Phi^T(y)$ satisfies $\Phi^T(0) = \Phi_0^T$ for some constant matrix Φ_0^T. Further, with $\bar{\Phi}(y) = \Phi(y) - \Phi_0$, and for continuous, smooth functions $\bar{\Psi}(y)$, and $\bar{\psi}_i$, $1 \le i \le n$,

$$\begin{aligned} |\Phi(y) - \Phi_0|_2 = |\bar{\Phi}(y)|_2 &\equiv y^2\bar{\Psi}(y) \\ ||\Phi_i(y) - \Phi_{0,i}||^2 = ||\bar{\Phi}_i(y)||^2 &\equiv y^2\bar{\psi}_i(y) \end{aligned} \tag{5}$$

where $\Phi_i^T(y)$ and $\Phi_{0,i}^T$ are the i-th rows of $\Phi^T(y)$ and Φ_0^T respectively.

3. THE CONTROLLER DESIGN

As the state is unavailable, a state estimate, $\hat{x}$, is introduced, and observer dynamics given by

$$\begin{aligned} \dot{\hat{x}} &= N_n\hat{x} + \phi(y) + k(y - \hat{y}) + b_n u \\ \hat{y} &= c_n^T\hat{x}. \end{aligned} \tag{6}$$

The gain k is chosen such that $A_0 = N_n - kc_n^T$ is a stable matrix. Using (6), the state estimation error $\tilde{x} = x - \hat{x}$, has the following dynamics

$$\dot{\tilde{x}} = A_0\tilde{x} + \Phi^T(y)\theta. \tag{7}$$

That is, a stable filter driven by the unknown θ. Now, introduce a new state z, and auxiliary input v, via a state transformation and full state feedback

$$z = \hat{x} + N_n^T f(\hat{x}, y, \hat{\beta}) + c_n(y - \hat{y}) \tag{8}$$

$$v = u + b_n^T f(\hat{x}, y, \hat{\beta}) \tag{9}$$

where $\hat{\beta}$ is an estimate of the unknown parameter $\beta = ||\theta||^2$, and f is a vector function to be determined. It is assumed that $f \in LTd(\hat{x})$, however no structure is assumed in y or $\hat{\beta}$.

By denoting the quantities $F_x(\hat{x}, y, \hat{\beta}) = \nabla_{\hat{x}} f(\hat{x}, y, \hat{\beta})$, $F_y(\hat{x}, y, \hat{\beta}) = \nabla_y f(\hat{x}, y, \hat{\beta})$, and $F_\beta(\hat{x}, y, \hat{\beta}) = \nabla_{\hat{\beta}} f(\hat{x}, y, \hat{\beta})$, and ignoring function dependence for simplicity, the z-dynamics are obtained by differentiating (8),

$$\begin{aligned} \dot{z} &= \dot{\hat{x}} + N_n^T(F_x\dot{\hat{x}} + F_y\dot{y} + F_\beta\dot{\hat{\beta}}) + c_n c_n^T\dot{\tilde{x}} \\ &= N_n z - f + \mathcal{X} + \mathcal{Y}c_n^T\Phi^T\theta + \mathcal{Z}\tilde{x} + \mathcal{W}\dot{\hat{\beta}} \end{aligned} \tag{10}$$

with $v = 0$, and where the terms $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{W} \in LTd(\hat{x}), SLTd(f)$ are defined as

$$\begin{aligned}\mathcal{X}(\hat{x},y,\hat{\beta},f) &= N_n^T F_x(N_n\hat{x}+\phi+k(y-\hat{y}))\\ &\quad +(\phi+k(y-\hat{y}))+N_n^T F_y c_n^T(N_n\hat{x}+\phi)\\ \mathcal{Y}(\hat{x},y,\hat{\beta},f) &= N_n^T F_y + c_n \\ \mathcal{Z}(\hat{x},y,\hat{\beta},f) &= N_n^T F_y c_n^T N_n + c_n c_n^T A_0\\ \mathcal{W}(\hat{x},y,\hat{\beta},f) &= N_n^T F_\beta\end{aligned} \tag{11}$$

From (10), the z-dynamics are dependent on the unknowns θ and $\tilde{x}$, and the yet to be determined update law $\dot{\hat{\beta}}$. A candidate Lyapunov function is introduced,

$$V(z,\tilde{\beta},\tilde{x}) = \frac{1}{2}z^T z + \frac{1}{2}\tilde{\beta}^2 + \frac{1}{2}\tilde{x}^T P\tilde{x} \tag{12}$$

where $\tilde{\beta} = \beta - \hat{\beta}$ is the parameter estimate error, and P is a positive definite matrix which satisfies $PA_0 + A_0^T P \leq -Q < 0$, for some Q.

The derivative of V is then given by

$$\begin{aligned}\dot{V}(z,\tilde{\beta},\tilde{x}) &\leq z^T\left(N_n z - f + \mathcal{X} + \mathcal{Y}\Phi_1^T\theta\right.\\ &\quad \left.+\mathcal{Z}\tilde{x} + \mathcal{W}\dot{\hat{\beta}}\right) - \frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T P\Phi^T\theta\end{aligned} \tag{13}$$

with $\Phi_1^T(y)$ denoting the first row of $\Phi^T(y)$. The product terms with respect to the unknown θ and $\tilde{x}$, can be appropriately bounded using Young's inequality. For positive constants ε and ε_0, then with some manipulation, we have

$$\begin{aligned} z^T\mathcal{Y}\Phi_1^T\theta &= z^T\mathcal{Y}\theta^T\bar{\Phi}_1 + z^T\mathcal{Y}\theta^T\Phi_{0,1} + \mu_1\\ &\leq \frac{\beta}{2}(\varepsilon+\varepsilon_0)z^T\mathcal{D}_2[\mathcal{Y}]z + \frac{n}{2\varepsilon}z^T c_n c_n^T\bar{\psi}_1(y)z\\ z^T\mathcal{Z}\tilde{x} &\leq \frac{\varepsilon}{2}z^T\mathcal{D}_2[\mathcal{Z}]z + \frac{n}{2\varepsilon}\tilde{x}^T\tilde{x}\\ \tilde{x}^T P\Phi^T\theta &= \sum_{i=1}^n\sum_{j=1}^n \tilde{x}_i p_{ij}\Phi_j^T\theta\\ &\leq \frac{n}{\varepsilon_0}\tilde{x}^T\tilde{x} + \mu_2 + \frac{\beta\varepsilon_0}{2}z^T c_n c_n^T\tilde{\psi}(y,P)z\end{aligned} \tag{14}$$

with

$$\begin{aligned}\tilde{\psi}(y,P) &= \sum_{i=1}^n \bar{p}_i\bar{\psi}_i(y)\\ \bar{p}_i &= \sum_{j=1}^n p_{ij}^2.\end{aligned} \tag{15}$$

Here, $\mu_1 = \dfrac{n}{2\varepsilon_0}\|\Phi_{0,1}\|^2$ and $\mu_2 = \dfrac{\beta\varepsilon_0}{2}\sum_{i=1}^n \bar{p}_i\|\Phi_{0,j}\|^2$ are constants, p_{ij} is the (i,j)-th element of P, and we have used $y = z_1 = z^T c_n$. Also importantly, it is easily confirmed that $\mathcal{D}_2[\mathcal{Y}], \mathcal{D}_2[\mathcal{Z}] \in LTd(\hat{x}), SLTd(f)$.

Now, returning to the Lyapunov function derivative, $\dot{V}$ now satisfies

$$\begin{aligned}&\dot{V}\;(z,\tilde{\beta},\tilde{x})\\ &\leq z^T\left[N_n z - f + \mathcal{X} + \frac{1}{2}\hat{\beta}\left((\varepsilon+\varepsilon_0)\mathcal{D}_2[\mathcal{Y}]\right.\right.\\ &\quad\left. +\varepsilon_0 c_n c_n^T\tilde{\psi}\right)z + \frac{n}{2\varepsilon}c_n c_n^T\bar{\psi}_1 z + \frac{\varepsilon}{2}\mathcal{D}_2[\mathcal{Z}]z\\ &\quad\left.+\mathcal{W}\dot{\hat{\beta}}\right] + \tilde{\beta}\left[\dot{\hat{\beta}} + \frac{1}{2}z^T\left((\varepsilon+\varepsilon_0)\mathcal{D}_2[\mathcal{Y}]\right.\right.\\ &\quad\left.\left.+\varepsilon_0 c_n c_n^T\tilde{\psi}\right)z\right] - \frac{1}{2}\left(q_m - \frac{2n}{\varepsilon_0} - \frac{n}{\varepsilon}\right)\tilde{x}^T\tilde{x}\end{aligned} \tag{16}$$

where q_m denotes the smallest eigenvalue of Q. The inequality (16) suggests the following parameter update law

$$\begin{aligned}\dot{\hat{\beta}} &= \frac{1}{2}z^T\left((\varepsilon+\varepsilon_0)\mathcal{D}_2[\mathcal{Y}] + \varepsilon_0 c_n c_n^T\tilde{\psi}\right)z - \sigma\hat{\beta}\\ &= z^T S(\hat{x},y,\hat{\beta},f)z - \sigma\hat{\beta}\end{aligned} \tag{17}$$

where $S = \frac{1}{2}((\varepsilon+\varepsilon_0)\mathcal{D}_2[\mathcal{Y}] + \varepsilon_0 c_n c_n^T\tilde{\psi})$, and σ is a positive constant. The inclusion of the $\sigma\hat{\beta}$ is a form of σ-modification.

With such a choice of update law, and with $\lambda = \frac{1}{2}(q_m - \frac{2n}{\varepsilon_0} - \frac{n}{\varepsilon})$, then from (16),

$$\begin{aligned}&\dot{V}\;(z,\tilde{\beta},\tilde{x})\\ &\leq z^T\left[N_n z - f + \mathcal{X} + \hat{\beta}Sz + \frac{n}{2\varepsilon}c_n c_n^T\bar{\psi}_1 z\right.\\ &\quad\left.+\frac{\varepsilon}{2}\mathcal{D}_2[\mathcal{Z}]z + \mathcal{W}z^T Sz\right] + \sigma\tilde{\beta}\hat{\beta} - \lambda\tilde{x}^T\tilde{x}\end{aligned} \tag{18}$$

To use the vector function f to cancel some of the terms multiplying z^T in (18), each term must have lower triangular structure. The product term $\mathcal{W}z^T Sz$ however does not have the required lower triangular structure necessary for cancellation. It is noted however that the term has "symmetric" lower triangular structure, so it can be split into its lower triangular part, $\mathcal{W}_l$, and strict upper triangular part, $\mathcal{W}_u$, such that

$$\mathcal{W}z^T Sz = \mathcal{W}_l z + \mathcal{W}_u z \tag{19}$$

where it can be confirmed that both $\mathcal{W}_l$ and $\mathcal{W}_u^T$ are LTd in $\hat{x}$ and SLTd in f. Defining a positive definite diagonal matrix $C(\hat{x},y) \in LTd(\hat{x})$, (18) becomes

$$\begin{aligned}\dot{V}(z,\tilde{\beta},\tilde{x}) &\leq z^T\left[N_n z - N_n^T z - Cz - f + \mathcal{H}\right.\\ &\quad\left.+\mathcal{W}_u z - \mathcal{W}_u^T z\right] - \lambda\tilde{x}^T\tilde{x}\end{aligned} \tag{20}$$

with $\mathcal{H}$ defined as

$$\begin{aligned}&\mathcal{H}\;(\hat{x},y,\hat{\beta},f) = N_n^T z + C(\hat{x},y)z + \mathcal{X}(\hat{x},y,\hat{\beta},f)\\ &\quad+\hat{\beta}Sz + \frac{n}{2\varepsilon}c_n c_n^T\bar{\psi}_1 z + \frac{\varepsilon}{2}\mathcal{D}_2[\mathcal{Z}]z\\ &\quad+\mathcal{W}_l(\hat{x},y,\hat{\beta},f) + \mathcal{W}_u^T(\hat{x},y,\hat{\beta},f)\end{aligned} \tag{21}$$

Finally, with $\mathcal{H} \in LTd(\hat{x}), SLTd(f)$, the function f is chosen such that $f(\hat{x},y,\hat{\beta}) = \mathcal{H}(\hat{x},y,\hat{\beta},f)$ has a unique solution and the transformation is completely defined.

The Lyapunov function derivative now satisfies the inequality

$$\begin{aligned}\dot{V}\ (z,\tilde{\beta},\tilde{x}) &\le -z^T C(\hat{x},y)z - \sigma\tilde{\beta}^2 - \lambda\tilde{x}^T\tilde{x} + \sigma\tilde{\beta}\beta \\ &\quad + \mu_1 + \mu_2 \\ &\le -\lambda_0 V(z,\tilde{\beta},\tilde{x}) + \frac{\sigma}{2}\beta^2 + \mu_1 + \mu_2\end{aligned} \tag{22}$$

with $\lambda_0 = \min\{c_{min}, \frac{\sigma}{2}, \frac{\lambda}{P_{max}}\}$ and c_{min} and P_{max} the minimum and maximum eigenvalues of C and P respectively.

Provided ε and ε_0 are chosen sufficiently large, such that $\lambda = \frac{1}{2}(q_m - \frac{2n}{\varepsilon_0} - \frac{n}{\varepsilon}) > 0$, then the inequality (22) is sufficient to ensure robust stability of the signals z, $\tilde{\beta}$, and $\tilde{x}$. In the event that the nonlinearity $\Phi^T(y)$ vanishes at the origin, we can discard the σ-modification by setting $\sigma = 0$. This achieves global stability of the states z, $\tilde{\beta}$, and $\tilde{x}$. Also under these conditions, from LaSalle's invariance theorem, we can further ascertain that both z and $\tilde{x}$ are asymptotically stable, so that $\lim_{t\to\infty} z(t) = 0$, and $\lim_{t\to\infty}\tilde{x}(t) = 0$. As $y = z_1$, we also then have the asymptotic stability of the output y, $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} x_1(t) = 0$. With $\phi(0)$ bounded, the system description implies that the original state is also bounded. As the state estimate converges to the true state, then $\hat{x}$ is also bounded. If we further assume that $\phi(0) = 0$, then it can be confirmed that the original state and thus the estimate is asymptotically stable. With further use of LaSalle's invariance theorem, it is not difficult to confirm that convergence of the parameter estimate to the true parameter however, cannot be guaranteed.

Alternatively, for $\Phi^T(0) \neq 0$, (22) ensures that the Lyapunov function V converges to the set

$$|V(z,\tilde{\beta},\tilde{x})| \le \frac{1}{2\lambda_0}\left(\sigma\beta^2 + 2(\mu_1+\mu_2)\right) \tag{23}$$

which given that $y^2 \le 2V$, then implies that the output converges to the set

$$|y| \le \sqrt{\frac{1}{\lambda_0}(\sigma\beta^2 + 2(\mu_1+\mu_2))}. \tag{24}$$

From (24), an appropriate choice of λ_0, dependent directly on design parameters, allows the output to converge arbitrarily close to the origin.

4. EXAMPLE

Consider the output regulation of a simple second order system with unknown constants θ_1 and θ_2, and with positive integer $r > 0$.

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta_1 y^r \\ \dot{x}_2 &= u + \theta_2 y\cos y \\ y &= x_1\end{aligned} \tag{25}$$

The matrix Φ^T is defined as

$$\Phi^T(y) = \begin{bmatrix} y^r & 0 \\ 0 & y\cos y\end{bmatrix} \tag{26}$$

and satisfies $\Phi^T(0) = 0$. With $s = 2(r-1)$, from Assumption 2.1

$$\begin{aligned}\bar{\psi}_1(y) &= y^s, \quad \bar{\psi}_2(y) = \cos^2 y \\ \bar{\Psi}(y) &= y^s + \cos^2 y.\end{aligned} \tag{27}$$

The observer and state transformation is given by the following equations:

$$\begin{aligned}\dot{\hat{x}} &= N_2\hat{x} + k(y-\hat{y}) + b_2 u \\ z_1 &= y \\ z_2 &= \hat{x}_2 + f_1(\hat{x}_1, y, \hat{\beta}) \\ v &= u + f_2(\hat{x}_1,\hat{x}_2,y,\hat{\beta}).\end{aligned} \tag{28}$$

with $k = [k_1 \quad k_2]^T$ and with the vector function f defined as:

$$\begin{aligned}& f_1(\hat{x}_1,y,\hat{\beta}) = (c_1 + k_1 + \frac{\varepsilon}{2})y - k_1\hat{y} + \frac{n}{2\varepsilon}y^{s+1} \\ & + \frac{\hat{\beta}\varepsilon}{2}\left(y + \bar{p}_1 y^{s+1} + \bar{p}_2 y\cos^2 y\right) \\ & f_2(\hat{x}_1,\hat{x}_2,y,\hat{\beta}) = y + c_2 z_2 + (k_2 - k_1^2)(y-\hat{y}) \\ & - k_1\hat{x}_2 - \frac{\partial f_1}{\partial y}\hat{x}_2 + (\hat{\beta}+1)\frac{\varepsilon}{2}z_2\left(\frac{\partial f_1}{\partial y}\right)^2 \\ & + \frac{\varepsilon}{2}y^2\left(1 + \bar{p}_1 y^s + \bar{p}_2\cos^2 y\right)\left(\frac{\partial f_1}{\partial\hat{\beta}}\right) \\ & + \frac{\varepsilon}{2}z_2^2\left(\frac{\partial f_1}{\partial\hat{\beta}}\right)\left(\frac{\partial f_1}{\partial y}\right)^2\end{aligned} \tag{29}$$

with $c_1, c_2, \varepsilon > 0$ positive constants, and with $\bar{p}_1 = ||P_1||^2$ and $\bar{p}_2 = ||P_2||^2$ denoting the 2-norm of the first and second row respectively, of a positive definite matrix P. The control law and parameter update law is then given by

$$u = -f_2(\hat{x}_1,\hat{x}_2,y,\hat{\beta}) \tag{30}$$

$$\dot{\hat{\beta}} = \frac{\varepsilon}{2}\left[y^2(1 + \bar{p}_1 y^s + \bar{p}_2\cos^2 y) + z_2^2\left(\frac{\partial f_1}{\partial y}\right)^2\right] \tag{31}$$

Notice that as the matrix Φ vanishes at the origin, there is no need for the σ-modification term in the update law, and hence $\sigma = 0$.

For simulation of the above system, the parameters and design constants in Table 1 are used. From Table 1, the true parameter $\beta = ||\theta||^2 = 0.26$, and with the gain k chosen as in Table 5.1, the eigenvalues of $A_0 = N_2 - kc_2^T$ are $\{-1,-1\}$.

Upon simulation, we allow the nonlinearity index r and the initial condition of the observer to vary:

System	
θ_1	0.1
θ_2	0.5
r	*variable*
Observer	
k	$[2 \quad 1]^T$
Q	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
P	$\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1.5 \end{bmatrix}$
Controller	
C	$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$
ε	4
Initial Conditions	
$y(0)$	1.5
$\hat{y}(0)$	*variable*

Table 1. Parameters and Design Constants

Case 1: $r = 1$, $\hat{y}(0) = 0$: The simulation results for this case are shown in Figures 1 and 2.

Case 2: $r = 2$, $\hat{y}(0) = 0$: The simulation results for this case are shown in Figures 3 and 4.

Case 3: $r = 2$, $\hat{y}(0) = y(0)$: The simulation results for this case are shown in Figures 5 and 6.

Note the reduced time scale in the figures showing the control effort.

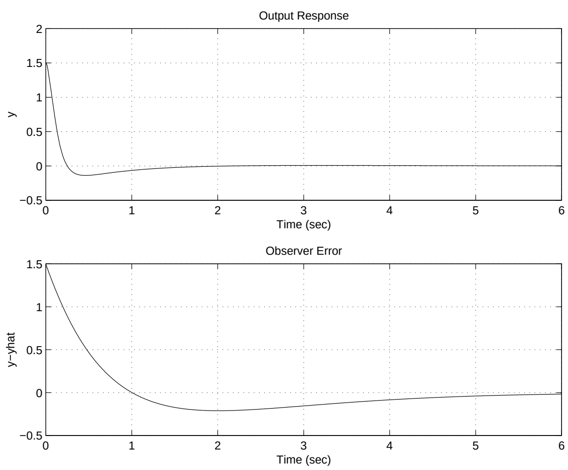

Fig. 1. $r = 1$ and $\hat{y}(0) = 0$

A significant characteristic of the simulation is the quite complicated control law expression that results, considering the order of the system. This unfortunately manifests itself in large initial control effort.

There is also a significant increase in the initial control effort required when changing from $r = 1$ to $r = 2$, that is, changing from a linear growth to quadratic growth. Althouth, at the same time, the control signal settles significantly quicker for the later case. The initial control effort is reduced again when matching the initial conditions of the observer to that of the system (although the control effort is still markedly larger than for $r = 1$). For this third case, the observer error remains quite small as the initial error is zero. It

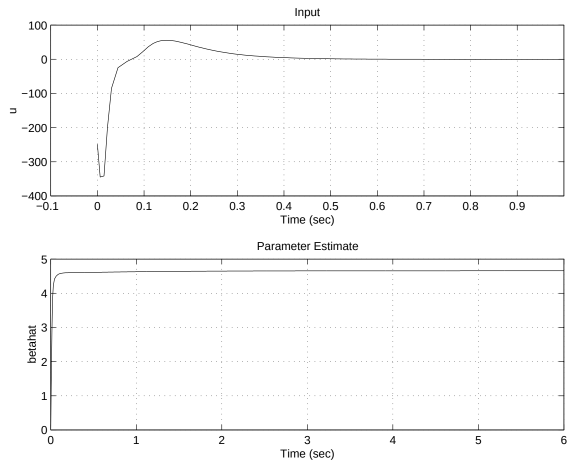

Fig. 2. $r = 1$ and $\hat{y}(0) = 0$

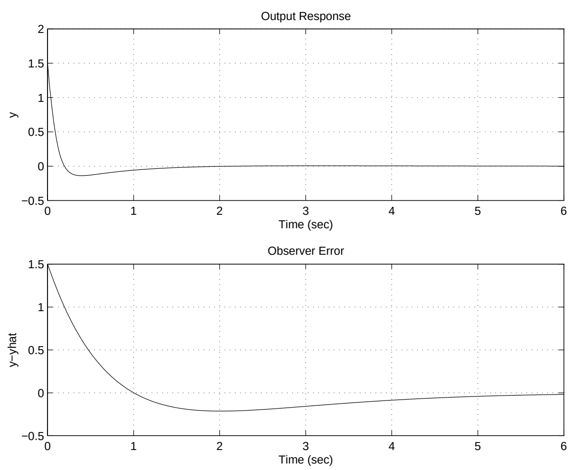

Fig. 3. $r = 2$ and $\hat{y}(0) = 0$

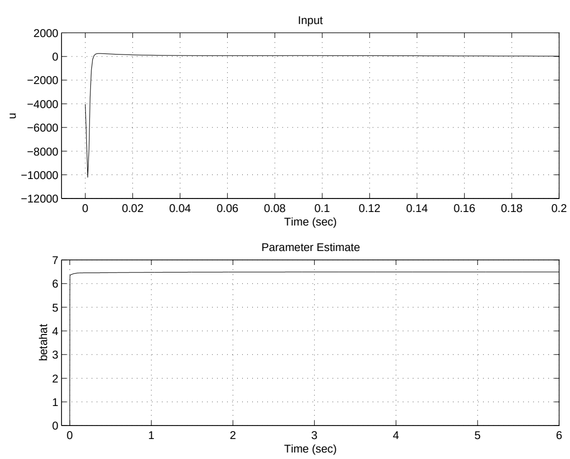

Fig. 4. $r = 2$ and $\hat{y}(0) = 0$

is also interesting to note that the parameter estimate converges to different values for each simulation, and not to the true parameter value. Once again, stability analysis shows that the controller cannot gaurantee convergence of the parameter estimate error to zero.

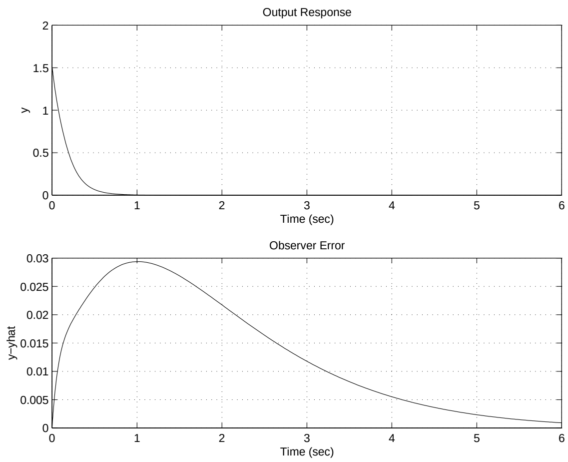

Fig. 5. $r = 2$ and $\hat{y}(0) = y(0)$

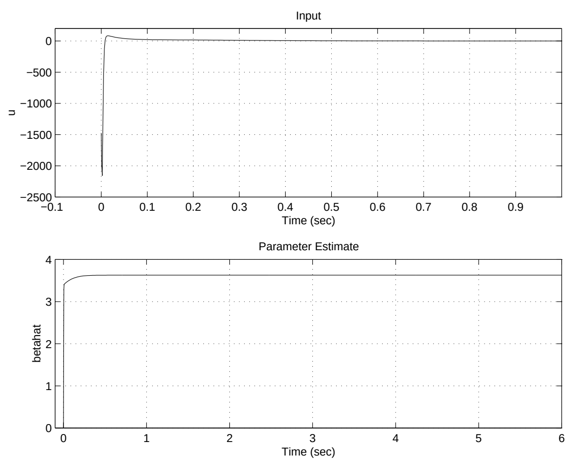

Fig. 6. $r = 2$ and $\hat{y}(0) = y(0)$

5. DISCUSSION

The method described offers a significant reduction in the total controller order. The entire controller consists of the state estimator, and the parameter update law, which gives a total controller dynamic order of 3. In comparison, for an adaptive backstepping solution using MT-Filters, the controller dynamics include the matrix filter, a state estimator, and a parameter update law for the whole parameter vector, resulting in a total controller order of 7.

6. CONCLUSION

A new tool for adaptive output feedback design using a structural, or state transformation approach to backstepping, has been demonstrated. This approach is conceptually simpler, and provides a clearer perspective of how backstepping achieves its goals. The controller is constructed by simply replacing those unknown and uncertain terms in the Lyapunov function derivative with bounding terms.

The class of systems considered here is not an extension of those considered already in the literature, but in fact a subclass. The method used however, offers an alternative design, which also provides a degree of improvement over existing approaches. The improvement is not only in the simplicity of the method, but importantly, the significant reduction in the controller dynamic order.

The large matrix filters required in existing designs, such as those which employ MT-Filters or K-Filters, are not required. As well, parametrisation is also minimised, whereby only a single scalar parameter estimate is required. Particularly for systems with several unknown parameters, this reduction in the total controller dynamic order is substantial and thus significant.

The price paid for this dramatic reduction is first of all, a requirement that for asymptotic stability of the output, the nonlinearities that multiply the unknown parameter vector must vanish at the origin. With reduced dynamic order, also comes a more conservative controller. But perhaps most importantly, as was seen in the simulation example, the resulting control law is potentially quite complicated, producing also potentially very high control activity.

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