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DYNAMIC POSITIONING $\mathscr{H}_\infty$ CONTROLLER TUNING BY GENETIC ALGORITHM

Decio C. Donha [*,1] **Reza M. Katebi** [**,2]

[*] *Mechanical Engineering Department of University of São Paulo*
Brazil
e-mail: decdonha@usp.br
[**] *ICC-Department of Electric and Eletronic Engineering University of Strathclyde, Glasgow, UK*
e-mail: r.katebi@eee.strath.ac.uk

Abstract: This paper is concerned with the development of a procedure to automatically calculate the parameters of the weighting functions used in $\mathscr{H}_\infty$ controller for a Dynamic Positioning System, seeking to achieve a desired system performance. The $\mathscr{H}_\infty$ synthesis problem is formulated as a multi-criterion optimization problem. A Genetic Algorithm is then employed to search for suitable solutions. To cope with the imprecision and vagueness that arises in the description of objective functions, and constraints of the process and actuators, concepts from the fuzzy logic are incorporated into the solution. A multi-objective fuzzy optimization is stated and a fuzzy convex decision-making is established.

Keywords: Dynamic positioning, Genetic algorithm, Weight selection,$\mathscr{H}_\infty$ control, Robust control, Multi-objective optimization, Optimal control.

1. INTRODUCTION

In this work, a GA based design procedure is employed to tune a $\mathscr{H}_\infty$ controller for dynamic ship positioning, aiming to achieve a desired performance. The $\mathscr{H}_\infty$ control design approach provides the necessary mathematical framework to obtain straightforward designs and efficient solutions for control systems involving uncertainties. This control technique also allows a means of easily shaping some of the well known frequency response functions of the system and at the same time maintaining stability and performance properties. Properties like stability robustness, disturbance rejection and command response behaviour can be jointly imposed in a certain measure. However, the design of a control system in general involves many constraints and competing objectives. To obtain an optimal solution, a trade-off between objectives and constraints is necessary, which formally needs a method for decision-making. Multi-objective fuzzy optimization techniques provide a means for the incorporation of the relative importance of competing objectives and problem restrictions and have been used in many similar contexts (Polkinghorne, 1996), and (Sutton, 1997). In this application, the control structure is keeped fix for practical and theoretical reasons. One of the design aims is to achieve controllers with the lowest order, therefore the space of search was restricted. This is of course not mandatory, and the procedure could be left free to search also the control structure. On the other hand, in $\mathscr{H}_\infty$ mixed-sensitivity control synthesis used in this work, the inverse of the weighting functions are expected ‘to adhere’ to the sensitivity functions. Therefore, in a large measure the structure of weighting functions are known. The beauty of the approach is that the tuning is basically achieved by the specification of low order weighting functions, which shape the sensitivity system responses.

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The paper is organised as follows. A brief description of the $\mathscr{H}_\infty$ problem and the effect of weighting functions are described in section 2. The Genetic Algorithm and the optimization problem is formulated in section 3. Results are given and analysed in section 4. Finally, the Conclusion is given in section 5.

2. $\mathscr{H}_\infty$ CONTROL DESIGN

A very usual approach to characterise the closed-loop performance objectives in the modern control theory is the measurement of certain closed-loop transfer matrices using different matrix norms (Zhou, 1995). These norms provide a measure of how large output signals can get for certain classes of input signals, which is a measure of the gain of the system. A mathematically convenient measure of a closed-loop matrix $T(s)$ in the frequency domain is the $\mathscr{H}_\infty$ norm defined as:

$$||T||_\infty := \max_{w \in \mathbf{R}} \overline{\sigma}(T(jw)) \tag{1}$$

where $\overline{\sigma}(T(jw))$ is the largest singular value of $T(jw)$ over the frequency range w.

The design of a control system involves two tasks: determining the structure of the controller and adjusting its parameters to give an 'optimal' system performance. There are several ways of setting up the control problem and consequently the selection of the weighting functions related to the system performance. One of the most popular procedures is the mixed sensitivity loop-shaping approach.

Consider the feedback control system in question represented in the standard two-port configuration shown in Figure 1, where R gives the reference dynamics, G represents the plant dynamics and K the controller dynamics. The system is forced by the command reference r, the sensor noise n, the plant input disturbance d_i and the plant output disturbance d. The weighting functions W_i, W_d and W_n reflect the available knowledge about the input and output disturbances and measurement noise, respectively. The weighting function W_s may be used to reflect requirements on the shape of the $\mathscr{H}_\infty$ controller. The weighting W_c may be used to reflect restrictions on the control signals, while W_t may be used to shape the complementary transfer function, to modify, for example, tracking features of the system.

The first step of the $\mathscr{H}_\infty$ design procedure in this case involves the minimization of a performance index, formulated as follows:

$$||T_{zw}|| = \left\| \begin{matrix} W_s S \\ W_c M \end{matrix} \right\|_\infty \tag{2}$$

where T_{zw} is the transfer function from $w = [\tilde{d} \quad \tilde{n} \quad \tilde{d}_i \quad c]^T$ to $z = [z_1 \quad z_2 \quad z_3]^T$ in the standard two port configuration; the weight W_s on the sensitivity function $S(s)$ will determine the tracking performance and the disturbance attenuation (Grimble, 1994); the weighting W_c on the control sensitivity function $M(s)$ will limit the actuator action at high frequencies, ensuring a desired controller roll-off frequency.

The state-space model used for control design is as follows:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{3}$$

$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t) \tag{4}$$

$$y(t) = C_2 x(t) + D_{21} w(t) + D_{22} u(t) \tag{5}$$

where $x(.)$ is the state vector; $x(t_0)$ is the known initial state; t is the time; $u(.)$ is the input vector; $w(.)$ is the dynamic disturbance, which may have random and deterministic components; $z(.)$ is the controlled state vector; $y(.)$ is the measured state vector. System matrices are assumed to have compatible dimensions.

Based on some well-known results an optimal controller $K(s)$ is defined as follows (Zhou 1995):

$$K(s) = \left[\begin{array}{c|c} A_\infty & -Z_\infty L_\infty \\ \hline F_\infty & 0 \end{array} \right] \tag{6}$$

where:

$$A_\infty \equiv A + \gamma^{-2} B_1 B_1^T X_\infty + Z_\infty L_\infty C_2 + B_2 F_\infty$$

$$F_\infty \equiv -B_2^T X_\infty; \qquad L_\infty \equiv -Y_\infty C_2^T$$

$$Z_\infty \equiv (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$$

where X_∞ and Y_∞ are the solutions of algebraic Riccati equations, respectively associated with the control and the estimator.

The second step in a $\mathscr{H}_\infty$ design amounts to the selection of different weighting functions in order to achieve performance objectives and practical requirements such as measurement noise attenuation. Suitable weighting functions will be determined in this work by a Genetic Algorithm (GA) search.

2.1 *Selection of Cost Weighting Functions*

Suitable cost functions can often be found after a trial and error procedure. However, more scientific methods of selecting weighting functions are required when the full potential of the dynamic cost weightings are to be exploited. Guidelines have been given by Grimble (1994). Assuring a small sensitivity function $S(s)$ leads to: a) good disturbance attenuation and b) good command following, related to the system bandwidth. Making the control sensitivity $M(s)$ small implies: a) moderated controller gains; b) limited influence of noises and c) robustness relatively to additive uncertainties. Assuring a small complementary sensitivity $T(s)$ will produce: a) moderated controller gains; b) limited influence of noises and c) robustness relatively to multiplicative uncertainties. The basic forms of the weighting functions are those shown in Figure 2.

3. MULTI-OBJECTIVE GENETIC ALGORITHM OPTIMIZATION

A brief description of the most important steps of the Genetic Algorithm follows.

3.1 *Decoding*

Each individual (string) of the population is decoded to produce four parameters for the weighting functions construction. The sensitivity weighting function W_s should be an integrator, possibly including a lead term and can be written as:

$$W_s(s) = \frac{(K_2)s+1}{ds+K_1} \quad (7)$$

where K_1 and K_2 are the parameters searched and d is a small constant resulting from the calculations. The GA algorithm decodes one string at a time producing $K_1$and K_2. It is then verified if the weighting functions are proper. If not the individual is discarded.

3.2 *Controller Evaluation*

If the weighting functions are proper the algorithm tries to define the $\mathscr{H}_\infty$ controller. If no solution is available the individual is discarded. If a $\mathscr{H}_\infty$ controller is available the system step response and the controller roll-off frequency are calculated. The procedure produces also the open loop frequency response, the sensitivity function, the control sensitivity function and the closed-loop frequency response. At this point informations like settling time, rise time, overshoot, actuator rate and actuator amplitude are available and are evaluated.

3.3 *Multi-objective Fuzzy Optimization*

Fuzzy sets can be used to define vague concepts such as those found when trying to evaluate the performance of a controller (Jang and Sun, 1994). The membership function fuzzy concept is what is mainly needed here. A membership function is a curve that defines how each point in an input space (universe of discourse) is mapped to a membership value (Dubois and Prade, 1980). It determines the degree to which an adjective, such as, e.g. too large in the statement: 'the overshoot is too large', truthfully describes the value of a variable and it is a measure of the designer's degree of satisfaction when faced with some result. The next stage is to fuzzify the objective functions and problem constraints and to calculate a membership value for each result produced in the previous step. The membership function for the fuzzy objective related, e.g., to the overshoot may be given by:

$$f(X,a,b,c) = max[min(\frac{X-a}{b-a},\frac{c-X}{c-b}),0] \quad (8)$$

where the parameters a and c locate the triangle basis and the parameter c locates its peak. The variable X is the overshoot value calculated before. If x is between a and c, a non-zero fitness value (μ) is assigned to the present solution ($\mu_{fi}(X) : R^n \to [0,1]$). Otherwise, the fitness value is zero. The fitness function is a measure of the degree of satisfaction for any solution available. If a constraint is violated the membership function is set to zero which means that no satisfaction was achieved. The results are passed to the fitness function which makes up part of the reproduction process.

3.4 *Reproduction*

The reproduction process is usually subdivided into two subprocesses:Fitness Evaluation and Selection. The fitness function is what drives the evolutionary process and its purpose is to determine how well a string (individual) solves a given problem allowing for the assessment of the relative performance of each population member. In this work the fitness function was established using convex decision-making. This technique provides a Pareto optimal solution, i.e. no unique optimal solution is found. Under the Pareto paradigm no improvement can be made in a determined objective without affecting others (Trebi-Ollenu and White, 1996). The optimal decision is made by selecting the best alternative from a fuzzy decision space A of alternatives. Convex decision-making based on the concept of arithmetic mean is used here. This allows a relative weighting between stated objectives and constraints. The multi-objective fuzzy optimization and fitness function can be defuzzified and formally stated as follows:

$$max\ \mu_A(X) = \sum_{i=1}^{k} \alpha_i \mu_{fi}(X) + \beta_i \mu_{gi}(X)$$
$$subject\ to\ g_j(X) \le a_j + d_j; \quad j = 1,2,....m \quad (9)$$

where $X \in R^n$ is any possible solution; $\mu_\bullet$ is the fitness value of a constraint or objective; α_i and β_i are weights attributed to an objective $f_i(X)$ or constraint $g_i(X)$, respectively; a_j is a constrained maximum value and d_j is the allowable tolerance for a fuzzy constraint. This means that an optimum solution is found in the crisp (non-fuzzy) domain which optimizes an a-priori defined fitness function. In this work the Tournament Selection with an Elitist Strategy (KrishnaKumar, 1994) was used to implement the selection operator.

4. RESULTS AND ANALYSIS

The main data of the ship used to evalute the designed controller are: Length overall (L): 68.88 m, Beam (B): 14.02 m, Design Draft (D): 3.96 m and Design Displacement ($\bigtriangledown$): 2038.68 m^3.

During station keeping DP systems are designed to hold the vessel inside a limiting radius known as the 'watch-circle' (usually defined as percentage of the local water depth) and not in a fixed position. The DP system is required to minimise the energy consumption and the wear and tear of the actuators. DP systems must also avoid the high frequency thruster modulation, induced by waves, wind gusts and sensor measurement noises. In addition, DP systems must tolerate sensor system transient errors and provide fast reconfiguration responses in the case of thruster failures.

A trade-off between track-keeping and wave filtering is accomplished using a two degree of freedom control design. Good tracking keeping features can be evaluated by the overshoot of linear step response, whereas good wave filtering characteristics are evaluated by the disturbance rejection of the controller. In this case, it was necessary to assign suitable weighting functions to the exogenous input signals representing the wave and wind disturbances, to the reference dynamics and to the noise in the sensor system to enhance the system performance.

To achieve faster numerical simulations, only the sensitivity cost weighting functions are searched for. The cost weighting functions for the control sensitivity are fixed in a way to guarantee a desired controller roll-off at high frequencies. This is possible since the control sensitivity represent the thruster input behaviour and the desired thruster performance can be accurately specified. Therefore, it is expected only a small drop of the final controller performance, compared to a full search including also the control sensitivity functions.

The GA parameters were set as follows: $N = 30$ binary strings, each one of length $l = 54$ bits. Crossover is performed with probability $p(X) = 0.9$ with three cross-over points in each string, mutation with probability $p(x) = 0.01$ and till 50 generations are evaluated if a high degree of satisfaction is not achieved. The following weightings ($\alpha_i,\ \beta_i$) were used in the generation of the fitness function: a) for the overshoots in all motions: 0.23; b) for the rise times in all motions: 0.05; c) for the settling times in all motions: 0.05 and d) for the value of $\gamma : 1/\gamma$. The last weight was set in order to achieve small γ values in equation 2, which is related to desired controller performance features. In this case a fitness value larger than one might be found. Polynomial Z-curves were used to evaluate the membership grades.

In the simulation the following weather conditions were used: Average wind velocity: $4m/s$; Absolute Wind Angle: 30^o; Average Current Velocity: $1m/s$; Absolute Current Angle: 45^o; Significant Wave Height: $3.0m$; Absolute Wave Angle: 60^o.

To evaluate the tracking together with the dynamic positioning characteristics of the system it is assumed that the vessel departs from the position $(X,Y) = (0,0)$ with zero degree heading and must go to position $(X,Y) = (15m, 15m)$ with at most 5 degrees heading deviation relatively to a desired trajectory and keep the final position.

Figure 3 is an example of the inverse sensitivity weighing functions found for surge, together with the resulting sensitivities and control sensitivities. Results for sway and yaw are very similar and due to lack of space are not presented. The sensitivities and control sensitivities are all bounded by the respective weightings, which is an indication that a good controller performance was achieved. The sensitivity functions are rolling-off at the low frequency range indicating a good system disturbance rejection. The control sensitivity is rolling-off at high frequencies indicating that the thruster will not be subjected to wear due to a high modulation.

In figure 4 it is depicted the close-loop function singular values. According to the small gain theorem a robust performance design was achieved, which is an advantage compared to the previous conventional design by Katebi, Grimble and Zhang(1997).

The following figures present the results of non-linear simulations using the designed controller. The tracking is very satisfactory as can be seen in figure 5. Though the heading is under the desired limits it could be improved by a better choice of the control sensitivity cost weighting function, which is held constant is this case. Finally, the thruster forces and moments commanded by the controller are presented in figure 6. It is clear that the controller has a good wave rejection since very smooth control signals were obtained. The control signal for sway is larger than for the other movements which can be explained by the larger environmental forces in this direction.

5. CONCLUSIONS

A simple method for the selection of suitable weighting functions for the $\mathscr{H}_\infty$ design using a multi-objective fuzzy GA for optimization has been proposed in this work. Results from a number of simulations proved that the approach can be valuable in reducing the design time necessary to tune a specific controller and that a good performance can effectively be achieved. The GA algorithm requires no previous knowledge of the search space. Of course, the solution is speed-uped if more information, such as the subset of the space of possible weighting parameters, is added to the search procedure.

The use of fuzzy logic in the multi-objective formulation may appear self-defeating, because instead of choosing weightings for the $\mathscr{H}_\infty$ problem, the designer should instead choose weightings of the fitness function. This is not true. The objective of the fuzzy logic is to emulate the behaviour of an experienced designer in the task of evaluation of the performance of a controlled system. Through the fitness function, the fuzzy algorithm emulates the evaluation of the designer when confronted with a new result. If the designer knows the relative importance of the specifications of a problem, the definition of the weightings of the fitness function is straightforward, and there is no need of a search here. In the case of the autopilot, for example, the designer will know the relative importance of the overshoot, of the settling-time, of the rise time and of the rudder rate. He will then, e.g., penalize more the overshoot and less the rise-time, if the overshoot is more important in the manoeuvres of the ship. Even in the unlikely case of a designer who does not know the relative importance of the specifications of a controlled system, the search of the weightings for the objective function would be far more easy then the search of the parameters of the weighting of the sensitivity functions, which have a very complex behaviour and consequences over the final result (Grimble,1994), (Whidborne et al., 1995).

The search approach developed here seems to suit very well with the $\mathscr{H}_\infty$ synthesis, considering the multivariable problem involved. The approach would also work in the tuning of other controllers, such a PID, for example. In the case of simpler controllers, when computer burden is smaller, the procedure would be speed-up, and on-line tuning seems possible.

The method also has some drawbacks. The velocity of convergence may vary from run to run due to the random generation of the initial population. When convergence does not occur rapidly it seems better to re-start the procedure than to let it run for a larger number of generations. The computer numerical burden is another disadvantage of the method. A parallel processing algorithm would also speed-up the procedure.

A development of this work would be to take into account advanced GA techniques with respect to real valued coding (Patton,and Liu, 1994).

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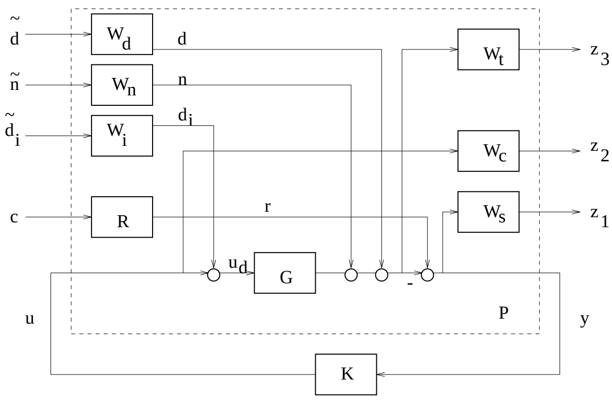

Fig. 1. Two-Port Configuration

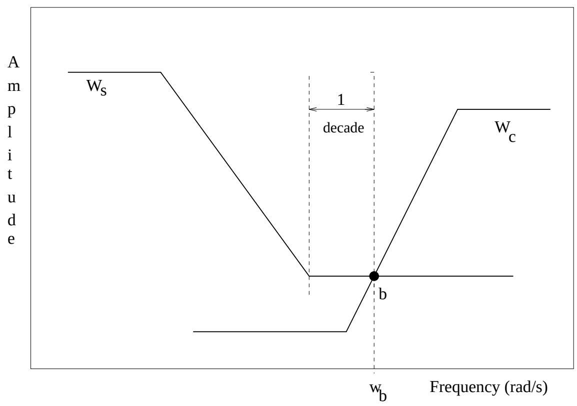

Fig. 2. Sensitivity Weighting Functions Forms

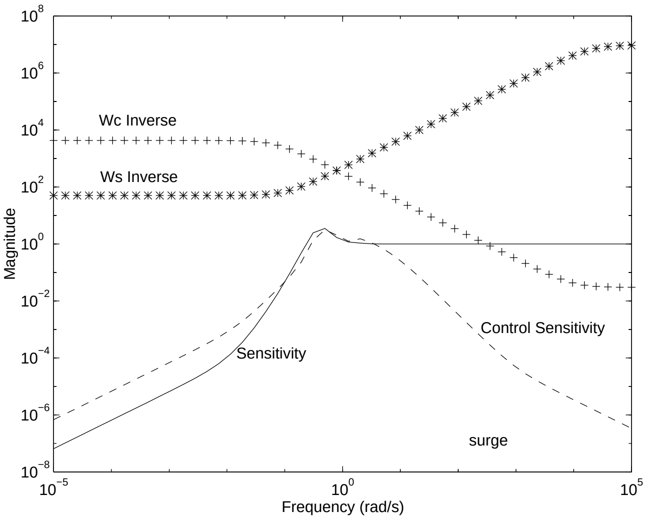

Fig. 3. Sensitivities and Inverse of Weightings - surge

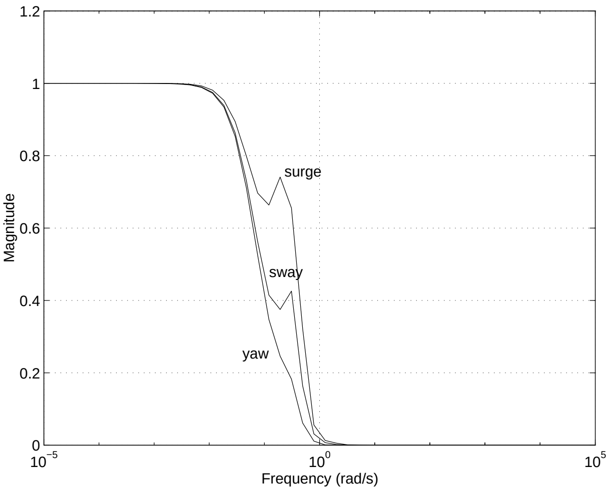

Fig. 4. Closed-loop Singular Values

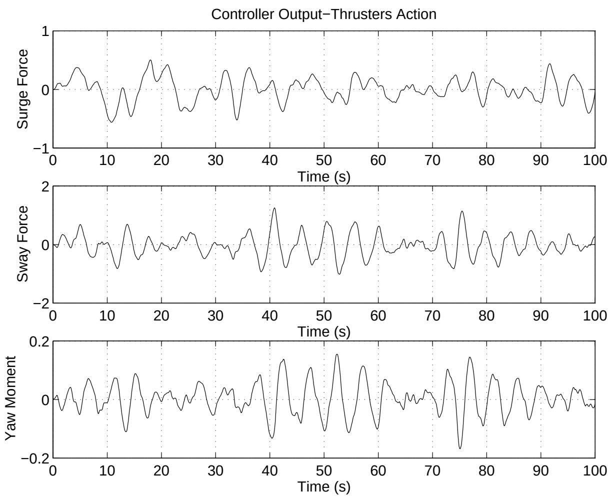

Fig. 6. Normalised Thruster Forces and Moment

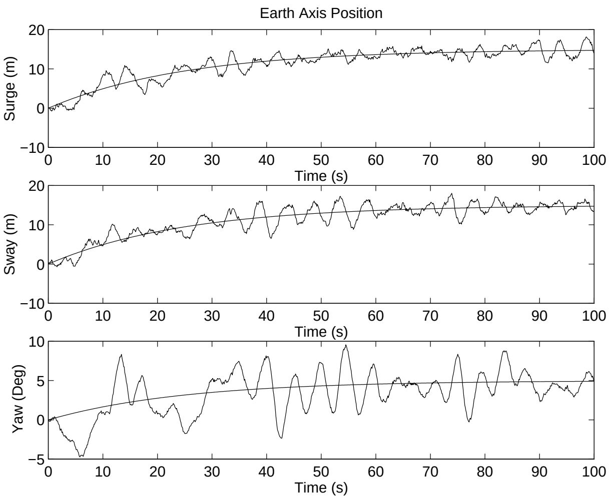

Fig. 5. Non Linear Tracking Performance