15th Triennial World Congress, Barcelona, Spain

ON THE ACHIEVABLE CLOSED LOOP PERFORMANCE FOR DECENTRALIZED CONTROL STRUCTURES

Henning Schmidt * **Elling W. Jacobsen** *

* *S3-Process Control, Royal Institute of Technology, S-10044 Stockholm, Sweden*

Abstract: The paper addresses the effect of interactions on the performance of decentralized control systems. Most existing tools for analyzing the effect of interactions assume perfect control up to the system bandwidth, and furthermore neglect the phase contributions from subsystem interactions. Here we show that for most systems, and in particular large scale systems, the assumption of perfect control is a poor one. Furthermore, we show that the phase contributions of the interactions can be crucial both for stability and performance. Based on this we develop a new tool, based on finite bandwidth control, for measuring interactions in decentralized control systems. We demonstrate how this tool may be used to select configurations that provide the best performance under independent tuning of the subsystems.

Keywords: Decentralized control, control structure selection, multivariable control, interactions, large scale systems

1. INTRODUCTION

Control systems in large scale systems, e.g. chemical plants, are usually highly structured. They are typically decomposed into one or more optimization layers and at least two control layers. The lowest control layer, the regulatory control, is usually highly decentralized and implements the setpoints received from the supervisory control layer, which consists mostly of local multivariable controllers.

Some advantages of this intensive structuring of a control system are 1) robustness (reduced modelling costs) and reduced cost of controller design and maintenance, 2) increased flexibility, 3) tolerance to measurement and actuator failures and 4) operator understanding.

Of course, when control systems are so highly structured, it is important to have tools that can assist in selecting a structure which can provide acceptable robustness and performance. In particular, the structural decisions made at the regulatory level are crucial since the performance of this layer sets the limit for the performance of the overall control system.

Control structure design (CSD) regarding the regulatory control layer involves 1) defining the controlled variables, 2) selecting measurements and actuators to use for control, 3) selecting a structure of interconnections between measurements and actuators and 4) selecting the type of controllers (see, e.g., Skogestad and Postlethwaite (1996)).

In this paper we focus on the problem of selecting a structure of interconnections which yields the best performance under independent controller tuning of the subsystems. Note that we here distinguish between independent and dependent controller tuning. In the first case, considered here, the aim is to find a structure which minimizes the effect of interactions on the performance of the subsystems and the overall system, such that the subsystems can be tuned more or less independently. In the second case the subsystem interactions are taken into account when tuning the individual controllers, and may be utilized to improve the control performance in some subsystems. For instance, interactions may be utilized to remove non-minimum phase performance limitations in a subsystem (Cui and Jacobsen, 2001). Central to both these approaches is obviously an understanding of the interactions among the different subsystems and how these affect the control performance.

The most commonly used tool for measuring interactions and selecting control structures for single-loop controllers is still the Relative Gain Array (RGA) introduced by Bristol (1966). Several authors, e.g. Skogestad *et al.* (1990), have demonstrated practical applications of the RGA. Important advantages of the RGA are that it depends on the plant model only and that it is scaling independent. However, it is important to stress that the common pairing rules based on the RGA merely are rules of thumb, and that all theoretical results based on the RGA relates to stability properties only, such as Decentralized Integral Controllability (DIC) (see, e.g. Grosdidier *et al.* (1985)). Indeed, as shown by Schmidt and Jacobsen (2001), the commonly used pairing rules based on the RGA will often fail to select the best structure with respect to performance, in particular for systems larger than 2×2. This result should be considered in the light of the fact that most examples and case studies used in papers on control structure selection are limited to 2×2 systems.

Hovd and Skogestad (1992) introduced a performance related interaction measure, the performance relative gain array (PRGA), which can be used to determine the compensation needed for interactions in each subsystem, i.e., using dependent tuning, to achieve a given performance. The tool can hence also be used to determine structures for which the least interaction compensation is needed. However, also this tool is based on the assumption of perfect control up to the desired control system bandwidth and does not take phase contributions from the interactions into account.

In the following we consider independent, decentralized and finite bandwidth control. To avoid a strong dependence on the type of controller type used, we simply specify performance using a few key parameters such as bandwidth and phase margin.

The paper considers mainly single-loop controller, i.e., scalar subsystems, but extension to block-diagonal controllers is straightforward, as discussed in the paper.

2. EFFECT OF INTERACTIONS ON PERFORMANCE

Given a general square multivariable $n \times n$ system $G(s)$, the transfer matrix can be divided into its diagonal $\tilde{G}$ and its off diagonal elements $\bar{G}$, such that

$$G = \tilde{G} + \bar{G} \qquad (1)$$

The system is to be controlled by a diagonal controller K. Independent design implies that K is designed, or tuned, based on $\tilde{G}$ only and that the off-diagonal elements in $\bar{G}$ should have as little influence on the closed loop performance as possible. The desired performance can then be expressed in terms of the diagonal sensitivity $S_{\tilde{G}} = (I + \tilde{G}K)^{-1}$.

However, the interactions caused by the off-diagonal elements $\bar{G}$ will cause the diagonal elements of the overall sensitivity $S = (1 + GK)^{-1}$ to deviate from the desired one. The relationship between S and $S_{\tilde{G}}$ can be written (see, e.g., Grosdidier and Morari (1986))

$$S = \left(I + S_{\tilde{G}}\bar{G}K\right)^{-1} S_{\tilde{G}} = S_{\tilde{G}}\left(I + ET_{\tilde{G}}\right)^{-1} \qquad (2)$$

With $E = \bar{G}\tilde{G}^{-1}$, $T_{\tilde{G}} = I - S_{\tilde{G}}$ and $S_{\tilde{G}} = \left(I + \tilde{G}K\right)^{-1}$.

The effect of interactions on the overall sensitivity is thus given by $(I + ET_{\tilde{G}})^{-1}$, and will obviously depend on the design of the controller K.

In order to avoid the controller dependence when dealing with interactions, the assumption of perfect control is commonly employed, e.g., (Bristol, 1966), (Hovd and Skogestad, 1992). However, since the critical frequency region, both for performance and stability, is around the control system bandwidth, and the control usually is far from perfect around this frequency, we here introduce finite bandwidth control and then evaluate later whether this is important for determining the best structure. Since it is the interactions around the desired bandwidth of the subsystems which is of main concern, the controllers can be defined using a few key specifications such as desired bandwidth (ω_b) or desired crossover frequency (ω_c), phase margin (ϕ_m) and roll-off (n_{ro}). These specifications can be achieved using essentially any controller type, or design methodology, and the specific type chosen is not important. For convenience we here choose to use IMC to derive the controllers that yield the desired characteristics around the bandwidth. Thus we specify

$$\hat{S}_{\tilde{G}}(s) = F(s) = diag(f_1(s), \ldots, f_n(s)) \qquad (3)$$

Here $F(s)$ consists of specific transfer-functions representing the desired performance around the desired bandwidth, in terms of the desired sensitivity $\hat{S}_{\tilde{G}}(s)$, and is chosen according to the desired ω_b or ω_c, ϕ_m and n_{ro}.

Based on $F(s)$ in (3) it is possible to derive an IMC-based controller $\tilde{K}$ for the system $\tilde{G}$.

$$\tilde{K} = \tilde{G}_m^{-1}\left(F^{-1} - I\right) \approx \tilde{G}_m^{-1}\left(\hat{S}_{\tilde{G}}^{-1} - I\right) \qquad (4)$$

The term $\tilde{G}_m$ results from the separation of the system $\tilde{G} = diag(G)$ into a diagonal allpass transfer matrix $\tilde{A}$ and a diagonal minimum phase system $\tilde{G}_m(s)$.

$$\tilde{G} = \tilde{A}\tilde{G}_m \qquad (5)$$

This is important since for independent controller design RHP-zeros in the diagonal elements of the transfer matrix do matter, while for multivariable and dependent decentralized control the same RHP-zeros might not pose a problem (Cui and Jacobsen, 2001). A generalization to blockdiagonal controllers is straightforward by factorizing the multivariable subsystems in terms of *Blaschke products* (see, e.g., Skogestad and Postlethwaite (1996)).

Now, by using $\tilde{K}$ as the controller K for the overall system in (2) we can derive a relationship between the desired sensitivity and the achieved sensitivity S, which is valid around the desired bandwidth.

$$S = \hat{S}_{\tilde{G}} \left(I + ET_{\tilde{G}}\right)^{-1} = \hat{S}_{\tilde{G}} X \tag{6}$$

with

$$\begin{aligned} X &= X(\omega_b, \phi_m, n_{ro}) = \\ &= \left(I + \bar{G}\tilde{K}\left(I + \tilde{G}\tilde{K}\right)^{-1}\right)^{-1} = \\ &= \left[I + \bar{G}\tilde{G}_m^{-1}\left(\left(F^{-1} - I\right)^{-1} + \tilde{A}\right)^{-1}\right]^{-1} \end{aligned} \tag{7}$$

Under the assumption of perfect control instead of real control, i.e. $F \approx 0$, the ratio X becomes equal to the PRGA Γ (see equation (8)).

$$\begin{aligned} X &= \left[I + \bar{G}\tilde{G}_m^{-1}\left(\left(F^{-1} - I\right)^{-1} + \tilde{A}\right)^{-1}\right]^{-1} \approx \\ &\approx \left[I + \bar{G}\tilde{G}_m^{-1}\left(0 + \tilde{A}\right)^{-1}\right]^{-1} = \\ &= \left[I + \bar{G}\tilde{G}^{-1}\right]^{-1} = \tilde{G}G^{-1} = \Gamma \end{aligned} \tag{8}$$

This is natural because the ratio X was derived in a similar way as the PRGA (see Hovd and Skogestad (1992)). And as the ratio $X(s)$, the PRGA can be seen to represent a measure for the interactions between the different subsystems. The main difference between the two measures lies in the assumption of finite bandwidth control and perfect control, respectively. Below we consider whether this assumption is important or not.

3. SELECTING CONTROL CONFIGURATIONS BASED ON INTERACTIONS USING THE RATIO X

In this section a simple pairing tool based on the ratio X is derived and is then applied to two examples in the next section. These examples allow for a comparison of the usefulness of the finite bandwidth control approach with the perfect control approach.

For independent design it is desirable that the overall sensitivity S is as close as possible to the designed sensitivity $S_{\tilde{G}}$. Thus, from (6), the matrix X should be as close to the identity matrix as possible. This lead us to the definition of the ξ-measure.

$$\xi = max\left(\bar{\sigma}\left(X(i\omega) - I\right)\right) \quad \omega \in [\omega_b, \infty] \tag{9}$$

Note that the measure is evaluated from the desired bandwidth ω_b and upwards in frequency only. The reason for this is that this is the frequency region where the impact of interactions expectedly will be most significant, and also the region we have focused upon in the controller design above. For frequencies well below the bandwidth the sensitivity will usually be relatively small and thus the effect of interactions will be less significant. Also note that the measure is based on the assumption that the desired bandwidth is the same for all subsystems. This is of course a limitation since one way to deal with interactions often is to use different bandwidths in different subsystems. However, we believe that in most regulatory control systems it is preferable to have approximately the same bandwidth in all subsystems.

Based on the above, we propose the following pairing rule based on the ξ-measure

> *Pairing rule: The pairing, for which the ξ-measure in equation (9) is the smallest, should be preferred, when closed loop performance and independent controller design is an issue.*

It has to be noted, that the ξ-measure does not say anything about the stability properties of the different pairings. Therefor it should always be checked, that $X(s)$ in equation (7) is stable for the chosen $F(s)$. Furthermore, since the controllers for the single loops are designed without taking care of interactions, it should also be made sure, that the pairings not being DIC are excluded. DIC might, e.g., be checked using the RGA or the Niederlinski Index (Niderlinski, 1971).

4. EXAMPLES

In this section two examples are given to demonstrate the importance of considering interactions under finite bandwidth control when selecting control structures for performance. Pairings based on the RGA and the PRGA are compared to pairings based on the ξ-measure defined above.

4.1 *Example 1*

Hovd and Skogestad (1992) introduced the following system $G(s)$ as a counter example to the conventional RGA pairing rule.

$$G(s) = \frac{1-s}{(1+5s)^2}\begin{pmatrix} 1 & -4.19 & -25.96 \\ 6.19 & 1 & -25.96 \\ 1 & 1 & 1 \end{pmatrix} \tag{10}$$

The RGA of the system is frequency independent and given by

$$\Lambda(i\omega) = \begin{pmatrix} 1 & 5 & -5 \\ -5 & 1 & 5 \\ 5 & -5 & 1 \end{pmatrix} \tag{11}$$

A common rule of thumb is to pair on subsystems corresponding to RGA-elements with magnitude closest to 1. Thus, the diagonal pairing should be preferred. However, Hovd and Skogestad found that pairing on the +1 RGA elements resulted in a poor closed loop performance with a maximum bandwidth of approx. 0.00086 rad/s and that pairing on the +5 elements led to a better performance with a bandwidth of approx. 0.0045 rad/s. In both cases PID-controllers were employed, and the tuning was based on maximizing the bandwidth subject to the maximum singular value of the sensitivity being restricted to be less than 2 ($\|S\|_\infty \leq 2$). It is clear to the authors that these bandwidths are far below the achievable bandwidth when using a full multivariable controller. Actually, when decentralized PI-control is used, a slightly higher bandwidth (0.003 rad/s) in each loop for the $+1$ pairing will make the closed loop system unstable. However, even if decentralized control may not seem appropriate for this system, a pairing tool should be able to select the best possible pairing.

Using the pairing rule from section 3 in a perfect control version (i.e. replacing $X(s)$ with the PRGA Γ) we get the ξ values given in Table 1. As seen from the table, the measure based on the PRGA supports the rule of thumb and proposes the +1-RGA element pairing to be the least interactive.

Pairing output/input	ξ_Γ
1/1 2/2 3/3 (+1 RGA elements)	7.15
1/2 2/3 3/1 (+5 RGA elements)	34.76

Table 1. The PRGA-ξ-measure computed for the different pairings of example 1.

If we instead of perfect control assume finite bandwidth control in the subsystems, with desired single loop bandwidths $\omega_{bi} = 0.001$, roll-off -1 and phase margin 90^o, we get the ξ-measures in Table 2. As seen from the table the pairing corresponding to the RGA-elements of $+5$ now appears to be significantly better than the pairing corresponding to $+1$ RGA-elements. Thus, relaxing the assumption of perfect control results in a recommendation contrary to that found with perfect control.

Pairing output/input	ξ
1/1 2/2 3/3 (+1 RGA elements)	19.05
1/2 2/3 3/1 (+5 RGA elements)	3.84

Table 2. The ξ-measure computed for the different pairings of example 1 using the measure X with desired bandwidths $\omega_i =$ 0.001 rad/s.

Figure 1 shows the maximum singular value of the difference between X and the identity matrix I, i.e. $\bar{\sigma}(X(i\omega) - I)$, for the two pairings and desired bandwidths $\omega_{bi} = 0.001$ and 0.01, evaluated over frequency. The figure shows more clearly the differences between the pairings. We can see that at low frequencies, where the control is close to perfect, the effect

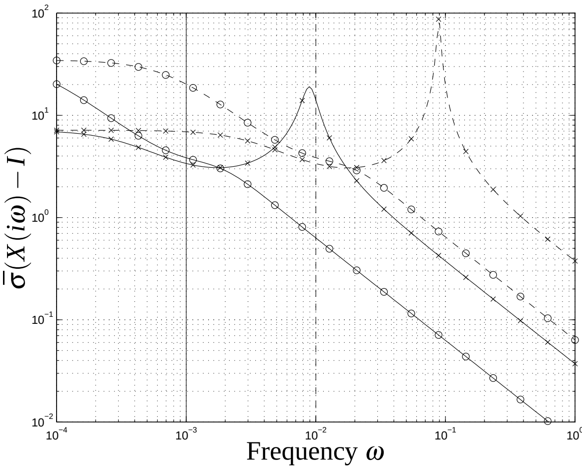

Fig. 1. Maximum singular values of $(X - I)$ plotted for pairings on +1 (x) and on +5 (o) RGA-elements for different bandwidths (solid: $\omega_i = 0.001$ rad/s and dashed: $\omega_i = 0.01$ rad/s)

of interactions is smaller for the pairing on $+1$ RGA-elements. At the desired bandwidth the effect of interactions are similar for the two pairings. However, for higher frequencies there are severe effects of the interactions for the $+1$-pairing but only small effects for the $+5$-pairing. The severity of the interactions are also seen to increase with increasing bandwidth for the $+1$-pairing, and as seen from the peak in $\bar{\sigma}(X - I)$ one can expect instability with this pairing for higher bandwidths. Also note that the severe effects of interactions for the $+1$-pairing implies that the achieved bandwidth in general will deviate significantly from the desired one.

Figure 2 shows the desired and achieved sensitivities, in the chosen subsystems, with the two pairings and desired bandwidths $\omega_{bi} = 0.001$ (PI-controllers were used). For low frequencies we again find that the RGA correctly predicts that the interactions have little effect on the subsystem performance when pairing on $+1$-elements. However, close to the desired bandwidth we see a significant effect of interactions for this pairing due to non-perfect control in the subsystems. However, for the $+5$ pairing the effect of interactions are relatively small around the bandwidth as predicted by the ξ-measure above.

In order to understand why the assumption of perfect control leads to erroneous conclusions with respect to the effect of interactions under finite bandwidth control, we employ the interaction measure dRGA proposed by Schmidt and Jacobsen (2001). The dRGA measures the effects of interactions on the magnitude and phase of a subsystem when the other systems are under finite bandwidth and decentralized control. We here consider the bandwidth $\omega_{bi} = 0.001$. In Figure 3 the magnitude and phase of the dRGA for the two pairings are shown. The magnitude plot shows the relative decrease in the gain of the subsystems (a value less than 1 implies a gain increase), while the phase plot shows the phase decrease (a positive

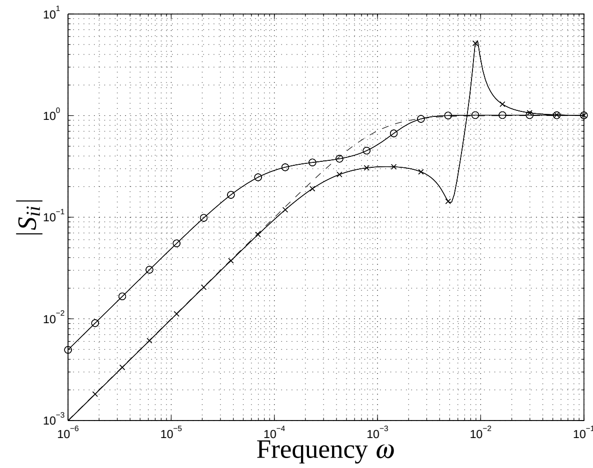

Fig. 2. Desired (dashed) and achieved subsystem sensitivities for pairings on +1 (x) and +5 (o) RGA/elements. PI-control realizing bandwidths of $\omega_{bi} = 0.001$ rad/s in the individual subsystems.

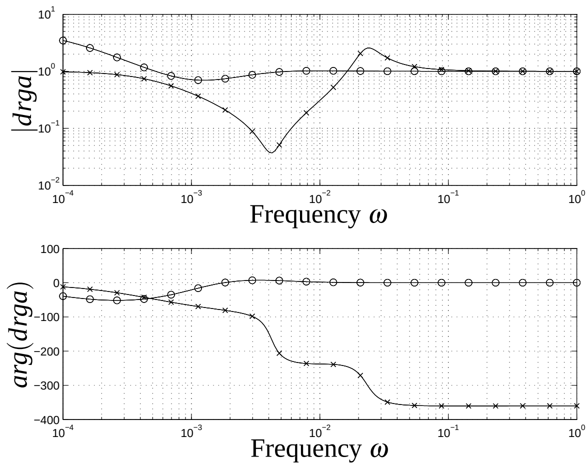

Fig. 3. Magnitude and phase of the dRGA for subsystems corresponding to +1 (x) and +5 (o) RGA-elements. Desired bandwidth in subsystem $i = 1,2$ $\omega_{bi} = 0.001$ rad/s

value implies phase loss) due to interactions. Note that for the two pairings the corresponding RGA elements would show magnitude 1 and 5, respectively, and the phase change prediction would be 0 for both pairings. However, from the dRGA we see that the interactions for the $+1$ pairing causes a very large gain increase and simultaneous phase loss at frequencies above the desired bandwidth. Compared to that, the effects for the $+5$ pairing around the desired bandwidth are relatively small.

It is important to stress that the above results can not be explained by the difference between perfect control and finite bandwidth control of a single scalar system. Rather, as shown by Schmidt and Jacobsen (2001), the main reason is that the performance of the $(n-1) \times (n-1)$ subsystems considered when computing the RGA and dRGA elements of an $n \times n$ system will be highly different from the performance of the individual scalar subsystems. This is due to the fact that also these $(n-1) \times (n-1)$ subsystems are under decentralized control, while the RGA and PRGA assume these subsystems to be under perfect full multivariable control. In fact, as discussed by Schmidt and Jacobsen (2001), for 2×2 systems the RGA usually gives good predictions of the effects of interactions, since in this case the $(n-1) \times (n-1)$ subsystems are scalar and hence under "full" control.

4.2 *Example 2*

As a second example we consider the following 3×3 system

$$G(s) = \frac{1+2s}{(1+10s)^2} \begin{pmatrix} 0.53 & -0.74 & 1.00 \\ 0.81 & 0.73 & 0.63 \\ -0.79 & 0.42 & 0.56 \end{pmatrix} \quad (12)$$

The performance parameters, defining the desired performance of the single controlled loops around the bandwidth are given by a crossover frequency of $\omega_c = 1$ rad/s, a roll-off of $n_{ro} = -1$ and a phase margin of $\phi_m = 70$ degrees for all loops. Using IMC, this corresponds to the desired diagonal sensitivities

$$[F]_{ii} = \frac{s^2 + 0.70s}{s^2 + 1.401s + 1} \quad , \quad i = 1, \ldots, n \quad (13)$$

Table 3 displays the ξ-measures calculated for the desired performance and all the 6 different pairings. The *DIC*-column in Table 3 indicates whether the pairing is decentralized integral controllable, this means, that all steady state RGA elements corresponding to the controlled elements are positive, when looking at the overall system G(s) and at all the 2x2 subsystems containing two controlled elements on their diagonal. This is important to check, since controllers having an integral part are going to be used and stability would not be achieved by independent design. Furthermore, because of zeros eventually crossing from the left half plane into the right half plane due to control (see Cui and Jacobsen (2001)), even for dependent controller design the achievable performance might not be very good. It can be seen, that the 4th pairing (also ranked

Pairing Nr	out/in	ξ	DIC
1	**1/1 2/2 3/3**	**86.97**	y
2	1/2 2/1 3/3	8.47	y
3	**1/3 2/2 3/1**	**3.09**	y
4	1/1 2/3 3/2	**1.21**	**n**
5	1/2 2/3 3/1	9.65	y
6	1/3 2/1 3/2	13.93	y

Table 3. ξ-measure pairing tool used for the 6 candidate pairings of the second example

as best one by the ξ-measure) has to be excluded from further consideration, since it is not DIC.

To see if the ξ-measure is able to predict the differences in the achievable performance for certain pairings, we compare the 3rd pairing, considered (second) best by the ξ-measure, with the 1st pairing, considered

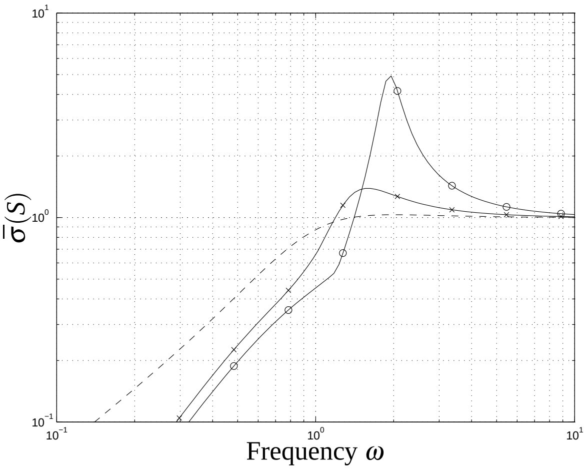

Fig. 4. Maximum singular value of the sensitivity function plotted for the two suggested pairings (x: 3rd, best pairing) and (o: 1st, worst pairing). The dashed line shows the desired performance in terms of $[F]_{ii}$. PI-controllers achieving crossover-frequencies of 1 rad/s and phase margins of 70 degrees and a roll of of -1 in the single loops were used.

worst. To do so, we design single-loop PI-controllers to achieve crossover-frequencies of 1 rad/s and phase margins of 70 degrees in the single loops. The maximum singular value of the diagonal sensitivities for the two pairings are displayed in Figure 4.

It can clearly be seen that the closed loop performance for the 1st pairing, which the ξ-measure identified as the worst pairing, is much further away from the desired performance, than the closed loop performance for the 3rd pairing. The desired closed loop performance is given by the dashed line. This result is supported by the closed loop step responses for the three outputs, shown in Figure 5. The same simple PI-controllers used for Figure 4 have been employed. The step responses for the corresponding off-diagonal elements are not shown.

The differences between the pairings become even more obvious if the PI controllers are tuned, such that the phase margins are smaller, e.g. 60 degrees. In this case the 1st pairing is very close to instability, while the 3rd pairing still shows good performance.

5. CONCLUSIONS

The paper considers closed loop performance under decentralized control. It is shown that the assumption of perfect control up to the bandwidth is a poor one for the evaluation of the achievable performance of a certain decentralized control structure, in particular for systems larger than 2×2. Since the interactions should be expected to have the most significant effect around the bandwidth of the subsystems, we have in this paper suggested a closed-loop interaction measure which takes finite bandwidth and decentralized control

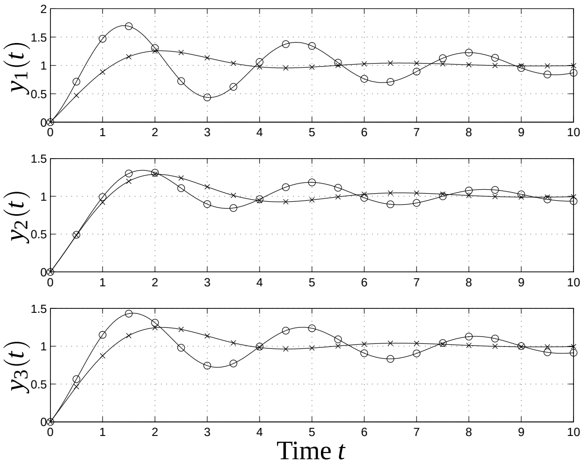

Fig. 5. Closed loop step responses for the two suggested pairings (x: 3rd, best pairing) and (o: 1st, worst pairing). PI-controllers achieving crossover-frequencies of 1 rad/s and phase margins of 70 degrees and a roll of of -1 in the single loops were used.

into account. The control has been defined in such a way, that, around the desired bandwidth, the behavior of the system is independent of the controller type employed. Based on the measure, a simple model based tool for control structure selection has been derived and successfully applied to two simple examples.

6. REFERENCES

Bristol, E.H. (1966). On a new measure of interactions for multivariable process control. In: *IEEE Trans. Autom. Control*.

Cui, H. and E.W. Jacobsen (2001). Performance limitations in decentralized control. *Journal of Process Control*. To appear.

Grosdidier, P. and M. Morari (1986). Interaction measures for systems under decentralized control. *Automatica*.

Grosdidier, P., M. Morari and B.R. Holt (1985). Closed-loop properties from steady-state gain information. *Industrial and Engineering Chemistry Process Design and Development*.

Hovd, M. and S. Skogestad (1992). Simple frequency dependent tools for control system analysis, structure selection and design. *Automatica*.

Niderlinski, A. (1971). A heuristic approach to the design of linear multivariable interacting control systems. *Automatica* **7**, 691–701.

Schmidt, H. and E. Jacobsen (2001). Selecting control configurations for performance. In: *ESCAPE 11*. Kolding, Denmark.

Skogestad, S. and I. Postlethwaite (1996). *Multivariable feedback control*. John Wiley and Sons.

Skogestad, S., P. Lundström and E. Jacobsen (1990). Selecting the best distillation control configuration. *AIChE Journal*.