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OUTPUT FEEDBACK SLIDING MODE CONTROL FOR MIMO DISCRETE TIME SYSTEMS

Chakravarthini M. Saaj and **B. Bandyopadhyay**

Interdisciplinary Programme in
Systems and Control Engineering,
Indian Institute of Technology Bombay,
Mumbai 400076, India.
chakru@ee.iitb.ac.in
bijnan@ee.iitb.ac.in
Tel:(91)-(22) 5767889
Fax:(91)-(22) 5723480

Abstract: In this paper we present a new algorithm for discrete sliding mode control of linear multivariable systems using fast output sampling feedback. Here we make use of a single switching surface regardless of the number of inputs. The main contribution of this work is that instead of using the system states, the output samples are used for designing the controller. Numerical example demonstrates the design technique.

Keywords: Discrete systems, Output feedback, Sliding mode control, Multirate sampling, Multivariable systems.

1. INTRODUCTION

The digital implementation of sliding mode control (SMC) for single-input single-output systems is done by Furuta (1990) and Gao, *et al.*, (1995). The solution to multi-input multi-output (MIMO) systems with matched parametric uncertainties, using switching sector is done by Wang, *et al.*, (1996). The application of discrete sliding mode control (DSMC) to linear dynamic systems is discussed in (Misawa, 1997). The DSMC for linear multivariable systems with unmatched additive uncertainties, utilizing a single hyperplane regardless of the number of inputs has been reported by Tang and Misawa (2000). This allows the use of well-established linear control design strategies under an eigenvalue constraint.

Most of the sliding mode control methods require full-state feedback. In practical situations measurement of all the system states might be neither possible nor feasible. Such situations would demand some observers or dynamic compensators which would make the overall system more complex. In (Tang and Misawa, 2000) the controller design is extended to output feedback using a prediction observer with bias estimation and the stability is examined. Since the output is available, output feedback can be used to design the controller. The static output feedback problem is one of the most investigated problems in control theory and application (Syrmos, *et al.*, 1997).

Output feedback can be realized using fast output sampling feedback (Werner, 1998) or by periodic output feedback (Werner and Furuta, 1995). Werner (1998) has used the fast output sampling (FOS) feedback which has the features of static output feedback and makes it possible to arbitrarily assign the system poles. Unlike static output feedback, fast output sampling feedback always guarantee the stability of the closed loop system. An output feedback sliding mode controller is developed by Chakravarthini and Bandyopadhyay (2001) for the two stage state feedback sliding mode controller proposed by Furuta (1990). A new algorithm for discrete-time sliding mode control using the fast output sampling feedback and reaching law approach is reported in (Chakravarthini, *et al.*, 2002).

This paper presents an new algorithm for sliding mode control of the discrete-time multivariable systems with unmatched additive uncertainties by converting the discrete sliding mode control proposed by Tang and Misawa (2000) into a fast output sampling sliding mode control. The proposed technique does not use any observer as is the case with output feedback in (Tang and Misawa, 2000). Also by this method both the switching function evaluation and feedback can be performed using the multirate output samples. The state feedback gain and the hyperplane design are carried out as proposed in (Tang and Misawa, 2000). The outline of this paper is as follows: a brief review of the preliminary results are presented in Section 2. In Section 3 the design procedure for fast output sampling sliding mode control is explained, followed by a numerical example in Section 4 to illustrate the advantages of the proposed method. Concluding remarks are drawn in Section 5.

2. PRELIMINARIES

2.1 *Fast Output Sampling Feedback*

Consider the single-input single-output discrete-time system $(\mathbf{\Phi}_\tau,\ \mathbf{\Gamma}_\tau,\ \mathbf{C})$ with sampling period τ. The matrices $\mathbf{\Phi}_\tau, \mathbf{\Gamma}_\tau$, and $\mathbf{C}$ are of appropriate dimensions. It is assumed that the pair $(\mathbf{\Phi}_\tau, \mathbf{\Gamma}_\tau)$ is controllable and the pair $(\mathbf{\Phi}_\tau, \mathbf{C})$ is observable. Let $(\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{C})$ be the system sampled at the rate $1/\Delta$, where $\Delta = \tau/N$. Let ν denote the observability index of $(\mathbf{\Phi}, \mathbf{C})$. N is chosen to be greater than or equal to ν. Output measurements are taken at time instants $t = l\Delta$, $l = 0, 1, \ldots N-1$. The control signal $u(t)$ which is applied during the interval $k\tau \leq t < (k+1)\tau$ is then constructed as a linear combination of the last N output observations. The control law is of the form (Werner, 1998)

$$u(k) = \mathbf{L}\mathbf{y}_k. \tag{1}$$

For $\mathbf{L}$ to realize the effect of the state feedback gain $\mathbf{F}$ it must satisfy the relation

$$\mathbf{L}\tilde{\mathbf{C}} = \mathbf{F}, \tag{2}$$

Here $\tilde{\mathbf{C}}$ is the fictitious measurement matrix defined as

$$\tilde{\mathbf{C}} = (\mathbf{C}_0 + \mathbf{D}_0\mathbf{F})(\mathbf{\Phi}_\tau + \mathbf{\Gamma}_\tau\mathbf{F})^{-1},$$

where

$$\mathbf{C}_0 = \begin{bmatrix} \mathbf{C} \\ \mathbf{C\Phi} \\ \vdots \\ \mathbf{C\Phi}^{N-1} \end{bmatrix}, \mathbf{D}_0 = \begin{bmatrix} 0 \\ \mathbf{C\Gamma} \\ \vdots \\ \mathbf{C}\sum_{j=0}^{N-2} \mathbf{\Phi}^j\mathbf{\Gamma} \end{bmatrix}.$$

2.2 *State feedback approach to DSMC*

Consider the discrete-time linear multivariable system

$$\mathbf{x}(k+1) = \mathbf{\Phi}_\tau\mathbf{x}(k) + \mathbf{\Gamma}_\tau\mathbf{u}(k) + \boldsymbol{\xi}\mathbf{w}(k),$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k),$$

where τ is the sampling period, $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^m$, $\mathbf{w} \in \mathbf{R}^q$ are the state, input and additive uncertainty vectors respectively, $\mathbf{\Phi}_\tau$, $\mathbf{\Gamma}_\tau$ and $\boldsymbol{\xi}$ are known constant matrices with appropriate dimensions and $\mathbf{\Gamma}_\tau$ is defined as $\mathbf{\Gamma}_\tau = [\mathbf{\Gamma}_{\tau_1} \mid \mathbf{\Gamma}_{\tau_2} \mid \ldots \mid \mathbf{\Gamma}_{\tau_m}]$, $\mathbf{\Gamma}_{\tau_i} \in \mathbf{R}^n$. The matching condition $rank\left(\begin{bmatrix} \mathbf{\Gamma}_\tau & \boldsymbol{\xi} \end{bmatrix}\right) = rank(\mathbf{\Gamma}_\tau)$ is not necessarily satisfied. The objective is to design a DSMC that would force the system state vector $\mathbf{x}(k)$ to track the reference trajectory defined by the vector $\mathbf{x}_d(k)$ i.e., to drive the tracking error $\tilde{\mathbf{x}}(k) = \mathbf{x}_d(k) - \mathbf{x}(k)$ to zero, or make it as small as possible (Tang and Misawa, 2000). Here it is assumed that the desired trajectory $\mathbf{x}_d(k+1)$ is "consistently generated" by a model-based $\mathbf{x}_d$-generator using the nominal system (Misawa, 1997)

$$\mathbf{x}_d(k+1) = \mathbf{\Phi}_\tau\mathbf{x}_d(k) + \mathbf{\Gamma}_\tau\mathbf{u}_d(k), \tag{3}$$

where $\mathbf{u}_d(k)$ is the hypothetical input. The switching variable $s \in \mathbf{R}$, the switching surface $\mathcal{S} \subset \mathbf{R}^n$, the boundary layer $\mathcal{B} \subset \mathbf{R}^n$ are respectively defined as

$$s(k) = \mathbf{c}^T\tilde{\mathbf{x}}(k), \tag{4}$$
$$\mathcal{S} = \{\tilde{\mathbf{x}} : s = \mathbf{c}^T\tilde{\mathbf{x}} = 0\},$$
$$\mathcal{B} = \{\tilde{\mathbf{x}} : \mid s \mid = \mid \mathbf{c}^T\tilde{\mathbf{x}} \mid \leq \phi\}.$$

where $\mathbf{c}^T \in \mathbf{R}^n$ and ϕ is the boundary layer thickness. The row vector $\mathbf{c}^T$ is determined such that $\mathbf{c}^T\mathbf{\Gamma}_{\tau_i} \neq 0, 1 \leq i \leq m$ and $\| \mathbf{c}^T \| = 1$. The control law that guarantees sliding motion proposed by Tang and Misawa (2000) is

$$\mathbf{u}(k) = (\mathbf{\Gamma}_\tau^T\mathbf{\Gamma}_\tau)^{-1}\mathbf{\Gamma}_\tau^T(\mathbf{x}_d(k+1) - \mathbf{\Phi}_\tau\mathbf{x}_d(k)) + \ldots \mathbf{\Psi}^{-1}\mathbf{M}(\mathbf{x}_d(k) - \mathbf{x}(k)) + \ldots \mathbf{\Psi}^{-1}\mathbf{K}sat\left(\frac{s(k)}{\phi}\right), \tag{5}$$

where

$$\mathbf{\Psi} = \mathrm{diag}(\mathbf{c}^T\mathbf{\Gamma}_{\tau_1}, \ldots, \mathbf{c}^T\mathbf{\Gamma}_{\tau_m}),$$
$$\mathbf{M} = [\mu_1 \ldots \mu_m]^T, \mu_i \in \mathbf{R}^n,$$
$$\mathbf{K} = [K_1 \ldots K_m]^T, K_i \in \mathbf{R},$$
$$\sum_{i=1}^n \mu_i^T = \mathbf{c}^T(\mathbf{\Phi}_\tau - \mathbf{I}), \quad \mathbf{I} = identity\ matrix,$$
$$\sum_{i=1}^m K_i = K_\Sigma = \gamma + 2\Delta t\epsilon, \epsilon > 0,$$
$$\phi > \gamma + \Delta t\epsilon,$$
$$\gamma \geq \mid \mathbf{c}^T\boldsymbol{\xi}\mathbf{w}(k) \mid .$$

For proof of stability see Tang and Misawa (2000). Simplifying (5) using (3) the control law reduces to the form

$$\mathbf{u}(k) = \mathbf{u}_d(k) + \mathbf{\Psi}^{-1}\mathbf{M}(\mathbf{x}_d(k) - \mathbf{x}(k)) + \mathbf{\Psi}^{-1}\mathbf{K}sat\left(\frac{s(k)}{\phi}\right). \quad (6)$$

3. FAST OUTPUT SAMPLING SLIDING MODE CONTROL

Case 1: $|s(k)| > \phi$
The state feedback control law (6) takes the form

$$\mathbf{u}(k) = \mathbf{F}_1\mathbf{x}(k) + \boldsymbol{\zeta}(k). \quad (7)$$

where

$$\boldsymbol{\zeta}(k) = -\mathbf{F}_1\mathbf{x}_d(k) + \mathbf{u}_d(k) \pm \boldsymbol{\rho},$$
$$\mathbf{F}_1 = -\mathbf{\Psi}^{-1}\mathbf{M},$$
$$\boldsymbol{\rho} = \mathbf{\Psi}^{-1}\mathbf{K}.$$

Now consider that a state feedback control of the form (7) is applied to the fictitious, lifted system (Werner, 1998)

$$\mathbf{x}(k+1) = \mathbf{\Phi}_\tau\mathbf{x}(k) + \mathbf{\Gamma}_\tau\mathbf{u}(k),$$
$$\mathbf{y}_{k+1} = \mathbf{C}_0\mathbf{x}(k) + \mathbf{D}_0\mathbf{u}(k).$$

Then the closed loop system is given by

$$\mathbf{x}(k+1) = (\mathbf{\Phi}_\tau + \mathbf{\Gamma}_\tau\mathbf{F}_1)\mathbf{x}(k) + \mathbf{\Gamma}_\tau\boldsymbol{\zeta}(k) \quad (8)$$
$$\mathbf{y}_{k+1} = (\mathbf{C}_0 + \mathbf{D}_0\mathbf{F}_1)\mathbf{x}(k) + \mathbf{D}_0\boldsymbol{\zeta}(k) \quad (9)$$

From (9) $\mathbf{x}(k)$ can be represented as

$$\mathbf{x}(k) = (\mathbf{\Phi}_\tau + \mathbf{\Gamma}_\tau\mathbf{F}_1)^{-1}\mathbf{x}(k+1) - \mathbf{\Phi}_\tau + \mathbf{\Gamma}_\tau\mathbf{F}_1)^{-1}\mathbf{\Gamma}_\tau\boldsymbol{\zeta}(k). \quad (10)$$

Substituting $\mathbf{x}(k)$ from (10) in (8), and simplifying one gets

$$\mathbf{y}_{k+1} = \tilde{\mathbf{C}}_1\mathbf{x}(k+1) + (\mathbf{D}_0 - \tilde{\mathbf{C}}_1\mathbf{\Gamma}_\tau)\boldsymbol{\zeta}(k),$$

where

$$\tilde{\mathbf{C}}_1 = (\mathbf{C}_0 + \mathbf{D}_0\mathbf{F}_1)(\mathbf{\Phi}_\tau + \mathbf{\Gamma}_\tau\mathbf{F}_1)^{-1}.$$

Let

$$\boldsymbol{\eta}(k) = (\mathbf{D}_0 - \tilde{\mathbf{C}}_1\mathbf{\Gamma}_\tau)\boldsymbol{\zeta}(k).$$

Therefore $\mathbf{y}_k$ can be represented as

$$\mathbf{y}_k = \tilde{\mathbf{C}}_1\mathbf{x}(k) + \boldsymbol{\eta}(k-1).$$

The equivalent output feedback control law for (7) is

$$\mathbf{u}(k) = \mathbf{L}_1\mathbf{y}_k + \boldsymbol{\delta}(k), \quad (11)$$

where

$$\mathbf{L}_1 = \mathbf{F}_1\tilde{\mathbf{C}}_1^{-1},$$
$$\boldsymbol{\delta}(k) = \boldsymbol{\zeta}(k) - \mathbf{L}_1\boldsymbol{\eta}(k-1).$$

The switching function is related to the output samples $\mathbf{y}_k$ by

$$s(k) = \mathbf{c}^T[\mathbf{x}_d(k) - \tilde{\mathbf{C}}_1^{-1}(\mathbf{y}_k - \boldsymbol{\eta}(k-1))]. \quad (12)$$

Case 2: $|s(k)| \leq \phi$
Inside the boundary layer the state feedback control law is

$$\mathbf{u}(k) = \mathbf{F}_2\mathbf{x}(k) + \boldsymbol{\alpha}(k), \quad (13)$$

where

$$\mathbf{F}_2 = -\mathbf{\Psi}^{-1}\mathbf{M} - \mathbf{\Psi}^{-1}\phi^{-1}\mathbf{K}\mathbf{c}^T,$$
$$\boldsymbol{\alpha}(k) = -\mathbf{F}_2\mathbf{x}_d(k) + \mathbf{u}_d(k).$$

The equivalent output feedback control law for (13) takes the form

$$\mathbf{u}(k) = \mathbf{L}_2\mathbf{y}(k) + \boldsymbol{\beta}(k), \quad (14)$$

where

$$\mathbf{L}_2 = \mathbf{F}_2\tilde{\mathbf{C}}_2^{-1},$$
$$\tilde{\mathbf{C}}_2 = (\mathbf{C}_0 + \mathbf{D}_0\mathbf{F}_2)(\mathbf{\Phi}_\tau + \mathbf{\Gamma}_\tau\mathbf{F}_2)^{-1},$$
$$\mathbf{y}_k = \tilde{\mathbf{C}}_2\mathbf{x}(k) + \boldsymbol{\sigma}(k-1),$$
$$\boldsymbol{\sigma}(k) = (\mathbf{D}_0 - \tilde{\mathbf{C}}_2\mathbf{\Gamma}_\tau)\boldsymbol{\alpha}(k),$$
$$\boldsymbol{\beta}(k) = \boldsymbol{\alpha}(k) - \mathbf{L}_2\boldsymbol{\sigma}(k-1).$$

The switching function is related to the output samples $\mathbf{y}_k$ by

$$s(k) = \mathbf{c}^T[\mathbf{x}_d(k) - \tilde{\mathbf{C}}_2^{-1}(\mathbf{y}_k - \boldsymbol{\sigma}(k-1))]. \quad (15)$$

Thus we see that the need to use the states of the system for feedback purpose as well as for switching function evaluation is completely eliminated by fast output sampling sliding mode control technique.
Remark The sliding mode using fast output sampling feedback will occur if the number of fast output samples $N_1, ...N_p$ for each of the p output channels of a $n-th$ order system with m inputs is chosen as $N_1 + N_2 + ... + N_p = n$.

4. NUMERICAL EXAMPLE

Consider the continuous-time system described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{w}(t),$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t),$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & -4 & -3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 3 & -2 & -3 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The equivalent discrete-time system $(\mathbf{\Phi}_\tau, \mathbf{\Gamma}_\tau, \mathbf{C})$ is obtained for $\tau = 0.2\,\text{sec}$. Let

$$\mathbf{w}(k) = \begin{bmatrix} 0 & \sin(3.5\pi k) & 0 \end{bmatrix}^T.$$

Here the disturbance is unmatched. Let the initial values of the states be

$$\mathbf{x}(0) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}^T$$

and the desired states be

$$\mathbf{x}_d(k) = \begin{bmatrix} 2 & 0 & 4 & 0 & 6 & 0 \end{bmatrix}^T.$$

The control objective is to make the tracking error small. The LQR technique is used to design $\mathbf{A}_{eq}$. The state feedback gain $\mathbf{F}$ is

$$\begin{bmatrix} 1.8 & 2.01 & 0.61 & 0.49 & -0.42 & 0.35 \\ 3.83 & 0.67 & 12.26 & 5.95 & 2.54 & 4.03 \end{bmatrix}.$$

The switching gain is

$$\mathbf{c}^T = \begin{bmatrix} -0.14 & -0.005 & -0.94 & -0.14 & -0.16 & -0.21 \end{bmatrix}.$$

Let

$$\mid w(k) \mid \leq 1, \gamma = 0.0471, \mathbf{K} = \begin{bmatrix} 0.001 & 0.1 \end{bmatrix}^T.$$

Then

$$K_\Sigma = 0.101, \epsilon = 0.1348, \phi = 0.1473$$

and $\mathbf{M}$ is

$$\begin{bmatrix} -0.007 & -0.008 & 0.004 & -0.001 & 0.003 & -0.0001 \\ -0.078 & -0.027 & 0.085 & -0.175 & -0.006 & -0.037 \end{bmatrix}.$$

Let $N = 6$ and $\Delta = 0.033$. The state feedback gains $\mathbf{F}_1$ is

$$\begin{bmatrix} -1.58 & -1.99 & 0.86 & -0.27 & 0.66 & -0.02 \\ -1.73 & -0.59 & 1.89 & -3.86 & -0.14 & -0.81 \end{bmatrix}$$

and $\mathbf{F}_2$ is

$$\begin{bmatrix} -1.8 & -2.1 & -0.6 & -0.5 & 0.4 & -0.4 \\ -3.8 & -0.7 & -12.2 & -5.9 & -2.5 & -4.1 \end{bmatrix}.$$

The equivalent output feedback gains obtained are $\mathbf{L}_1 = 1.0e + 005*$

$$\begin{bmatrix} 0.03 & 0.01 & 0.02 & -0.001 & 0.002 & 0.001 \\ 1.42 & -3.51 & 2.09 & 0.05 & -0.02 & -0.06 \end{bmatrix}$$

and $\mathbf{L}_2 = 1.0e + 005*$

$$\begin{bmatrix} -0.3 & 0.6 & -0.4 & -0.01 & 0.01 & 0.012 \\ 1.7 & -4.2 & 2.5 & 0.06 & -0.03 & -0.08 \end{bmatrix}.$$

The simulation results of the sixth order two-input two-output discrete time system in the presence of bounded disturbance is shown in Figure 1. The results are satisfactory. Figure 1(a) and (b) show the response of the states. The states $x_1(k)$, $x_3(k)$,and $x_5(k)$ are tracked to the desired values 2, 4 and 6 respectively. The states $x_2(k)$, $x_4(k)$ and $x_6(k)$ are tracked very close to zero. The effect of the disturbance is evident in the transient. The state variables approach the origin despite the presence of any perturbation. Though the tracking error is not exactly zero, it is acceptably small. The control function plot is shown in Figure 1(c). The plot of the switching variable shown in Figure 1(d) approaches zero from the initial value and stays within the boundary layer $\mathcal{B}$. Inside the boundary layer the amplitude of the switching variable decreases towards zero. It is clearly evident from the plot of the switching variable that the steady state value is very much interior to the set boundary layer thickness 2ϕ. Thus we see that the boundary layer $\mathcal{B}$ is attractive and invariant, and good disturbance rejection is attained. The plots of the outputs $y_1(k)$ and $y_2(k)$ are shown in Figure 1 (e) and (f).

5. CONCLUDING REMARKS

A new approach for the design of fast output sampling sliding mode control using one switching hyperplane for linear discrete-time MIMO systems has been presented in this paper. The key advantage is that in this technique the system states are neither used for feedback purpose nor for switching function evaluation. Only the multirate output samples are used for the above purposes. Since the effect of the state feedback gain is realized using an output feedback gain, the stability is guaranteed. Also the use of observer is eliminated thereby the complexity of the overall system is reduced.

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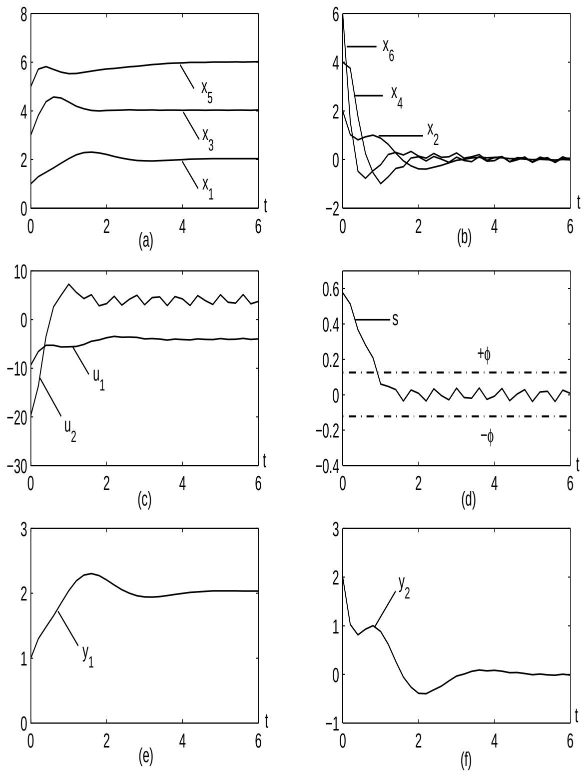

Fig. 1. Simulation results: (a) System states x_1, x_3, x_5 (b) System states x_2, x_4, x_6 (c) Control inputs u_1 and u_2 (d) Switching variable s (e) Plant output y_1 (f) Plant output y_2

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