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DEVELOPMENT OF A FULL ACTIVE SUSPENSION SYSTEM

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Abstract: In this work we obtain a new description of the vertical motion of the car including the active suspension. It consists of 4 *completely decoupled* subsystems. The first 3 ones are of order 4 each and they are analogous to standard quarter car suspension models. The 4th subsystem is of order 2 and it introduces a new abstract variable denoted by γ. One of the novelties of this modeling result is that the full suspension system is no more seen as 4 decoupled quarter car suspension models with sprung mass for each suspension being equal to a quarter of the total vehicle mass. *Copyright 2002 IFAC*

Keywords: Active vehicle suspension; Dynamic modelling; Automotive control; Nonlinear systems; Control systems

1. INTRODUCTION

Car active suspension has been the object of many studies in the last decade. The basic objectives of car active suspension are: increasing the passenger comfort and providing a good road handling, see for instance (Butsuen, 1989; Hrovat, 1990; Alleyne and Hedrick, 1995; Lin and Kanellakopoulos, 1997; Campos *et al.*, 1999). Most of the control strategies we find in the literature use an over-simplified model of the suspension system. In this model the vehicle is supposed to consist of 4 decoupled so-called quarter car suspension models with sprung masses equal to a quarter of the vehicle mass (Hrovat, 1990; Alleyne and Hedrick, 1995; Lin and Kanellakopoulos, 1997). The only works we have found which really address the control of the complete vehicle suspension system are (Butsuen, 1989; Campos *et al.*, 1999).

The vehicle consists of one rigid mass suspended on four contact points by means of the suspensions (dampers and springs) and the wheels. For instance, it is not clear at all how to set up the active suspension actuators in order to regulate the roll without disturbing the pitch and the heave motions.

In addition to this decoupling problem, there is a major issue in car suspension control: roughly, we have an over-actuated system with 3 degrees of freedom controlled by 4 actuators.

In this work we obtain a new description of the vertical motion of the car including the active suspension. It consists of 4 *completely decoupled* subsystems. The first 3 ones are of order 4 each and they are analogous to standard quarter car

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suspension models. The 4th subsystem is of order 2; it introduces a new abstract variable denoted by γ. The variable γ will actually be seen as not completely decoupled from the previous 3 virtual suspension variables in that the latter appear in its dynamics. But this is of no concern in general. One of the novelties of this modeling result is that the full suspension system is no more seen as 4 decoupled quarter car suspension models with sprung mass for each suspension being equal to a quarter of the total vehicle mass. We note that the latter equal sprung mass assumption is, actually, *never valid*. The fact that the sprung masses are different is precisely the reason why the center of gravity of the vehicle is not located at the geometric center of the chassis. In the new modeling we do not make such an assumption on the distribution of the vehicle mass. There is precisely no restrictive hypothesis on the vertical dynamic model of the car. The 3 fourth order subsystems of the new model may be seen as *virtual* suspensions which are truly *decoupled*, and able to describe the modification of the sprung mass distribution during vehicle accelerations. These 3 virtual suspensions are decoupled in the sense that they may be controlled independently. In particular we may use classic control laws designed for quarter car suspensions, say, "skyhook" (Butsuen, 1989). How these virtual actuators are linked to the 4 real ones? Here is where the 4th subsystem, that is, the dynamics of γ, comes into play. It insures a rational distribution of the 3 virtual suspension efforts to the 4 real ones.

This new model allows the design of control laws which satisfy the basic objectives of vehicle dynamics: passenger comfort and car handling.

The remainder of this paper is organized as follows. In the next section we derive our new full suspension model. In section 3 we provide some interpretations of the new model. In section 4 we describe a potential new suspension system with the particularity of having asymmetric front rear suspensions. The last section is devoted to numerical simulations using classical controls such as skyhook.

2. DERIVATION OF THE FULL SUSPENSION MODEL

We refer to Figure 1 for the notations. The motion of the chassis consists of the rotation about the X and Y axes and the vertical displacement along the Z axis. The frame $G\,X\,Y\,Z$ which is used to write the dynamics equations is defined as follows. The plane $G\,X\,Y$ is parallel to the road's plane. The origin of the frame, G, is the position of the vehicle's center of gravity at rest. This frame is then moving along the road but it is able to capture the rotational and vertical translation motions of the vehicle. The three variables defining these three motions are the *roll*, θ, the *pitch*, φ, and the *heave*, z.

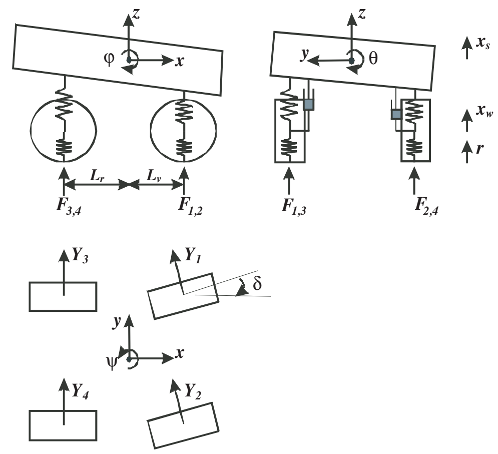

Figure 1. A schematic view of the car

Applying the laws of dynamics we obtain

$$\begin{cases} m_{\mathrm{s}}\ddot{z} = F_1 + F_2 + F_3 + F_4 + d_{\mathrm{z}} \\ I_{\mathrm{xx}}\ddot{\theta} = v_{\mathrm{f}}F_1 - v_{\mathrm{f}}F_2 + v_{\mathrm{r}}F_3 - v_{\mathrm{r}}F_4 + d_\theta \\ I_{\mathrm{yy}}\ddot{\varphi} = -L_{\mathrm{f}}F_1 - L_{\mathrm{f}}F_2 + L_{\mathrm{r}}F_3 + \\ \qquad L_{\mathrm{r}}F_4 + d_\varphi \end{cases} \tag{1}$$

where the symbols are defined in Table 1. The term d_{z} is created by the lateral and longitudinal accelerations which result from the kinematic quantities anti-roll, anti-dive and anti-squat. The terms d_θ and d_φ are kinematic moments created by the longitudinal and lateral accelerations. We shall not specify them: they are considered as perturbing terms.

The previous equations may be written in matrix form as follows

$$M\,\ddot{z}_{\mathrm{s}} = \mathrm{A}\,F + d_{\mathrm{s}} \tag{2}$$

with

$$M = \begin{pmatrix} m_{\mathrm{s}} & 0 & 0 \\ 0 & I_{\mathrm{xx}} & 0 \\ 0 & 0 & I_{\mathrm{yy}} \end{pmatrix},$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ v_{\mathrm{f}} & -v_{\mathrm{f}} & v_{\mathrm{r}} & -v_{\mathrm{r}} \\ -L_{\mathrm{f}} & -L_{\mathrm{f}} & L_{\mathrm{r}} & L_{\mathrm{r}} \end{pmatrix}.$$

$$F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}, \quad d_{\mathrm{s}} = \begin{pmatrix} d_{\mathrm{z}} \\ d_\theta \\ d_\varphi \end{pmatrix}, \quad z_{\mathrm{s}} = \begin{pmatrix} z \\ \theta \\ \varphi \end{pmatrix}.$$

The F_i's are defined by

$$F_i = -k_{\mathrm{s}}(x_{\mathrm{s},i} - x_{\mathrm{w},i}) - b_{\mathrm{s}}(\dot{x}_{\mathrm{s},i} - \dot{x}_{\mathrm{w},i}) + u_i \tag{3}$$

where

$$x_{\mathrm{s}} = \begin{pmatrix} z - v_{\mathrm{f}}\sin\theta - L_{\mathrm{f}}\sin\varphi \\ z + v_{\mathrm{f}}\sin\theta - L_{\mathrm{f}}\sin\varphi \\ z - v_{\mathrm{r}}\sin\theta + L_{\mathrm{r}}\sin\varphi \\ z + v_{\mathrm{r}}\sin\theta + L_{\mathrm{r}}\sin\varphi \end{pmatrix},$$

Symbol	Signification
m_s	Vehicle sprung mass, or chassis mass
I_{xx}	Inertia moment with respect to the GX axis
I_{yy}	Inertia moment with respect to the GY axis
F_1	Vertical force created by suspension 1 (Front left wheel)
F_2	Vertical force created by suspension 2 (Front right wheel)
F_3	Vertical force created by suspension 3 (Rear left wheel)
F_4	Vertical force created by suspension 4 (Rear right wheel)
k_s	Spring coefficient
b_s	Damping coefficient
u_i	Force created by the active suspension at the ith wheel (control input)
v_f	Half distance between front wheels
v_r	Half distance between rear wheels
x_w	Vertical displacement of the center of gravity of the wheels

Table 1. Notations

and

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} .$$

Assuming θ and φ to be small we have

$$x_s = \begin{pmatrix} z - v_f\theta - L_f\varphi \\ z + v_f\theta - L_f\varphi \\ z - v_r\theta + L_r\varphi \\ z + v_r\theta + L_r\varphi \end{pmatrix}$$

so that

$$x_s = A' z_s \tag{4}$$

where the prime denotes the transposition of matrices. Equation (2) then becomes

$$M\,\ddot{z}_s = -k_s A(x_s - x_w) - b_s A(\dot{x}_s - \dot{x}_w) + Au + d_s .$$

Given equation (4) we have

$$\begin{aligned} M\ddot{z}_s \;=\; & -b_s A\,A'\dot{z}_s - k_s A\,A' z_s + b_s A\dot{x}_w + \\ & k_s\,A\,x_w + A\,u + d_s \end{aligned} \tag{5}$$

which represents the chassis dynamics equation.

2.1 *Model of the wheels dynamics*

The wheels dynamics are given by

$$m_w \ddot{x}_w = -F - k_t(x_w - \mathrm{r})$$

with

$$x_w = \begin{pmatrix} x_{w,1} \\ x_{w,2} \\ x_{w,3} \\ x_{w,4} \end{pmatrix}, \quad r = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{pmatrix} .$$

Using the previous notations we obtain

$$\begin{aligned} m_w\ddot{x}_w \;=\; & b_s A'\dot{z}_s + k_s A' z_s - b_s\dot{x}_w - k_s x_w - \\ & u - k_t(x_w - \mathrm{r}) . \end{aligned} \tag{6}$$

2.2 *The new model of the full suspension*

The vertical motion of the vehicle is described by the combination of equations (5) and (6)

$$\left\{ \begin{aligned} M\ddot{z}_s \;=\; & -b_s A\,A'\dot{z}_s - k_s A\,A' z_s + \\ & b_s A\dot{x}_w + k_s A x_w + Au + d_s , \\ m_w\ddot{x}_w \;=\; & b_s A'\dot{z}_s + k_s A' z_s - b_s\dot{x}_w - \\ & k_s x_w - u - k_t(x_w - \mathrm{r}) . \end{aligned} \right. \tag{7}$$

These equations are the ones usually considered in the literature, see for instance (Butsuen, 1989; Ikanaga *et al.*, 2000; Campos *et al.*, 1999). They are interpreted as a dynamic system of order 14.

In this communication we present a quite new vision of these equations. The main point of this development is to rewrite them into the full suspension model consisting of

- three decoupled subsystems of order 4. Each one of those represents a standard quarter car suspension model.
- An extra subsystem of order 2 which describes the interrelation between the previous three quarter car suspensions.

We first recall the form of a quarter car suspension model.

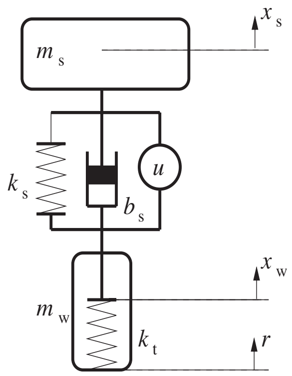

Figure 2. The quarter car suspension model

$$\left\{ \begin{aligned} m_s\,\ddot{x}_s \;=\; & -b_s\,\dot{x}_s - k_s\,x_s + b_s\,\dot{x}_w + \\ & k_s\,x_w + u + d_s \\ m_w\ddot{x}_w \;=\; & b_s\,\dot{x}_s + k_s\,x_s - b_s\dot{x}_w - \\ & k_s x_w - u - k_t(x_w - \mathrm{r}) \end{aligned} \right. \tag{8}$$

Comparing system (7) and system (8) we see that the former lacks some of the symmetries in the latter. In other words the full suspension model (7) does not present itself as an assembly of four quarter car suspension models.

As we may, we multiply the second equation in system (7) by an *invertible* matrix T to obtain

$$\begin{aligned} m_w\,T\,\ddot{x}_w \;=\; & b_s\,T\,A'\dot{z}_s + k_s\,T\,A' z_s - b_s\,T\,\dot{x}_w - \\ & k_s\,T\,x_w - T\,u - k_t\,T\,(x_w - \mathrm{r}) \end{aligned}$$

Assume for a moment that there is a real vector Γ of $\mathbb{R}^4$ such that

$$T = \begin{pmatrix} A \\ \Gamma' \end{pmatrix} \tag{9}$$

is invertible. We shall exhibit a specific Γ with that property. If we denote

$$\gamma = \Gamma' x_{\rm w}$$

the previous equation now reads as

$$\begin{cases} m_{\rm w}\, A\, \ddot{x}_{\rm w} = b_{\rm s}\, A\, A' \dot{z}_{\rm s} + k_{\rm s}\, A\, A' z_{\rm s} - b_{\rm s}\, A\, \dot{x}_{\rm w} - \\ \qquad k_{\rm s}\, A\, x_{\rm w} - A\, u - k_{\rm t}\, A\, (x_{\rm w} - {\rm r})\,, \\ m_{\rm w}\, \ddot{\gamma} = b_{\rm s}\, \Gamma' A' \dot{z}_{\rm s} + k_{\rm s}\, \Gamma' A' z_{\rm s} - b_{\rm s}\, \dot{\gamma} - \\ \qquad k_{\rm s}\, \gamma - \Gamma' u - k_{\rm t}(\gamma - \Gamma' {\rm r})\,. \end{cases}$$

We adopt the following notations

$$z_{\rm w} = A\, x_{\rm w}, \quad u_{\rm w} = A\, u, \quad r_{\rm w} = A\, r, \quad u_\gamma = \Gamma' u, \quad r_\gamma = \Gamma' r\,.$$

The previous equations then become

$$\begin{cases} m_{\rm w}\, \ddot{z}_{\rm w} = b_{\rm s}\, A\, A' \dot{z}_{\rm s} + k_{\rm s}\, A\, A' z_{\rm s} - b_{\rm s}\, \dot{z}_{\rm w} - \\ \qquad k_{\rm s}\, z_{\rm w} - u_{\rm w} - k_{\rm t}\, (z_{\rm w} - r_{\rm w})\,, \\ m_{\rm w}\, \ddot{\gamma} = b_{\rm s}\, \Gamma' A' \dot{z}_{\rm s} + k_{\rm s}\, \Gamma' A' z_{\rm s} - b_{\rm s}\, \dot{\gamma} - \\ \qquad k_{\rm s}\, \gamma - u_\gamma - k_{\rm t}(\gamma - r_\gamma)\,. \end{cases}$$

Therefore the full suspension model then takes the following form

$$\begin{cases} M \ddot{z}_{\rm s} = -b_{\rm s} A\, A' \dot{z}_{\rm s} - k_{\rm s} A\, A' z_{\rm s} + \\ \qquad b_{\rm s}\, \dot{z}_{\rm w} + k_{\rm s}\, z_{\rm w} + u_{\rm w} + d_{\rm s} \\ m_{\rm w}\, \ddot{z}_{\rm w} = b_{\rm s}\, A\, A' \dot{z}_{\rm s} + k_{\rm s}\, A\, A' z_{\rm s} - \\ \qquad b_{\rm s}\, \dot{z}_{\rm w} - k_{\rm s}\, z_{\rm w} - u_{\rm w} - \\ \qquad k_{\rm t}\, (z_{\rm w} - r_{\rm w}) \end{cases} \tag{10a}$$

$$m_{\rm w}\, \ddot{\gamma} = b_{\rm s}\, \Gamma' A' \dot{z}_{\rm s} + k_{\rm s}\, \Gamma' A' z_{\rm s} - b_{\rm s}\, \dot{\gamma} - k_{\rm s}\, \gamma - u_\gamma - k_{\rm t}(\gamma - r_\gamma)\,. \tag{10b}$$

A necessary and sufficient condition for

$$\Gamma = \begin{pmatrix} a \\ b \\ c \\ b \end{pmatrix}$$

to make T invertible is the following one

$$v_{\rm r}\, (b - a) \neq v_{\rm f}\, (d - c)\,.$$

The matrix $A\, A'$ may be seen as

$$A\, A' = \Lambda + \Delta$$

where

$$\Lambda = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2(v_{\rm f}^2 + v_{\rm r}^2) & 0 \\ 0 & 0 & 2(L_{\rm f}^2 + L_{\rm r}^2) \end{pmatrix}$$

and

$$\Delta = \begin{pmatrix} 0 & 0 & 2(L_{\rm r} - L_{\rm f}) \\ 0 & 0 & 0 \\ 2(L_{\rm r} - L_{\rm f}) & 0 & 0 \end{pmatrix}$$

With the following pre-compensation

$$u_{\rm w} = v_{\rm w} + k_{\rm s} \Delta z_{\rm s} + b_{\rm s} \Delta \dot{z}_{\rm s}$$

the dynamics of the chassis (10a) becomes one of a system of *three single input single output*

$$\begin{cases} M \ddot{z}_{\rm s} = -b_{\rm s} \Lambda \dot{z}_{\rm s} - k_{\rm s} \Lambda z_{\rm s} + \\ \qquad b_{\rm s}\, \dot{z}_{\rm w} + k_{\rm s}\, z_{\rm w} + v_{\rm w} + d_{\rm s}\,, \\ m_{\rm w}\, \ddot{z}_{\rm w} = b_{\rm s}\, \Lambda \dot{z}_{\rm s} + k_{\rm s}\, \Lambda z_{\rm s} - \\ \qquad b_{\rm s}\, \dot{z}_{\rm w} - k_{\rm s}\, z_{\rm w} - v_{\rm w} - \\ \qquad k_{\rm t}\, (z_{\rm w} - r_{\rm w})\,, \end{cases} \tag{11a}$$

$$m_{\rm w}\, \ddot{\gamma} = b_{\rm s}\, \Gamma' A' \dot{z}_{\rm s} + k_{\rm s}\, \Gamma' A' z_{\rm s} - b_{\rm s}\, \dot{\gamma} - k_{\rm s}\, \gamma - u_\gamma - k_{\rm t}(\gamma - r_\gamma)\,. \tag{11b}$$

Remark 1. The system (10a) is observable (in the absence of disturbances, i. e., $r = 0$) in the sense that we may estimate $z_{\rm s}$, $\dot{z}_{\rm s}$, $z_{\rm w}$, and $\dot{z}_{\rm w}$ in order to implement any control strategy which is a feedback on the latter variables.

Indeed, the suspension quantities which are usually assumed available as measurements are 4 accelerations of the 4 wheels and the 4 following variables

$$\begin{pmatrix} x_{\rm s,1} - x_{\rm w,1} \\ x_{\rm s,2} - x_{\rm w,2} \\ x_{\rm s,3} - x_{\rm w,3} \\ x_{\rm s,4} - x_{\rm w,4} \end{pmatrix}.$$

If we take

$$y = A\, (x_{\rm s} - x_{\rm w})$$

we have the following measurements for the system (10a)

$$\begin{cases} a_w = \ddot{z}_{\rm w}\,, \\ y = A\, A' z_{\rm s} - z_{\rm w}\,. \end{cases} \tag{12}$$

Now the control system consisting of the dynamics (10a) together with the measurements (12) is observable as may be easily checked.

Remark 2. The suspension control problem is then reduced to the control of three decoupled single input single output systems, and the control of an extra linear system which is stable.

3. INTERPRETATION

The lower level control variables take the form

$$u = T^{-1} u_{\rm d} + T^{-1} \begin{pmatrix} v_{\rm w} \\ u_\gamma \end{pmatrix}$$

where

$$u_{\rm d} = \begin{pmatrix} k_{\rm s} \Delta\, z_{\rm s} + b_{\rm s} \Delta\, \dot{z}_{\rm s} \\ 0 \end{pmatrix}$$

is the part of the control which achieves the decoupling.

The matrix T^{-1} is the *effort distribution matrix.* It may be explicitly computed in terms of a, b, c and d. With the decoupling of the system we can design 3 independent linear single input single outputs higher level controls, namely, the $v_{\rm w}$. The role of T^{-1} is precisely to indicate how these decoupling controls are distributed to the lower level controls, i. e., the active suspension actuators. For instance, for an active roll suspension control the distribution made is quite intuitive: it is symmetric with respect to the actuators. But, for controls designed for pitch and heave the distribution achieved by means of T^{-1} is quite sophisticated. Note also that, once $v_{\rm w}$ is designed for some specified purpose, the choice of Γ still provides us with some freedom on the distribution of these controls.

For instance, we may be fortunate to be able to find a, b, c and d in order to keep the controls u below some saturation bounds. Another example of utilization of a, b, c and d is to minimize the energy of the control u.

4. ON AN ASYMMETRIC SUSPENSION

We show that the previous analysis can be extended to the case of an asymmetric suspension for which the coefficients of the two front wheel suspensions are the same, the coefficients of the two rear suspensions are also the same, but the corresponding front and rear coefficients may be different. The basic model (1) remains the same, only the equations of the F_i's, namely, equations (3), change according to the following ones

$$F_i = -k_{{\rm s},i}(x_{{\rm s},i} - x_{{\rm w},i}) - b_{{\rm s},i}(\dot{x}_{{\rm s},i} - \dot{x}_{{\rm w},i}) + u_i\,, \quad (13)$$

where

$$k_{{\rm s},1} = k_{{\rm s},2} \quad \text{and} \quad k_{{\rm s},3} = k_{{\rm s},4}\,,$$

and

$$b_{{\rm s},1} = b_{{\rm s},2} \quad \text{and} \quad b_{{\rm s},3} = b_{{\rm s},4}\,.$$

Equations (7) become

$$\left\{\begin{array}{rl} M\ddot{z}_{\rm s} = & -A\,b_{\rm s}A'\dot{z}_{\rm s} - A\,k_{\rm s}A'z_{\rm s} + \\ & A\,b_{\rm s}\dot{x}_{\rm w} + A\,k_{\rm s}x_{\rm w} + Au + d_{\rm s} \\ m_{\rm w}\ddot{x}_{\rm w} = & b_{\rm s}A'\dot{z}_{\rm s} + k_{\rm s}A'z_{\rm s} - b_{\rm s}\dot{x}_{\rm w} - \\ & k_{\rm s}x_{\rm w} - u - k_{\rm t}(x_{\rm w} - {\rm r}) \end{array}\right. \quad (14)$$

where $b_{\rm s}$ and $k_{\rm s}$ are now diagonal *matrices*.

In order to recover our decoupled form (11) of the latter dynamics we use the following trick. We define

$$\rho_1 = \frac{k_{s,1}}{k_{s,1}+k_{s,3}}\,; \quad \overline{\rho}_1 = \frac{k_{s,3}}{k_{s,1}+k_{s,3}}$$

$$\rho_2 = \frac{b_{s,1}}{b_{s,1}+b_{s,3}}\,; \quad \overline{\rho}_2 = \frac{b_{s,3}}{b_{s,1}+b_{s,3}}$$

so that

$$\rho_1 + \overline{\rho}_1 \;=\; 1\,; \qquad \rho_2 + \overline{\rho}_2 \;=\; 1$$

Now $k_{\rm s}$ and $b_{\rm s}$ take the form

$$k_{\rm s} = \overline{k}_{\rm s}\Lambda_1 \quad b_{\rm s} = \overline{b}_{\rm s}\Lambda_2$$

with

$$\overline{k}_{\rm s} = k_{{\rm s},1} + k_{{\rm s},3}\,, \quad \overline{b}_{\rm s} = b_{{\rm s},1} + b_{{\rm s},3}$$

and

$$\Lambda_1 = \begin{pmatrix} \rho_1 & 0 & 0 & 0 \\ 0 & \rho_1 & 0 & 0 \\ 0 & 0 & \overline{\rho}_1 & 0 \\ 0 & 0 & 0 & \overline{\rho}_1 \end{pmatrix},$$

$$\Lambda_2 = \begin{pmatrix} \rho_2 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 \\ 0 & 0 & \overline{\rho}_2 & 0 \\ 0 & 0 & 0 & \overline{\rho}_2 \end{pmatrix}.$$

Defining

$$\overline{\Lambda}_1 = I - \Lambda_1\,, \quad \overline{\Lambda}_2 = I - \Lambda_2$$

where I is the unit matrix of order 4, then precompensating with the following control

$$u = v - \overline{k}_{\rm s}\overline{\Lambda}_1(x_{\rm s} - x_{\rm w}) - \overline{b}_{\rm s}\overline{\Lambda}_2(\dot{x}_{\rm s} - \dot{x}_{\rm w}) \quad (15)$$

we recover the type of model we started with at (7), that is,

$$\left\{\begin{array}{rl} M\ddot{z}_{\rm s} = & -\overline{b}_{\rm s}A\,A'\dot{z}_{\rm s} - \overline{k}_{\rm s}A\,A'z_{\rm s} + \\ & \overline{b}_{\rm s}A\dot{x}_{\rm w} + \overline{k}_{\rm s}Ax_{\rm w} + Av + d_{\rm s} \\ m_{\rm w}\ddot{x}_{\rm w} = & \overline{b}_{\rm s}A'\dot{z}_{\rm s} + \overline{k}_{\rm s}A'z_{\rm s} - \overline{b}_{\rm s}\dot{x}_{\rm w} - \\ & \overline{k}_{\rm s}x_{\rm w} - v - k_{\rm t}(x_{\rm w} - {\rm r}) \end{array}\right. \quad (16)$$

The procedure we used to obtain the decoupled form of the model (11) can now be applied to have

$$\left\{\begin{array}{rl} M\ddot{z}_{\rm s} = & -\overline{b}_{\rm s}\Lambda\dot{z}_{\rm s} - \overline{k}_{\rm s}\Lambda z_{\rm s} + \overline{b}_{\rm s}\,\dot{z}_{\rm w} + \\ & \overline{k}_{\rm s}\,z_{\rm w} + v_{\rm r} + d_{\rm s} \\ m_{\rm w}\,\ddot{z}_{\rm w} = & \overline{b}_{\rm s}\Lambda\dot{z}_{\rm s} + \overline{k}_{\rm s}\Lambda z_{\rm s} - \overline{b}_{\rm s}\,\dot{z}_{\rm w} - \\ & \overline{k}_{\rm s}\,z_{\rm w} - v_{\rm r} - k_{\rm t}\,(z_{\rm w} - r_{\rm w}) \end{array}\right. \quad (17a)$$

$$\begin{array}{rl} m_{\rm w}\,\ddot{\gamma} = & \overline{b}_{\rm s}\,\Gamma'\,A'\dot{z}_{\rm s} + \overline{k}_{\rm s}\,\Gamma'\,A'z_{\rm s} - \overline{b}_{\rm s}\,\dot{\gamma} - \\ & \overline{k}_{\rm s}\,\gamma - v_\gamma - k_{\rm t}(\gamma - r_\gamma)\,. \end{array} \quad (17b)$$

5. SIMULATIONS

To illustrate the usefulness of the new modeling of the active suspension, we have chosen to apply standard control laws one may find in the literature to our new model. In the simulations below we have selected the so-called "skyhook" control strategy.

A more sophisticated utilization of the variable γ is done in (Lakehal-Ayat *et al.*, 2002) where we propose a new control strategy for the yaw rate.

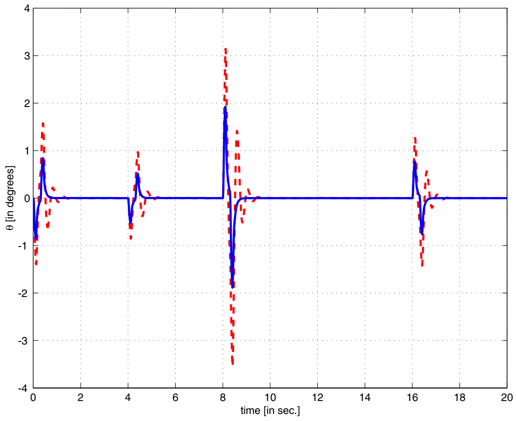

Figure 3. The roll for the passive (in red discontinuous line) and active (in blue continuous line) suspension.

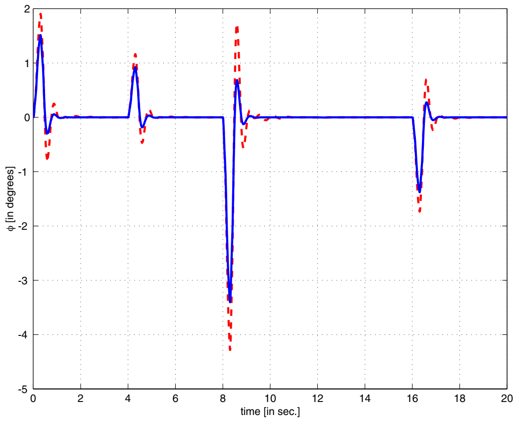

Figure 4. The pitch for the passive (in red discontinuous line) and active (in blue continuous line) suspension.

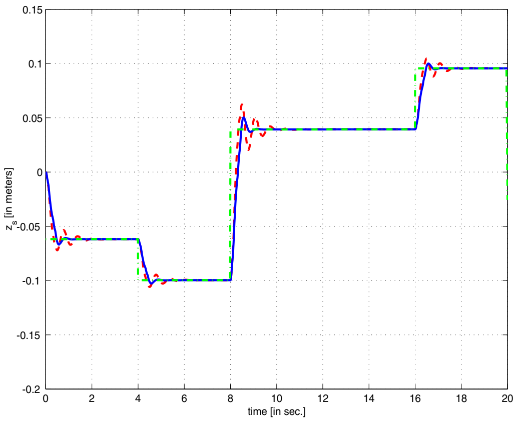

Figure 5. The heave for the passive (in red discontinuous line) and active (in blue continuous line) suspension.

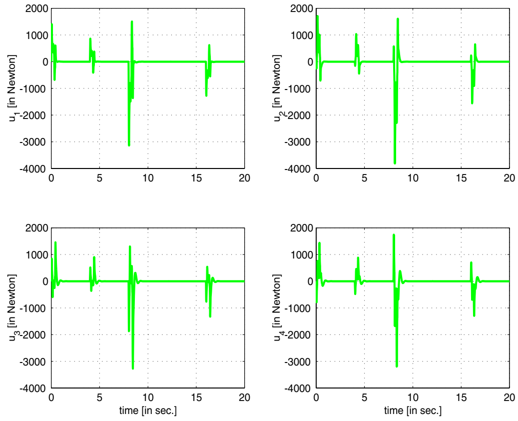

Figure 6. The actuator forces on the 4 wheels

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