15th Triennial World Congress, Barcelona, Spain

A POLYTOPIC SYSTEM APPROACH FOR GAIN SCHEDULED CONTROL OF A DIESEL ENGINE

S. Bengea[⊥] **R. DeCarlo**[⊥] **M. Corless**[⊥⊥] **G. Rizzoni**[*] **S. Yurkovich**[*]

[⊥]*School of ECE,* [⊥⊥]School *of AAE, Purdue University, West Lafayette, IN, 47907-1285*
[*]*Center for Automotive Research, The Ohio State University, Columbus, Ohio*

Abstract: This paper develops a gain scheduled controller to move a diesel engine through a driving profile represented as a set of 6 operating points in the state space of a 7th order nonlinear state model. The calculations for the control design are based on a 3rd order(reduced) model of the Diesel engine on which state space are projected the 6 operating points. About each operating point, we generate a 3rd order nonlinear error models of the Diesel engine. Using the error model for each operating point, a linear feedback control is computed as a solution to a set of LMIs. The solution of each system of LMIs produces a norm bounded controller guaranteeing that $x_{i-1}^d \to x_i^d$ where x_i^d is the i-th desired operating point in the 3-dimensional state space. The LMIs incorporate a term which requires that x_{i-1}^d be in the region of attraction of x_i^d. The gain scheduled controller performance is evaluated on the 7th order model. *Copyright © 2002 IFAC*

Keywords: Engine Control, Nonlinear Control Systems, Lyapunov Methods, Automotive Control, Point-to-point control, Piecewise Linear Controllers, Linear Matrix Inequalities

1. INTRODUCTION.

This paper develops a gain scheduled control law for a diesel engine having a VGT/EGR(Variable Geometry Turbocharger/Exhaust Gas Recirculation) represented by a 7th order nonlinear state model. (Guzzella and Amstutz, 1998) The objective of the controller is to drive the engine operating point from an initial value to a desired value along a drive cycle and stabilize the engine around the desired equilibrium. This equilibrium is computed so that it satisfies driver needs while achieving a reasonable trade-off between undesirable emissions of nitrogen oxides (NO_x) and smoke emissions on the one hand, and fuel consumption on the other hand. The control design will be achieved using polytopic system methods. Here a chain of overlapping compact regions of the state space is formed so that each region contains an equilibrium point common to the next polytopic region in the chain. Given appropriate continuity, the induced image of each region in the model vector fields is bounded by a polytope. Using Lyapunov methods applied to each region, a feedback control and an ellipsoidal domain of attraction is obtained by solving a set of LMIs. Each controller will move the state through the associated region to an operating point common to the domain of attraction of current region and the next region along the chain. The controller for the next equilibrium state is invoked when the system is sufficiently close to the preceding equilibrium state. This gain scheduled controller represents the lowest type of hybrid control.

To preserve geometric understanding of the control construction and to simplify the calculations associated with the LMIs, the design algorithm builds on a 3rd order nonlinear diesel engine model. However, the control and its performance is evaluated on the 7th order nonlinear diesel engine model.

2. MODEL OF THE DIESEL ENGINE WITH VGT/EGR.

For environmental and legislative reasons, emission control of engines is critical. Diesel engines offer superior fuel economy but reducing their nitrogen oxide (NO_x) emission control remains challenging, because the conventional Three-Way Catalyst utilized in gasoline-powered vehicles is inefficient for NO_x conversion at lean air-to-fuel ratios where diesels typically operate. To be competitive with gasoline engines, new generation diesel engines are equipped with exhaust gas recirculation (EGR) systems to reduce NO_x emission and variable geometry turbochargers (VGT) to reduce transient smoke. Combination of EGR and VGT provides an important avenue for NO_x emission reduction. Traditionally, turbocharging has been used to increase the power density of diesel engines. VGT is accomplished by a turbine with adjustable guide vanes. By adjusting the guide vanes, the exhaust gas energy to the turbocharger can be regulated, thus controlling the compressor mass airflow and exhaust manifold pressure. Initially, the rationale for using the VGT was to increase engine torque output at tip-ins and reduce turbo-lag. Now, VGT has emerged as an important way to reduce NO_x emissions because it can be used to increase the exhaust gas recirculation rates.

The primary exhaust gas recirculation is accomplished by an EGR valve that directs some of the exhaust gas back into the intake manifold. This dilutes the cylinder charge and lowers the combustion temperatures thereby impeding the process of NO_x formation. Because the flow through the EGR valve depends on the pressure drop across the valve, and because the VGT can affect this pressure drop, the turbocharger can also be utilized to increase the EGR flow. Thus, these two devices are strongly coupled and the system exhibits internal instability, requiring advanced control algorithms. The controller has to keep EGR flow rate and air-fuel ratio at the desired levels such that NO_x emission, as well as transient smoke can be lowered to meet future regulations. Additional factors that burden the application of conventional control design methods are: lack of information about the system states; only a limited number of physical coordinates (air flow rate and intake manifold pressure) can be measured directly, while knowledge of the variable "EGR flow rate" to be controlled is not available; parameters of the process (intake burnt gas fraction, intake charge flow rate and exhaust pressure) are unknown, due to aging and deposits on the flow ways/valves, and may vary over a wide range for different operation modes; system behavior is governed by a high order system of nonlinear equations and as a result new control strategies should be developed (traditional methods based on PI or PID controllers are no longer effective).

In light of this background, the model of the VGT/EGR Diesel engine (Utkin *et al.*, 2000; Kolmanovsky *et al.*,1999a; Kolmanovsky *et al.*, 1999b) is obtained through the application of the mass and energy balances for the intake and exhaust manifolds. For control design we will use a simplified 3rd order model (developed by differentiating the universal gas law assuming locally constant manifold temperatures) whereas the 7th order model will serve to evaluate and fine tune the control design.

2.1 The 7th order model for the diesel engine.

The definitions of the state variables used in the nonlinear state model below are: (i) ρ_1, ρ_2 (gas density in intake (subscript 1) and exhaust (subscript 2) manifold), (ii) F_1, F_2 (burnt gas fractions in intake and exhaust manifolds), (iii) p_1, p_2 (pressures in intake and exhaust manifolds), and (iv) P_C (compressor power). For convenience we define the state vector $x(t) = [\rho_1(t), F_1(t), p_1(t), \rho_2(t), F_2(t), p_2(t), P_C(t)]^T$. In general, the indices 1, 2, e, and C stand for intake manifold, exhaust manifold, engine and compressor respectively. The symbols W_{ij} and u_{ij} have the meaning of flows from index i to index j where i, j $\in$ {1, 2, e, C}. T_i has the meaning of temperature in "compartment" i, whereas T_{ij} means the temperature of the mixture flowing from i to j. The intake manifold equations are:

$$\begin{aligned}
\dot{\rho}_1 &= \left[W_C + u_{21}(\alpha) - W_{1e} - u_{12}(\alpha)\right] / V_1 \\
\dot{F}_1 &= \left[(F_2 - F_1) u_{21}(\alpha) - F_1 W_C\right] / \rho_1 V_1 \qquad (2.1) \\
\dot{p}_1 &= \frac{\gamma R}{V_1}\left[W_C T_C + u_{21}(\alpha) T_{21} - W_{1e} T_1 - u_{12}(\alpha) T_1\right]
\end{aligned}$$

The exhaust manifold equations:

$$\begin{aligned}
\dot{\rho}_2 &= \frac{\left(W_{e2} - u_{2t}(\beta) - u_{21}(\alpha) + u_{12}(\alpha)\right)}{V_2} \\
\dot{F}_2 &= \frac{(F_{e2} - F_2) W_{e2}(W_f) + (F_1 - F_2) u_{12}(\alpha)}{\rho_2 V} \qquad (2.2) \\
\dot{p}_2 &= \gamma R \frac{W_{e2}(W_f) T_{e2} - u_{2t}(\beta) T_2 - u_{21}(\alpha) T_2 + u_{12}(\alpha) T_1}{V_2}
\end{aligned}$$

The power transfer from the turbine to the compressor equation:

$$\dot{P}_C = (-P_C + \eta_m P_t) / \tau \qquad (2.3)$$

The parameters that appear in this model above have the following meanings: (i) γ is the specific heat ratio, (ii) R is the difference of specific heats, (iii) τ is turbine to compressor power transfer time constant, (iv) k_e is the pumping rate, (v) η_m is the turbocharger mechanical efficiency, (vi) V_1, V_2 are the volumes of the intake and exhaust manifolds respectively. Equation (2.3) above represents the approximation of the turbocharger delay in the transfer of energy to the compressor. The controlled inputs to the model are α and β which represent the valve position of the EGR, $W_{egr} = W_{egr}(\alpha)$, and VGT respectively. Both α and β have values in the interval [0,1] with 1 meaning fully opened. Note that W_f is fuel flow and $W_{e2}(W_f)$ means that the flow from the engine to the exhaust manifold depends on fueling rate W_f. W_C (the flow from the compressor to the intake manifold) and W_{1e} depend on the state of the system. Finally, we set $u_{12}(\alpha) = 0$, i.e., the EGR flow from the intake manifold to the exhaust manifold is set to zero, although there are circumstances when this is not the case. When implementing the control we use W_{egr} and u_{2t} rather than α and β.

2.2 Reduced order model

A reduced (3rd order) model of the diesel engine follows by differentiating the ideal gas law for the intake and exhaust manifolds under the assumptions that the temperatures in the intake and exhaust manifolds are constant for local operation and that the there is no dependence of thermodynamic properties on composition (i.e. all thermodynamic properties are referenced to air $\gamma = \gamma_a$ and $R = R_a$). Set points for this model are projections of those of the 7th order model. From (Utkin *et al*., 2000) the 3rd order reduced model can be written as:

$$\dot{p}_1 = k_1 \frac{\eta_C}{T_a C_p} \frac{P_C}{\left(\left(\frac{p_1}{p_a}\right)^{\mu} - 1\right)} - k_1 k_e p_1 + k_1 W_{egr}$$

$$\dot{p}_2 = k_2\left(k_e p_1 + W_f^d - W_{egr} - u_{2t}\right) \qquad (2.4)$$

$$\dot{P}_C = -\frac{1}{\tau}\left[P_C - \eta_m \eta_t T_2 C_p \left(1 - \left(\frac{p_a}{p_2}\right)^{\mu}\right) u_{2t}\right]$$

2.3 Operating Points

Generating the set of operating points for the 7th order system is done by specifying a specific air-to-fuel ratio, EGR, and the fueling rate for a given engine speed and load. These (input) quantities are then mapped (Utkin *et al.*, 2000; Upadhyay, 2001) into desired (equivalent input) values of the flow rates, i.e., the fueling rate, W_C^d and W_{egr}^d. From these "inputs" we determine consistent equilibrium states in the 7th order model which are then projected onto the 3rd order model state space assuming locally constant T_1 and T_2 for each operating point.

3. A POLYTOPIC/LYAPUNOV/LMI METHOD FOR STABILIZING NONLINEAR SYSTEMS

Several notions from Lyapunov stability theory (Khalil, 1996) are pertinent. To state the results more simply, we consider the usual nonlinear state model

$$\dot{x}(t) = f(t, x(t)), x(t_0) = x_0 \qquad (3.1)$$

where $x(t) \in R^n$ and $t \in R$ and assume that the origin is a equilibrium point, i.e. $f(t,0) = 0$ for all t. The origin is uniformly exponentially stable with rate of convergence α > 0 if there exists R > 0 and β > 0 such that whenever $\|x(t_0)\| < R$, $\|x(t)\| < \beta\|x(t_0)\| e^{-\alpha(t-t_0)}$ for all $t \geq t_0$. Assuming the origin is an exponentially stable equilibrium point for 3.1, the set Ω is a region of attraction for the origin if $x(t_0) \in \Omega \Rightarrow \lim_{t\to\infty} x(t) = 0$. Finally, a subset Λ of the state space R^n is called an invariant set for 3.1 if $x(t_0) \in \Lambda \Rightarrow x(t) \in \Lambda$ for all $t \geq t_0$. Thus:

Theorem 1. [Lyapunov] With respect to the system 3.1 suppose that there is a continuously differentiable function V and positive scalars α, β_1, β_2, and c such that for all x in $\Omega = \{x \in R^n : V(x) < c\}$, $\beta_1\|x\|^2 \leq V(x) \leq \beta_2\|x\|^2$ and $DV(x) f(t,x) \leq -2\alpha V(x)$ for all t. Then the origin is a uniformly exponentially stable equilibrium point with invariant region of attraction Ω and rate of convergence α. The main theoretical result of this paper states sufficient conditions for the origin to be uniformly exponentially stable for a system having the following polytopic form:

$$\dot{x}(t) = A(t,x)x(t) + B(t,x)u(t), x(t_0) = x_0 \qquad (3.2)$$
$$A(t,x) = A_0 + \Psi(t,x)\Delta A, B(t,x) = B_0 + \Psi(t,x)\Delta B$$

for constant A_0, B_0, ΔA, and ΔB. The scalar valued function $\Psi(t,x)$ has the property that whenever $\|Cx\| \leq \mu$, for a matrix C and a positive scalar μ, then

$$a(\mu) \leq a(\|Cx\|) \leq \Psi(t,x) \leq b(\|Cx\|) \leq b(\mu) \qquad (3.3)$$

where $a(\circ)$ and $b(\circ)$ are non-increasing function and non-decreasing functions respectively.

Theorem 2 (Corless, 2001). With respect to the system 3.2 with property 3.3, suppose there exists a matrix L and a positive definite matrix S such that

$$A_1 S + B_1 L + S A_1^T + L^T B_1^T < 0 \qquad (3.4a)$$
$$A_2 S + B_2 L + S A_2^T + L^T B_2^T < 0 \qquad (3.4b)$$
$$C S C^T \leq 1 \qquad (3.4c)$$

where $A_1 = A_0 + a(\mu)\Delta A$, $A_2 = A_0 + b(\mu)\Delta A$, $B_1 = B_0 + a(\mu)\Delta B$, and $B_2 = B_0 + b(\mu)\Delta B$. Then for the closed loop system, system 3.2 with the linear state feedback $u = Kx = LS^{-1}x$, the origin is a uniformly exponentially stable equilibrium point with Ω = $\{x \in R^n \mid x^T S^{-1} x < \mu^2\}$ an invariant region of attraction.

The previous result can be generalized to systems described by 3.2 so that the origin is an exponentially stable equilibrium when the time/state dependent matrices $A(t,x)$ and $B(t,x)$ have the following more general structure: $A(t,x) = A_0 + \sum_{i=1}^{l} \Psi_i(t,x)\Delta A_i$ and $B(t,x) = B_0 + \sum_{i=1}^{l} \Psi_i(t,x)\Delta B_i$ where the $\Psi_i(\circ,\circ)$ are scalar valued functions of t and x, A_0, $\Delta A_1, \ldots, \Delta A_l$ are constant $n \times n$ matrices and $B_0, \Delta B_1, \ldots, \Delta B_l$ are $n \times m$ matrices. This requires that, whenever $\|C_i x\| \leq \mu$,

$a_i(\mu) \le \Psi_i(t,x) \le b_i(\mu)$ for constant matrices C_i, a positive scalar μ, and nonincreasing and nondecreasing $a_i(.)$ and $b_i(.)$, $i = 1 \ldots l$. The region of attraction is $\Omega = \{x \in R^n : x^T S^{-1} x < \mu^2\}$, the matrices L and S and the scalar μ must satisfy

$$\begin{aligned} &AS + BL + SA^T + L^T B^T < 0 \\ &CSC^T \le 1 \end{aligned} \qquad (3.5a)$$

for all $C \in \{C_i\}$ and all matrix pairs

$$(A,B) \in \left\{ \left(A_0 + \sum_{i=1}^{l} \Psi_i \Delta A_i, B_0 + \sum_{i=1}^{l} \Psi_i \Delta B_i \right), \Psi_i = \left\{ \begin{matrix} a_i(\mu) \\ b_i(\mu) \end{matrix} \right\} \right\} \qquad (3.5b)$$

for i = 1,...,*l*. This more general formulation of Theorem 2, represented as a family of LMIs by the relations of 3.5, will be used to derive a linear state feedback for the reduced order model of the Diesel engine. The existence of L, S, and μ in the LMI 3.5 provides state feedback that is sufficient for stability. We must further impose two more constraints on the variables S and L for a practical solution to our control problem. The first constraint permits inclusion of a starting point x_o in the invariant ellipsoid Ω centered at the origin which can be expressed as an LMI using the Schur complement:

$$\left(x_o - x_1^d\right)^T S^{-1} \left(x_o - x_1^d\right) < \mu^2 \Leftrightarrow \begin{bmatrix} \mu^2 & \left(x_o - x_1^d\right)^T \\ \left(x_o - x_1^d\right) & S \end{bmatrix} > 0 \qquad (3.6)$$

where $\left(x_o - x_1^d\right)$ is to be driven to zero. A second constraint (Boyd *et al.*, 1994) upper bounds the norm of $u(t)$ by γ. For $x(t) \in \Omega$, we require that for all t

$$\|u(t)\|^2 = \|Kx(t)\|^2 = x^T(t) K^T K x(t) \le \gamma^2 \qquad (3.7)$$

But if

$$K^T K \le \left(\gamma^2 / \mu^2\right) S^{-1} \qquad (3.8)$$

holds, then $\|u(t)\|_2 \le \gamma$ as desired. To convert 3.8 to LMI form, observe that

$$K^T K = S^{-1} L^T L S^{-1} \le \frac{\gamma^2}{\mu^2} S^{-1} \Leftrightarrow L^T L \le \frac{\gamma^2}{\mu^2} S \qquad (3.9)$$

Using the Schur complement 3.18 can be written as the LMI:

$$\begin{bmatrix} S & L^T \\ L & \gamma^2/\mu^2 \end{bmatrix} \ge 0 \qquad (3.10)$$

In summary, the LMI system formed by inequalities 3.5, 3.6 and 3.10 will be used in the next section to develop a gain scheduled controller for the error system set forth in the next section.

4. CONTROL DESIGN

This section details the application of the general version of Theorem 2 to the design of a control law for the reduced order model of the Diesel engine. The controller drives the system state through a sequence of operating points. For each operating point we generate a nonlinear error system amenable to polytopic form. For each such system, a system of LMIs is formulated so that the previous equilibrium is included in the region of attraction of the current error system. The solution of each system of LMIs generates the needed control.

4.1 Derivation of the error system

The reduced order model of the EGR-VGT Diesel engine (Upadhyay, 2001) is given in the set of equations 2.4. Assuming that the desired operating point is $\left[p_1^d, p_2^d, P_C^d\right]^T$ (see section 2.3), the equilibrium equations are obtained by setting the derivative in equation 2.4 to zero and solving, i.e.,

$$\begin{aligned} 0 &= \frac{\eta_C}{T_a C_p} \frac{P_C^d}{\left(\left(\frac{p_1^d}{p_a}\right)^{\mu} - 1\right)} - k_e p_1^d + W_{egr} \\ 0 &= k_e p_1^d + W_f^d - W_{egr}^d - u_{2t}^d \\ 0 &= P_C^d - \eta_m \eta_t T_2 C_p \left(1 - \left(\frac{p_a}{p_2^d}\right)^{\mu}\right) u_{2t}^d \end{aligned} \qquad (4.1)$$

where the superscript d has the meaning of the desired operating point. Since the control inputs for the model (2.4) are W_{egr} and u_{2t} we denote them by u_1 and u_2, respectively. Define the error relative to the desired operating point as $\Delta x = [\Delta p_1, \Delta p_2, \Delta P_C]^T = [p_1 - p_1^d, p_2 - p_2^d, P_C - P_C^d]$ and from equations 2.4 and 4.1 we can obtain a polytopic form of the error system as:

$$\dot{\Delta x} = A(\Delta x)\Delta x + B(\Delta x)\Delta u \qquad (4.2)$$

where

$$A(\Delta x) = A_0 + \sum_{i=1}^{4} \Psi_i(\Delta x) \Delta A_i \qquad (4.3)$$

$$B(\Delta x) = B_0 + \sum_{i=1}^{4} \Psi_i(\Delta x) \Delta B_i \qquad (4.4)$$

The associated functions in 4.3 and 4.4 are:

$$\Psi_1(\Delta x) = \frac{k_1 \eta_C P_C^d}{T_a C_p \Delta p_1} \left(\frac{1}{\left(\frac{p_1^d + \Delta p_1}{p_a}\right)^{\mu} - 1} - \frac{1}{\left(\frac{p_1^d}{p_a}\right)^{\mu} - 1} \right),$$

$$\Psi_2(\Delta x) = \frac{\eta_C}{T_a C_p} \frac{1}{\left(\frac{p_1^d + \Delta p_1}{p_a}\right)^{\mu} - 1}$$

$$\Psi_3(\Delta x) = -\frac{1}{\tau} \frac{\eta_m \eta_t T_2 C_p u_2^d}{\Delta p_2} \left[\left(\frac{p_a}{p_2^d + \Delta p_2} \right)^{\mu} - \left(\frac{p_a}{p_2^d} \right)^{\mu} \right],$$

$$\Psi_4(\Delta x) = -\frac{1}{\tau} \eta_m \eta_t T_2 C_p \left(\frac{p_a}{p_2^d + \Delta p_2} \right)^{\mu}$$

with $\Delta A_4 = [0]$,

$$A_0 = \begin{bmatrix} -k_e & 0 & 0 \\ k_2 k_e & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \Delta A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Delta A_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Delta A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

and with $\Delta B_1 = \Delta B_2 = \Delta B_3 = [0]$,

$$B_0 = \begin{bmatrix} k_1 & 0 \\ -k_2 & -k_2 \\ 0 & \frac{1}{\tau}\eta_m \eta_t T_2 C_p \end{bmatrix} \text{ and } \Delta B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The above defines the polytopic form of the reduced order model of the Diesel engine. Observe that there exist functions $\Phi_i(\circ), i = 1,2,3,4$ such that $\Psi_i(\Delta x) = \Phi_i(C_i \Delta x), i = 1,2,3,4$ where $C_1 = C_2 = [1 \;\; 0 \;\; 0]$ and $C_3 = C_4 = [0 \;\; 1 \;\; 0]$. Since in a sufficiently small region about the origin the functions $\Phi_i(\circ), i = 1,2,3,4$ are monotone then, for a suitably chosen small $\mu > 0$, the functions $\Phi_i(C_i \Delta x)$ can be included in the interval defined by $\Phi_i(-\mu)$ and $\Phi_i(\mu)$ whenever $\|C_i \Delta x\| \le \mu$. It follows that the constants $a_i = a_i(\mu)$ and $b_i = b_i(\mu)$ can now be explicitly chosen such that $a_i \le \Psi_i(\Delta x) \le b_i$ whenever $\|C_i \Delta x\| \le \mu$ for $i = 1,2,3,4$. Thus the LMIs formed by the above inequalities is now completely specified and the gain K of the linear state feedback controller $u = K \times \Delta x$ is then obtained as a function of the solution of this system of LMIs.

5. GAIN SCHEDULED CONTROL LAW

In a typical drive cycle, the diesel engine transitions through different reference states determined by the ECM according to driver demands, road conditions, and appropriate VGT and EGR associated with pre-computed (mapped) exhaust gas emissions constraints. Associated with each reference state $x_i^d \in D$ is the triple (L_i, S_i, Ω_i), $i = 1,\ldots,n_D$, where D is the (finite) set of all reference states which may occur during the operation of the engine and n_D is the number of elements in D. Assuming that the engine is in state x_k^d and the next desired state is x_{k+1}^d the following situation may occur: $x_k^d \notin \Omega_{k+1}$. Here the engine cannot be driven from x_k^d to x_{k+1}^d using the control law $u_k = L_k S_k^{-1} x$. It is necessary to compute additional intermediate reference states $x_k^1, x_k^2, \ldots, x_k^{i_k}$ so that the engine state passes through the whole chain of regions of attractions $\Omega_k^1, \Omega_k^2, \ldots, \Omega_k^{i_k}$ until it reaches Ω_{k+1}. By applying the control $u_k = L_k S_k^{-1} x$ at the appropriate point, the engine will then be driven asymptotically to x_{k+1}^d and it will remain there as long as the reference state remains unchanged.

The above techniques were applied to the 3rd order engine model using a total of 6 equilibrium states including the initial and final points. The resulting control was then applied to the 7th order model and simulated for evaluation as shown in figure 2. The results here are similar to those reported in (Stefanopoulou *et al.*, 1998) although this work does not illustrate a return to a lower load. An advantage here is that we utilize the nonlinear error model similar to (Utkin *et al.*, 2000) although in (Utkin *et al.*, 2000) a robust variable structure control was implemented in contrast to the polytopic/LMI development here.

In implanting the control, a relative error

$$\frac{\|x - x_{i-1}^d\|}{\|x_{i-1}^d\|} < error$$

of 0.5 was used for switching to the next controller. The relative error significantly impacts the time of convergence. A relative error of 10^{-3} took upwards of 70 seconds. Further reduction of convergence time is possible by allowing a larger norm on the control. It is necessary to explore these tradeoffs relative to fuel economy and emissions between the intermediate equilibrium states.

7. CONCLUSIONS.

This paper has provided an algorithm for designing a set of linear gain scheduled feedback controllers that will drive a the state from an initial operating point to a desired one. The controls are obtained via the solution of a family of LMI's developed from a polytopic form of the 3rd order error model of the engine. The polytopic form is not unique although in this case we felt the given equations were natural. Investigation of other forms is a possible area of future research.

Besides robustness, flexibility in the controller design permeates our approach. Flexibility means that around an equilibrium point several regions of attractions that include the same starting point can exist simultaneously. If the origin is the equilibrium point (as in the error model) this is equivalent to saying that there exist positive definite matrices S_i, $i = 1{:}N$, such that the sets

$\Omega_i = \left\{ x : x^T S_i^{-1} x < 1 \right\}$, $i = 1{:}N$, are invariant regions of attractions for the closed loop system, that contain the same starting point. A controller K_i corresponds to each region of attraction Ω_i which in the error model is the origin. Hence, for the same equilibrium point there often exist several controllers with different regions of attraction (which contain the same starting point), providing us with flexibility in selecting a controller that has a better performance in terms of a trade-off of the response speed, variations of the transient response, variations of the control input, etc. This trade-off has been taken into account for determining a sequence of controllers that drive the 7th order model Diesel engine from a low load operating point to a medium load operating point.

The controller performance on the 7th order model was reasonable, although EPA standards necessitate a more stringent trajectory following. The ideas in (Landau, 2001) would appear to provide an avenue for the improved controller design. The idea is to simulate the 7th order nonlinear model with control and using the results, implement a parameter ID on the closed loop 3rd order model to have its performance better approximate the closed loop performance of the 7th order model. Because the 3rd order model is not a singular perturbation of the 7th, such an approach would seem warranted.

REFERENCES.

Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*. SIAM, Philadelphia.

Corless, M. (2001). Class notes, *AAE 666*. Purdue University, West Lafayette, IN, 47907.

Guzzella, L. and A. Amstutz(1998). Control of Diesel Engines. *IEEE Control Systems*, **Vol. 18**, No. 5, October.

Khalil, H. (1996). *Nonlinear Systems*. Upper Saddle River: Prentice Hall.

Kolmanovsky, I., P. Moraal, M. van Nieuwstadt, A. Stefanopoulou (1999a). Issues in Modeling and Control of Intake Flow in Variable Geometry Turbocharged Engines. Proceedings of 18th IFIP Conference on System Modeling and Optimization, Detroit, July, 1997; published in *System Modeling and Optimization*, Eds. M.P. Polis, A.L. Dontchev, P. Kall, I. Lasiecka and A.W. Olbrot, Chapman and Hall/CRC Research Notes in Mathematics, pp. 436-445.

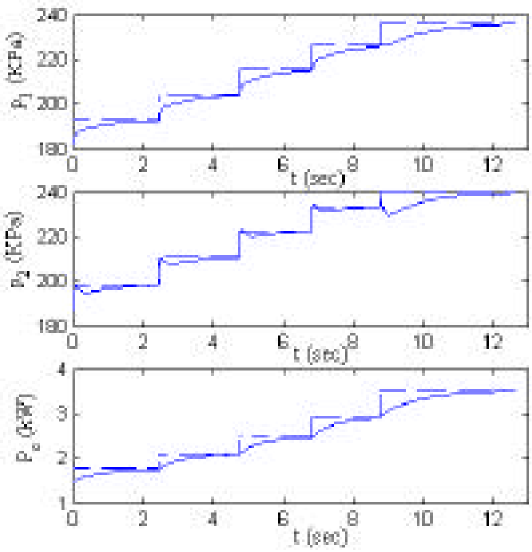
Kolmanovsky, I., M. van Nieuwstadt, and P. Moraal (1999b), Optimal control of variable geometry turbocharged diesel engines with exhaust gas recirculation. DSC-**Vol. 67**, *Proceedings of the ASME Dynamic Systems and Control Division* , pp. 265-273.

Landau, I.D(2001). Identification in closed loop: a powerful design tool (better design models, simpler controllers). *Control Engineering Practice*, **Vol 9**, pp. 51-65.

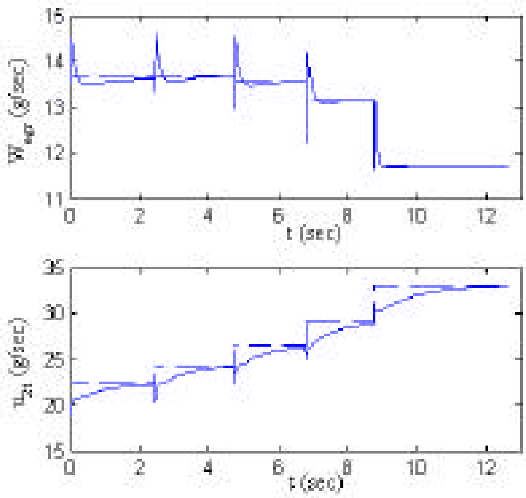
Stefanopoulou, A.G., I.V. Kolmanovsky and J.S. Freudenberg (1998). Control of variable geometry turbocharged diesel engines for reduced emissions. *Proceedings of the American Control Conference*, Philadelphia, pp. 1383-1388.

Upadhyay, D. (May 2001).*Modeling and Model based Control Design of the VGT-EGR System for Intake Flow Regulation in Diesel Engines*, Ph. D. Dissertation, Ohio State University, Columbus Ohio.

Utkin, V., H.-C. Chang, I. Kolmanovski, J. Cook (2000). Sliding mode control for the variable geometry turbocharged diesel engines. *Proceedings of the 2000 American Control Conference*, Chicago.



(a)



(b)

Figure 2. (a) Plot of the pertinent state trajectorie from 7th order model simujlaiton. (b) Control inputs: with units in grams per second.