

## ROBUST ADAPTIVE PREDICTIVE CONTROL OF WATER DISTRIBUTION IN IRRIGATION CANALS

Rivas Pérez R. \*, Prada Moraga C. \*\*, Perán González J.R. \*\*, Kovalenko P.I. \*

\* *Department of Automatica and Computer Science, Havana Polytechnic University, 127 S/N, CUJAE, Marianao, C. Habana, Cuba, C.P. 11500, [rivas@electronica.ispjae.edu.cu](mailto:rivas@electronica.ispjae.edu.cu)*

\*\* *Department of Systems Engineering and Automatica, University of Valladolid, 47011 Valladolid, Spain, [prada@autom.uva.es](mailto:prada@autom.uva.es)*

**Abstract:** Applying automatic control methods to irrigation canals is a way to improve the management of irrigation systems. The dynamic difficulties that involved this hydraulic systems such as non-linearity, varying time delay, and uncontrolled perturbations make the choice of a suitable automatic control method a challenge. To this purpose, this paper presents a methodology for design of decentralized robust adaptive predictive control system of water distribution in irrigation canals. The objective of this system is to maintain the downstream end water level of each pool at constant target value under external uncontrolled perturbation.

**Keywords:** Predictive Control, Adaptive Control, Robust Control, Robust Identification, Irrigation Canals Control, Water Distribution.

### 1. INTRODUCTION

Automatic control of water distribution in irrigation open canals is becoming increasingly obvious due to necessity of improving water efficiency, to reduce water losses, to supply water users in due time, as well as to protect the environment and to augment the agricultural production to satisfy the growing necessities of food. Nowadays most of irrigation canals are manually controlled (Malaterre, 1998).

Significant research efforts has been recognized in the literature in the development of new automatic control methods for open canals operation (Akouz et al., 1996; Beauchamb Baez et al., 1998; Cardona et al., 1997; Malaterre et al., 1998; Rivas Pérez et al.,

2000; Rodellar et al., 1993). Many authors have proposed different techniques and strategies either monovariable and multivariable, downstream and upstream or centralized and decentralized methods.

One approach followed in the literature (which will be used in this paper), consists in stating the control problem of water distribution in the canal as if it was made of a number of subsystems, controlling each one of them in a monovariable setting. Recently predictive control strategy was considered by several authors (Akouz et al., 1998; Cardona et al., 1997; Malaterre, 1997; Sawadogo et al., 1998).

Designing a control strategy leading to a practical controller is a arduous task because the hydraulic

behavior of irrigation canals shows that these systems are distributed over long distances, with a dynamic characterized by important varying time delay, strong nonlinearity, numerous interactions between different consecutive sub-systems and the existence of others dynamics parameters that change over time during operation (Malaterre, 1998; Rivas Pérez et al. 1998). Also different structures, like gates, intakes/offtakes, are placed along the canal at particular positions, interacting with the natural dynamics of the canals.

Thus, the whole canal has to be regarded as a system with complex dynamical behavior (Rodellar et al., 1993). A good knowledge of system dynamic is needed to design an effective automatic controller for irrigation canals.

Many studies have shown that classical regulators such as the PID controller seem to be unsuitable to solve the problem of effective control of water distribution in irrigation canals due to the complex dynamical behavior that characterize to these systems (Malaterre et al., 1998).

Model Based Predictive Controllers (MBPC) have been previously applied successfully to solve industrial problems control of process whit complex dynamical behavior (Camacho and Bordons, 1999; Clarke, 1994; Richalet, 1989). There are a few published results on the application of Predictive Control in irrigation systems.

The necessity to have a good process model constitutes one of the inconveniences that the MBPC presents. Obviously, the performance of the predictive control systems will depend significantly of the precision made by the model (Martin Sanchez, 1996).

It is well-known that in the irrigation systems exist many canals whose dynamic parameters vary in the time in a non predictable way and in a wide range around the operation point (Beauchamb Baez et al., 1998; Rivas Pérez et al., 2000). This implies that the application of the MBPC leads to no satisfactory predictions of the process output due to unadjusted model parameters. This originates an imprecise calculation of the control signal.

In these cases it would be convenient to have an adaptive predictive controller that allows to adjust the parameter of the model from the error in comparison between process and model outputs.

The presence of process model uncertainties, external disturbances and noise can influence significantly in the effectiveness of the adaptive predictive

controllers. Of here the importance of robustness analysis of this class of controllers.

In this paper a methodology for the design of decentralized robust adaptive predictive control system of water distribution in irrigation canals is presented, which facilitates to solve the problem of the effective control of processes under consideration.

## 2. MATHEMATICAL MODEL OF IRRIGATION CANALS

An typical irrigation canal is an open water hydraulic system whose mainly objective is to convey water from is source down to is final users (farmers). This system is integrated by several pools separated by cross structures (mainly hydraulic gate), that are operated for regulating the water levels (flows), discharges and/or volumes from one pool to the next one (see Fig. 1).

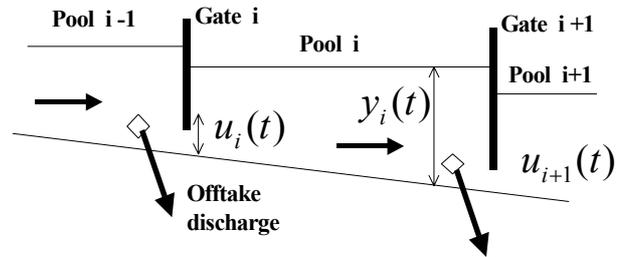


Fig. 1. Schematic of an open irrigation canal.

The dynamical behavior of irrigation canals can be correctly approximated by Saint-Venant's equations (Saint-Venant, 1891), which are nonlinear partial derivative hyperbolic equations (distributed model) and are given by:

$$\begin{aligned} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} &= q; \\ \frac{\partial Q}{\partial t} + \frac{\partial Q^2 / A}{\partial x} + gA \frac{\partial z}{\partial x} &= -gAS_f, \end{aligned} \quad (1)$$

where:

A - cross section area; t - time; Q - discharge; x - longitudinal abscissa in the direction of the flow; q - lateral inflow or outflow; g - gravity acceleration; z - water surface absolute elevation;  $S_f$  - friction slope.

Nowadays different methods exist for the solution of Saint-Venant's equations, however the common for all they, are the serious mathematical complexities that the same ones present. These equations are also very difficult to use for the prediction and control.

In this paper we present a simplified discrete-time model for a one-pool of irrigation canal, that reflects in adequate form the dynamic behavior of this process in the mensuration point of water level (the downstream end of the pool), which was obtained using system identification and present the following CARIMA structure:

$$A(q^{-1})y(t) = B(q^{-1})u(t-d-1) + \frac{T_c(q^{-1})}{\Delta} \xi(t), \quad (2)$$

where:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{na} q^{-na};$$

$$B(q^{-1}) = b_1 q^{-1} + \dots + b_{nb} q^{-nb};$$

$T_c(q^{-1}) = 1 + t_{c1} q^{-1} + \dots + t_{cnt} q^{-nt}$  - noise coloring polynomial;  $y(t)$  - downstream water level;  $u(t)$  - gate opening;  $\xi(t)$  - uncorrelated random noise sequence whit zero mean;  $\Delta = 1 - q^{-1}$  - difference operator;  $d$  - time delay.

The model (2) it is not as universal as the Saint-Venant's equations however, it is characterized by its simplicity and possibility of being used for predictive control.

### 3. ROBUST ADAPTIVE PREDICTIVE CONTROL

The process to be controlled is a one-pool open irrigation canal receiving water from a source located upstream. The control objective is to regulate water level at the downstream end of each pool of canal by modifying upstream discharge and thus the gate opening.

The robust adaptive predictive control system of water distribution in irrigation canals that is proposed is based on the application of Generalized Predictive Controller (GPC). The GPC algorithms consist of applying a control sequence that minimizes a multistage cost function (Clark et al., 1987; Camacho and Bordons, 1999) of the form:

$$J = \sum_{j=N_1}^{N_2} [\hat{y}(t+j/t) - \omega(t+j)]^2 + \sum_{j=1}^{N_u} \lambda(j) [\Delta u(t+j-1)]^2, \quad (3)$$

where:

$\hat{y}(t+j/t)$  - optimum  $j$ -step ahead prediction of the system output on data to time  $t$ ;  $\omega(t+j)$  - desired

reference trajectory;  $N_1$  and  $N_2$  - minimum and maximum costing horizons respectively;  $N_u$  - control horizon;  $\lambda$  - weighing coefficient.

At the instant  $(t+j)$  the optimal prediction of the downstream water level is given by:

$$\hat{y}(t+j) = G_j(q^{-1})\Delta u(t+j-d-1) + F_j(q^{-1})y(t), \quad (4)$$

where:

$$G_j(q^{-1}) = E_j(q^{-1})B(q^{-1}), \quad (5)$$

$E_j(q^{-1}), F_j(q^{-1})$  - polynomials that are uniquely defined by  $A(q^{-1})$  and  $j$ , and satisfy the following Diophantine equation:

$$T_c(q^{-1}) = A(q^{-1})\Delta E_j(q^{-1}) + q^{-j}F_j(q^{-1}); \quad (6)$$

$$\deg E_j(q^{-1}) = j-1; \deg F_j(q^{-1}) = \deg A(q^{-1}).$$

$\hat{y}(t+j/t)$  for  $j \leq d+1$  depends on available data and for  $j > d+1$  depend of variable which have to be determined.

If the desired reference trajectory  $\omega(t+j)$ ,  $j=1,2,\dots$ , is available, then a set of predicted systems errors  $\mathcal{E}(t+j)$ ,  $j=1,2,\dots$ , is generated:

$$\mathcal{E}(t+j) = \hat{y}(t+j/t) - \omega(t+j), \quad j=1,2,\dots \quad (7)$$

Considering that the irrigation canals present a significant time delay  $d$ , the values  $N_1, N_2$  and  $N_u$  are defined by  $N_1 = d+1$ ,  $N_2 = d+N$  and  $N_u = N$ .

For  $j$  varying from minimum to maximum horizon is obtained the following vectorial equation:

$$\hat{y} = G\tilde{u} + p, \quad (8)$$

where:

$$\hat{y} = [\hat{y}(t+d+1/t), \dots, \hat{y}(t+d+N/t)]^T;$$

$$\tilde{u} = [\Delta u(t), \dots, \Delta u(t+N-1)]^T;$$

$$G = \begin{bmatrix} g_0 & 0 & 0 \\ g_1 & g_0 & 0 \\ \vdots & \vdots & \vdots \\ g_{N-1} & g_{N-2} & \dots & g_0 \end{bmatrix};$$

$$p = F(q^{-1})y(t) + G'(q^{-1})\Delta u(t-1).$$

The minimization of the criterion (3), assuming there are not constraints on the control signals, allows to get the analytic optimal control expression given by:

$$\tilde{u} = (G^T G + \lambda I)^{-1} G^T (w - p), \quad (9)$$

where:

$$w = [\omega(t+d+1), \dots, \omega(t+d+N)]^T.$$

The control signal that is actually sent to the process is the first element of vector  $\tilde{u}$  expressed as:

$$\Delta u(t) = K(w - p), \quad (10)$$

where:

$$K - \text{the first row of the matrix } (G^T G + \lambda I)^{-1} G^T.$$

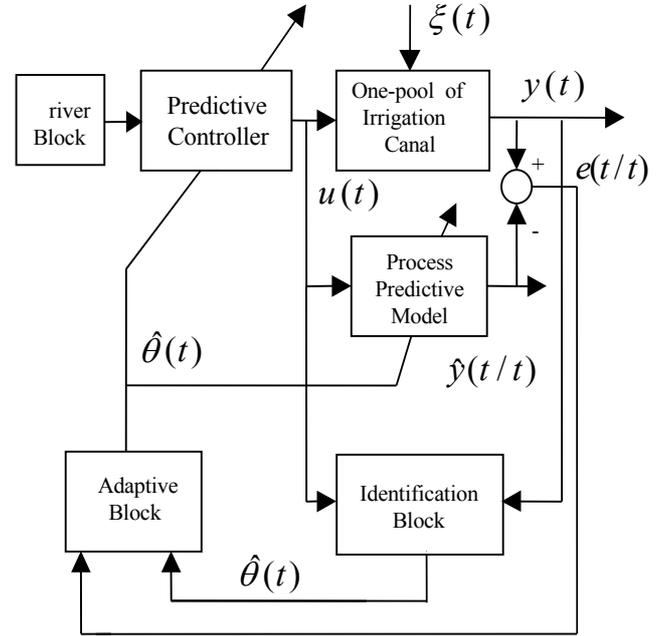
The above features of the predictive control are based on the implicit assumption that the process dynamics is known and that this knowledge is contained in the predictive model (Martin Sanchez, 1996). However, in most cases, as in the irrigation canals, is difficult to obtain precise information about the process a priori due their complex dynamic behavior. The purpose of adding an adaptive block to the predictive controller is to reach, in time varying environment, the satisfactory results that would be obtained by predictive controller if the process dynamics were known.

The adaptive control systems may become unstable due to unmodeled dynamics (Ioannou and Sun, 1988). It is for it, that this systems has to be designed to be insensitive, i.e. robust with respect to the class of plant uncertainties that are likely to be encountered in real life. Some of the researches on adaptive control has been directed towards robust estimation. Normalization and relative dead zone are among the useful ideas which have emerged (Ioannou and Sun, 1996).

Normalization is used to prevent the variables inside an estimator from growing unbounded. Relative dead zone guarantees that the estimates are bounded and convergent. The common and essential idea of both (normalization and relative dead zone) is that potentially unbounded disturbances can be bounded by an overbounding signal (Yoon and Clarke, 1993).

The block diagram of robust adaptive predictive control system of water distribution for one-pool of irrigation canals that is proposed in this paper is represented in the Fig. 2. In this system, on line recursive robust estimation of the process parameters

is developed using the information available on the process input and output up to instant  $t$ .



**Fig. 2. Block diagram of robust adaptive predictive control system of water distribution in a one-pool of irrigation canal.**

For recursive estimation the CARIMA model (2) is considered whit  $T_c(q^{-1})$  replaced by design polynomial  $T(q^{-1})$ . The resulting CARIMA model leads to the following prediction error:

$$e(t) = \frac{\Delta}{T(q^{-1})} [A(t, q^{-1})y(t) - B(t, q^{-1})u(t-d-1)]. \quad (11)$$

Equation (11) can be rewritten in the regressor form:

$$e(t) = \Delta y_f(t) - \phi(t-1)^T \hat{\theta}(t-1), \quad (12)$$

where, subscript  $f$  denotes a quantity passed through the filter  $1/T(q^{-1})$ , the parameter vector  $\hat{\theta}(t)$  comprises the coefficients of  $1 - A(t, q^{-1})$  and  $B(t, q^{-1})$ , and the regressor  $\phi(t)$  is given by:

$$\phi(t) = [\Delta y_f(t) \dots \Delta y_f(t-na+1) \Delta u_f(t-d) \dots \Delta u_f(t-nb-d)]^T. \quad (13)$$

Using the estimation error (12), the least-squares estimator whit normalization is described as:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-1)\phi(t-1)}{c_2(t)^2 + \phi(t-1)^T P(t-1)\phi(t-1)} e(t); \quad (14)$$

$$\bar{P}(t) = P(t-1) - \frac{P(t-1)\phi(t-1)\phi(t-1)^T P(t-1)}{c_2(t)^2 + \phi(t-1)^T P(t-1)\phi(t-1)}; \quad (15)$$

$$P(t) = \left(1 - \frac{\lambda_0}{\lambda_1}\right) \bar{P}(t) + \lambda_0 I, \quad (P(0) = \lambda_1 I); \quad (16)$$

$$c_2(t) = \max(m(t), 1); \quad (17)$$

$m(t)$  - bounding function;

$$m(t) = \sigma_0 m(t-1) + \sigma_1 \|\phi(t-1)\|; \quad (18)$$

$0 < \sigma_0 < 1; \sigma_1 > 1$  - design constants, which have been chosen such that:

$$|\Delta \xi_f(t)| \leq \varepsilon m(t); \quad (19)$$

$\varepsilon$  - finite constant that represent the size of unmodelled dynamics;

$\lambda_0 \geq 0; \lambda_1 > 0$  - design constant, which are introduced to keep the eigenvalues of  $P(t)$  in the interval  $[\lambda_0, \lambda_1]$ .

The adaptive block, whit the objective of transferring a bigger robustness to the system, adjust the parameters of the predictive process model only in those time intervals, in which the estimation error surpasses a certain threshold, that is to say:

$$e(t/t) = y(t) - \hat{y}(t/t); \quad (20)$$

$$e(t/t) > e_r \quad (21)$$

#### 4. SIMULATIONS RESULTS

The robust adaptive predictive control algorithms discussed previously have been implemented in C. The controller was tuned for good response varying the parameters in the expected variation range.

Simulation results presented are performed whit the following design parameters:  $N_1=1, N_2=8, N_u=1,$

$$\lambda = 0.8, \lambda_0 = 0.1, \lambda_1 = 1, \sigma_0 = 0.3, \sigma_1 = 2$$

$$T_c = T = 1 - 0.7q^{-1}.$$

Figure 3 shows the results of simulation of the proposed robust adaptive predictive control system of water distribution in a one-pool of irrigation canals. The evolutions of the water level at the downstream end of pool due the set-point variation and disturbance (offtake discharges) respectively are presented.

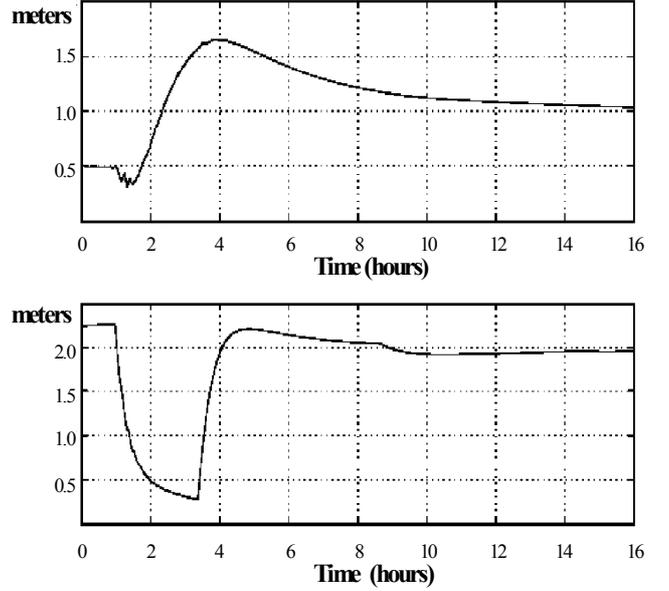


Fig. 3. Evolution of water level at the downstream end of pool.

Figure 4 shows the results of the application of proposed control system to follow an explicitly given setpoint sequence starting whit parameters at arbitrary initial values. These result clearly illustrate that the system is stable, i. e. the input/output vector  $\phi$  is bounded, the parameter converge in a finite time and the estimation error is bounded. As can be seem, the proposed system behaves well in the whole range of operation.

#### 5. CONCLUSIONS

A methodology for design a decentralized robust adaptive predictive control system of water distribution in irrigation canals has been presented. This system based on predictive control theory and uses a robust identification and adaptive methods. The control system uses the pool's downstream end water level as feedback information. The system has been validated by simulation and prove its effectiveness. Compared whit conventional controllers present a quicker and more exact evolution to target water levels and is able to deal with unknown perturbations

such as unpredicted water withdrawals. Thus, the control methodology presented in this paper seems well suited for real-time operations.

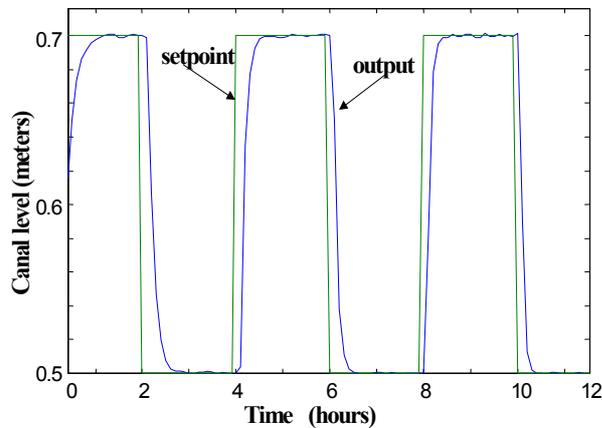


Fig. 4. Set-point and water level at the downstream end of pool.

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