15th Triennial World Congress, Barcelona, Spain

DELAY-DEPENDENT ROBUST H_∞ FILTERING FOR UNCERTAIN STATE DELAYED SYSTEM

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Abstract: The delay-dependent robust H_∞ Filtering for uncertain state delay system is addressed in this paper. The parameter uncertainties are time-varying and unknown but norm-bounded. The present robust filtering methods for time-delay system are nearly all independent of time-delay, which is conservative, and often can't reflect the characters of system accurately. Therefore, to design the delay-dependent filtering method is significant. Based on appropriate Lyapunov function, the necessary condition that the satisfactory filter exists is given by the form of LMI, and the concrete expression of filter is also obtained. *Copyright © 2002 IFAC*

Keywords: Robust estimation, Time-delay, Uncertain dynamic systems, H-infinite, Lyapunov function,

1. INTRODUCTION

In the past decade, the H_∞ filter design received a considerable amount of attention. Unlike the conventional Kalman filter, H_∞ filter does not need the statistical information on noise, and the only requirement for the noise is that it has bounded energy. The H_∞ norm, which reflects the worst-case gain of the system, is minimized in H_∞ filter design, therefore the filter tends to have good robust property. Owning to the widespread uncertainties of system model and noise disturbance, robust H_∞ filtering for uncertainty system has emerged. To solve robust H_∞ filter design problem, algebraic Riccati equation (ARE) approach, see (Fu, *et al*., 1992 and Zhang, 1999). for norm-bounded uncertainty is most often used. But for complex system such as with high-order or with some constraints, ARE is difficult to solve. Recently, linear matrix inequalities (LMIs) approach have emerged as a powerful computational design tool in system and control field because of their computational efficiency and flexibility, see(Gahinet *et al*.,1994 and Iwasaki *et al*.,1994). Using LMIs approach, Li and Fu (see Li *et al*., 1998) designed a kind of robust H_∞ filtering algorithm for uncertain linear systems described by the so-called integral quadratic constraints. Yang (see Yang *et al*., 2000) designed reduced-order robust H_∞ filtering for system with parameters uncertainty.

But the above systems they researched contain no time-delay. In fact, many systems, especially industrial process contains time delay and parameter uncertainty. The time delay and parameter uncertainty often causes system unstable, and the filter design for such systems is more difficult. Pila (see Pila *et al*.,1999) investigated the H_∞ filtering problem for linear system with time-delayed only in measurement, and the system contains no uncertainty. However, the filter they obtained is delay-independent, and we find that most of papers about robust filtering for time-delay system are delay-independent. But this kind of filter is too conservative, and it can't reflect the characteristic of system accurately. Considering that infinite time delay is scarce, and it's always bounded in real process, we want to construct a kind of delay-dependent filter in this paper.

After constructing a Lyapunov function, we obtain the criterion that a given delay-dependent filter can satisfy the H_∞ norm constraint. Then we give the sufficient condition that the set of satisfactory filters is nonempty, and we also give the parameterization of all such satisfactory delay-dependent filters which can guarantee the H_∞ norm constrain. Another main advantage of this method we develop is that it can produce reduced-order filter easily. The reduced-order filter would reduce computation complexity and improve computation speed, and it is superior to the filter with full-order. In this paper, we use the tool of LMI technology, and there are some effective algorithms such as alternating projection algorithm to solve it. A numerical example will be included to demonstrate the advantages and effectiveness of the method.

The notation to be used is as follows. The H_∞ norm of a rational transfer function $T(s)$ is denoted by $\|T(s)\|_\infty$. Given a real $n\times m$ dimensional matrix Q with rank r, the orthogonal complement $Q^\perp$ is defined as $(n-r)\times n$ matrix that satisfies $Q^\perp Q=0$. The notation $A>(<)B$ means that $A-B$ is positive (negative) definite matrix.

2. PROBLEM FORMULATION

Consider a stable linear system with state delay and parameter uncertainties

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_d+\Delta A_d(t))x(t-\tau)+Bw(t),\qquad x(t_0)=x_0 \tag{1}$$

$$y(t)=(C+\Delta C(t))x(t)+Dw(t) \tag{2}$$

$$x(t)=\phi(t),\qquad t\in[-\tau,0] \tag{3}$$

where $x(t)$ is n- dimensional state vector, $y(t)$ is the measured output, and $w(t)$ is a disturbance vector containing both process and measurement noise. τ is the time delay item. A, A_d, B, C, D are known matrices that describe the nominal system, and $\Delta A, \Delta A_d, \Delta C$ are real continuous functions, which denote the uncertainties. In this paper the admissible uncertainties are assumed to be of the form

$$\begin{bmatrix}\Delta A(t)\\ \Delta A_d(t)\\ \Delta C(t)\end{bmatrix}=\begin{bmatrix}M_1\\ M_2\\ M_3\end{bmatrix}F(t)N \tag{4}$$

and M_1, M_2, M_3, N are known constant matrices of appropriate dimensions. $F(t)$ is an uncertain time-varying matrix bounded by

$$F^T(t)F(t)\le I \tag{5}$$

The r-dimensional signal to be estimated is

$$z(t)=Lx(t) \tag{6}$$

In this paper, we focus on the design of robust linear estimator for z with guaranteed performance in the sense of H_∞ norm of transfer function from the noise $w(t)$ to the estimated error. More specifically, we want to design a linear filter

$$\dot{\hat{x}}=G\hat{x}+Hy \tag{7}$$

$$\hat{z}=J\hat{x}+Ky \tag{8}$$

where $\hat{x}$ is an n-dimensional state vector of filter, $\hat{z}$ is the estimate of $z(t)$. And the estimation error is

$$e=z-\hat{z} \tag{9}$$

which will satisfy that the H_∞ norm of the transfer function T_{we} from disturbance w to estimation error e, is strictly less than γ. That is

$$\|T_{we}\|_\infty<\gamma \tag{10}$$

So the filter is also called as γ-suboptimal filter.

3. MAIN RESULTS

Gathering all parameters of filter into the single variable

$$\Theta:=\begin{bmatrix}K & J\\ H & G\end{bmatrix} \tag{11}$$

and the augmented system can be changed into

$$\dot{\bar{x}}(t)=(\bar{A}+\bar{E}_1\Theta\bar{C})\bar{x}(t)+\bar{A}_d\bar{x}(t-\tau)+(\bar{B}+\bar{E}_1\Theta\bar{D})w \tag{12}$$

$$e=z-\hat{z}=(\bar{L}+\bar{E}_2\Theta\bar{C})\bar{x}+(\bar{E}_2\Theta\bar{D})w \tag{13}$$

where

$\bar{x}=\begin{bmatrix}x\\ \hat{x}\end{bmatrix}$, $\bar{A}=\begin{bmatrix}A+\Delta A & 0\\ 0 & 0\end{bmatrix}$, $\bar{E}_1=\begin{bmatrix}0 & 0\\ 0 & I\end{bmatrix}$, $\bar{B}=\begin{bmatrix}B\\ 0\end{bmatrix}$, $\bar{C}=\begin{bmatrix}C+\Delta C & 0\\ 0 & I\end{bmatrix}$, $\bar{D}=\begin{bmatrix}D\\ 0\end{bmatrix}$, $\bar{L}=\begin{bmatrix}L & 0\end{bmatrix}$, $\bar{E}_2=\begin{bmatrix}-I & 0\end{bmatrix}$ $\bar{A}_d=\begin{bmatrix}A_d+\Delta A_d & 0\\ 0 & 0\end{bmatrix}$ $\hat{A}_d=\begin{bmatrix}A_d & 0\\ 0 & 0\end{bmatrix}$, $\hat{C}=\begin{bmatrix}C & 0\\ 0 & I\end{bmatrix}$, $\hat{A}=\begin{bmatrix}A & 0\\ 0 & 0\end{bmatrix}$, $\begin{bmatrix}\Delta A & 0\\ 0 & 0\end{bmatrix}=\bar{M}_1F\bar{N}$, $\begin{bmatrix}\Delta A_d & 0\\ 0 & 0\end{bmatrix}=\bar{M}_2F\bar{N}$, $\begin{bmatrix}\Delta C & 0\\ 0 & 0\end{bmatrix}=\bar{M}_3F\bar{N}$,

$\eta=\bar{E}_2\Theta\bar{M}_2$.

Lemma 1: (Gahinet *et al*.,1994 and Iwasaki *et al*.,1994) Given a symmetric matrix $\phi\in R^{m\times m}$ and two matrices P and Q of column dimension m, consider the problem of finding some matrix Θ of compatible dimensions such that

$$\phi+P^T\Theta^TQ+Q^T\Theta P<0 \tag{14}$$

Denote by W_P, W_Q any matrices whose columns form bases of the null bases of P and Q respectively. Then (10) is solvable for Θ if and only if

$$\begin{cases}W_P^T\psi W_P<0\\ W_Q^T\psi W_Q<0\end{cases}$$

In this case all solution matrices Θ are parameterized by

$$\Theta=-R^{-1}P^TTQ^TQ+\Omega^{1/2}\beta\Lambda^{1/2}$$

where T, R and β are free parameters subject to $T=(PR^{-1}P^T-\phi)^{-1}>0$, $R>0$, $\|\beta\|<1$

and Ω and Λ are defined by

$$\Omega = R^{-1} - R^{-1}P^T(T - TQ^T\Lambda QT)PR^{-1}$$
$$\Lambda = (QTQ^T)^{-1}.$$

Lemma 2 The matrix

$$S = \begin{bmatrix} A & C \\ C^T & B \end{bmatrix}$$

where $A = A_{11} - A_{12}A_{22}^{-1}A_{12}^T$, $B = B_{11} - B_{12}B_{22}^{-1}B_{12}^T$, $A_{11}, A_{22}, B_{11}, B_{22}$ are symmetric matrices and A_{22}, B_{22} are negative matrices, is negative definite if and only if

$$\begin{bmatrix} A_{11} & A_{12} & C & 0 \\ A_{12}^T & A_{22} & 0 & 0 \\ C^T & 0 & B_{11} & B_{12} \\ 0 & 0 & B_{12}^T & B_{22} \end{bmatrix} < 0 \qquad (15)$$

Proof: Using the property of Schur complement, the inequality (15) can be easily derived. □

Next, we will give the criterion that a given delay-dependent filter can satisfy the H_∞ norm constraint.

***Theorem 1**:* A given filter Θ of uncertain system (1)-(3), can satisfy $\|T_{we}\|_\infty < \gamma$, if there exists parameters $\varepsilon_i (i = 1,2,\cdots,7)$ and a positive definite matrix P such that the following LMI holds .

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12}^T & -\alpha_{22} \end{bmatrix} < 0 \qquad (16)$$

where

$$\alpha_{11} = P(\hat{A} + \overline{E}_1\Theta\hat{C} + \hat{A}_d) + (\hat{A} + \overline{E}_1\Theta\hat{C} + \hat{A}_d)^T P + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + 2\tau\varepsilon_4 + \tau\varepsilon_5 + 2\varepsilon_7^{-1})\overline{N}^T\overline{N}$$

$$\alpha_{12} = [P\overline{E}_1\Theta\overline{M}_3 \quad (\hat{A} + \overline{E}_1\Theta\hat{C})^T \quad (\overline{L} + \overline{E}_2\Theta\hat{C})^T \quad P\overline{M}_1 \quad \hat{A}_d^T \quad P\hat{A}_d \quad P\hat{A}_d\overline{N}^T \quad P\overline{M}_2 \quad P\tilde{B}]$$

$$\alpha_{22} = diag[\varepsilon_2^{-1}I, S_1, S_2, \varepsilon_1^{-1}I, \varepsilon_3^{-1}I, \tau(I - \varepsilon_5\overline{M}_2\overline{M}_2^T), 3I, 3(\varepsilon_6 I - \overline{N}\overline{N}^T), 3\varepsilon_6 I, \gamma^2 I - \tilde{B}\tilde{B}^T - 2(\overline{E}_2\Theta\overline{D})^T(\overline{E}_2\Theta\overline{D})]$$

$$S_1 = \tau[I - \varepsilon_4\overline{M}_1\overline{M}_1^T - \varepsilon_4\overline{E}_1\Theta\overline{M}_3(\overline{E}_1\Theta\overline{M}_3)^T]$$

$$S_2 = 2[I - \varepsilon_7\overline{E}_2\Theta\overline{M}_3(\overline{E}_2\Theta\overline{M}_3)^T].$$

Proof. In order to make the augmented system (12) stable, we construct the Lyapunov function

$$V(\overline{x}, t) = \overline{x}^T P\overline{x} + W(\overline{x}, t) \qquad (17)$$

where

$$W(\overline{x}, t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} \overline{x}^T(s)\tilde{A}_0^T(s)\tilde{A}_0(s)\overline{x}(s)dsd\theta + \int_{-\tau}^{0}\int_{t-\tau+\theta}^{t} \overline{x}^T(s)\tilde{A}_d^T(s+\tau)\tilde{A}_d(s+\tau)\overline{x}(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} w^T(s)\tilde{B}^T(s)\tilde{B}(s)w(s)dsd\theta$$

$\tilde{A}_0(t) = \overline{A} + \overline{E}_1\Theta\overline{C}$, $\tilde{A}_d(t) = \overline{A}_d(t)$, $\tilde{B} = \overline{B} + \overline{E}_1\Theta\overline{D}$.

Owning to $\overline{x}(t-\tau) = x(t) - \int_{-\tau}^{0} x(t+\theta)d\theta$, we obtain

$$\dot{V}(x,t) = \dot{\overline{x}}^T(t)P\overline{x}(t) + \overline{x}^T(t)P\dot{\overline{x}}(t) + \dot{W}(t)$$
$$= \overline{x}^T\left[P(\tilde{A}_0 + \tilde{A}_d) + (\tilde{A}_0 + \tilde{A}_d)^T P\right]\overline{x} - 2\overline{x}^T P\tilde{A}_d(t) \cdot \int_{-\tau}^{0}\{\tilde{A}_0(t+\theta)x(t+\theta) + \tilde{A}_d(t+\theta)x(t-\tau+\theta) + \tilde{B}w(t+\theta)\}d\theta + \overline{x}(t)^T P\tilde{B}w(t) + w^T(t)\tilde{B}^T P\overline{x}(t) + \dot{W}(t)$$

$$\dot{W}(t) = \tau\overline{x}^T(t)\tilde{A}_0^T(t)\tilde{A}_0(t)\overline{x}(t) + \tau\overline{x}^T(t)\tilde{A}_d^T(t+\tau) \cdot \tilde{A}_d(t+\tau)\overline{x}(t) + \tau w^T(t)\tilde{B}^T(t)\tilde{B}(t)w(t) - \int_{-\tau}^{0}\overline{x}^T(t+\theta) \cdot \tilde{A}_0^T(t+\theta)\tilde{A}_0(t+\theta)\overline{x}(t+\theta)d\theta - \int_{-\tau}^{0}\overline{x}^T(t-\tau+\theta)\tilde{A}_d^T(t-\tau+\theta) \cdot \tilde{A}_d(t-\tau+\theta)\overline{x}(t-\tau+\theta)d\theta - \int_{-\tau}^{0} w^T(t+\theta)\tilde{B}^T(t+\theta) \cdot \tilde{B}(t+\theta)w(t+\theta)d\theta$$

It's obvious that

$$\beta = -2\overline{x}^T P\tilde{A}_d(t) \cdot \int_{-\tau}^{0}\{\tilde{A}_0(t+\theta)x(t+\theta) + \tilde{A}_d(t+\theta)x(t-\tau+\theta) + \tilde{B}w(t+\theta)\}d\theta$$
$$- \int_{-\tau}^{0}\overline{x}^T(t+\theta)\tilde{A}_0^T(t+\theta)\tilde{A}_0(t+\theta)\overline{x}(t+\theta)d\theta$$
$$- \int_{-\tau}^{0}\overline{x}^T(t-\tau+\theta)\tilde{A}_d^T(t-\tau+\theta) \cdot \tilde{A}_d(t-\tau+\theta)\overline{x}(t-\tau+\theta)d\theta$$
$$- \int_{-\tau}^{0} w^T(t+\theta)\tilde{B}^T(t+\theta)\tilde{B}(t+\theta)w(t+\theta)d\theta$$
$$- 3\overline{x}^T(t)P\tilde{A}_d(t)\tilde{A}_d^T(t)P\overline{x}(t) \le 0$$

Therefore

$$\dot{V}(\overline{x},t) \le \phi(\overline{x},t) = \overline{x}^T\left[P(\tilde{A}_0 + \tilde{A}_d) + (\tilde{A}_0 + \tilde{A}_d)^T P\right]\overline{x} + \tau\overline{x}^T(t)\tilde{A}_0^T(t)\tilde{A}_0(t)\overline{x}(t) + \tau\overline{x}^T(t)\tilde{A}_d^T(t+\tau)\tilde{A}_d(t+\tau)\overline{x}(t) + \tau w^T(t)\tilde{B}^T(t)\tilde{B}(t)w(t) + 3\overline{x}^T(t)P\tilde{A}_d(t)\tilde{A}_d^T(t)P\overline{x}(t)$$

We can construct following function.

$$J = \int_0^\infty\left[e^T e - \gamma^2 w^T w + \frac{d}{dt}V(\overline{x},t)\right]dt - \overline{x}^T(\infty)P\overline{x}(\infty)$$
$$< \int_0^\infty\{\phi(\overline{x},t) + \begin{bmatrix}\overline{x} \\ w\end{bmatrix}^T \begin{bmatrix} (\overline{L}+\overline{E}_2\Theta\overline{C})^T(\overline{L}+\overline{E}_2\Theta\overline{C}) & (\overline{L}+\overline{E}_2\Theta\overline{C})^T(\overline{E}_2\Theta\overline{D}) \\ (\overline{E}_2\Theta\overline{D})^T(\overline{L}+\overline{E}_2\Theta\overline{C}) & (\overline{E}_2\Theta\overline{D})^T(\overline{E}_2\Theta\overline{D}) - \gamma^2 I \end{bmatrix}\begin{bmatrix}\overline{x} \\ w\end{bmatrix}\}dt$$

So we can draw a conclusion that if

$$\phi(\overline{x},t) + 2\overline{x}^T(\overline{L} + \overline{E}_2\Theta\overline{C})^T(\overline{L} + \overline{E}_2\Theta\overline{C})\overline{x} + w^T[2(\overline{E}_2\Theta\overline{D})^T(\overline{E}_2\Theta\overline{D}) - \gamma^2 I]w < 0 \qquad (18)$$

$$\vartheta = \begin{bmatrix} -(\overline{L}+\overline{E}_2\Theta\overline{C})^T(\overline{L}+\overline{E}_2\Theta\overline{C}) & (\overline{L}+\overline{E}_2\Theta\overline{C})^T(\overline{E}_2\Theta\overline{D}) \\ (\overline{E}_2\Theta\overline{D})^T(\overline{L}+\overline{E}_2\Theta\overline{C}) & -(\overline{E}_2\Theta\overline{D})^T(\overline{E}_2\Theta\overline{D}) \end{bmatrix} < 0$$

then $J(\overline{x}, t) < 0$.

It is obvious that $\vartheta \le 0$. Then the only condition is (18) holds. Considering the uncertainties, we obtain

$$P(\tilde{A}_0 + \tilde{A}_d) + (\tilde{A}_0 + \tilde{A}_d)^T P \le P(\hat{A}_0 + \overline{E}_1\Theta\hat{C} + \hat{A}_d) + (\hat{A}_0 + \overline{E}_1\Theta\hat{C} + \hat{A}_d)^T P + \varepsilon_1^{-1}P\overline{M}_1\overline{M}_1^T P + \varepsilon_2^{-1}P\overline{E}_1\Theta\overline{M}_3(\overline{E}_1\Theta\overline{M}_3)^T P + \varepsilon_3^{-1}P\overline{M}_3\overline{M}_3^T P + (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)\overline{N}^T\overline{N}$$

$$\tau\tilde{A}_0^T\tilde{A}_0 \le \tau(\hat{A}+\bar{E}_1\Theta\hat{C})^T[I-\varepsilon_4\bar{M}_1\bar{M}_1^T - \varepsilon_4\bar{E}_1\Theta\bar{M}_3(\bar{E}_1\Theta\bar{M}_3)^T]^{-1}(\hat{A}+\bar{E}_1\Theta\hat{C})+2\tau\varepsilon_4\bar{N}^T\bar{N}$$

$$\tau\tilde{A}_d^T\tilde{A}_d \le \tau\hat{A}_d^T[I-\varepsilon_5\bar{M}_2\bar{M}_2^T]^{-1}\hat{A}_d+\tau\varepsilon_5\bar{N}^T\bar{N}$$

$$3P\tilde{A}_d^T\tilde{A}_dP \le 3P\hat{A}_d\hat{A}_d^TP+3P\hat{A}_d\bar{N}^T(\varepsilon_6I-\bar{N}\bar{N}^T)^{-1}\times\bar{N}\hat{A}_d^TP+3\varepsilon_6P\bar{M}_2\bar{M}_2^TP$$

$$(\bar{L}+\bar{E}_2\Theta\bar{C})^T(\bar{L}+\bar{E}_2\Theta\bar{C}) \le (\bar{L}+\bar{E}_2\Theta\hat{C})^T[I-\varepsilon_7\bar{E}_2\Theta\bar{M}_3(\bar{E}_2\Theta\bar{M}_3)^T]^{-1}(\bar{L}+\bar{E}_2\Theta\hat{C})+\varepsilon_7^{-1}\bar{N}^T\bar{N}$$

Considering Lemma 3, the following inequality can be tested

$$\phi(\bar{x},t)+2\bar{x}^T(\bar{L}+\bar{E}_2\Theta\bar{C})^T(\bar{L}+\bar{E}_2\Theta\bar{C})\bar{x}+w^T[2(\bar{E}_2\Theta\bar{D})^T(\bar{E}_2\Theta\bar{D})-\gamma^2I]w<\alpha .$$

We know that $J(\bar{x},t)<0$ if there exist parameter $\varepsilon_i\,(i=1,2,3,\cdots,7)$ and P, satisfying the LMI $\alpha<0$. □

From theorem 1, we get the condition (16) under which the given filter Θ is γ-suboptimal filter. Now we will discuss the condition that the set of such filter Θ satisfying $\|T_{we}\|_\infty<\gamma$ is nonempty, which is just the condition that (16) exists.

Theorem 2: The set of filter Θ satisfying $\|T_{we}\|_\infty<\gamma$ is nonempty, if there exists an admissible parameters $\varepsilon_i\,(i=1,2,3,\cdots,8)$, and matrices $0<X\le Y$, such that the following LMIs exist

$$\begin{bmatrix} S_1 & 0 & [XA\ 0] & B & M_1 & M_3 & [XA_d\ \ 0] & [A_d\ \ 0] & A_dN & M_2 \\ 0 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ [XA\ 0]^T & 0 & Z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B^T & 0 & 0 & Z_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_1^T & 0 & 0 & 0 & Z_6 & 0 & 0 & 0 & 0 & 0 \\ M_3^T & 0 & 0 & 0 & 0 & Z_7 & 0 & 0 & 0 & 0 \\ [XA_d\ \ 0]^T & 0 & 0 & 0 & 0 & 0 & Z_8 & 0 & 0 & 0 \\ [A_d\ \ 0]^T & 0 & 0 & 0 & 0 & 0 & 0 & Z_9 & 0 & 0 \\ NA_d^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{10} & 0 \\ M_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{11} \end{bmatrix}<0 \quad (19)$$

$$V\cdot\begin{bmatrix} S_2 & 0 & YB & [A\ \ 0] & \bar{L}^T & YM_1 & YM_3 & [A_d\ \ 0] & YA_d & YA_dN & YM_2 \\ 0 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (YB)^T & 0 & Z_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ [A\ \ 0]^T & 0 & 0 & Z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{L} & 0 & 0 & 0 & Z_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ (YM_1)^T & 0 & 0 & 0 & 0 & Z_6 & 0 & 0 & 0 & 0 & 0 \\ (YM_3)^T & 0 & 0 & 0 & 0 & 0 & Z_7 & 0 & 0 & 0 & 0 \\ [A_d\ \ 0]^T & 0 & 0 & 0 & 0 & 0 & 0 & Z_8 & 0 & 0 & 0 \\ (YA_d)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_9 & 0 & 0 \\ (YA_dN)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{10} & 0 \\ (YM_2)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{11} \end{bmatrix}\cdot V^T<0 \quad (20)$$

where $V=\begin{bmatrix}\begin{bmatrix}\hat{C}^T \\ \bar{M}_3^T \\ \bar{D}\end{bmatrix}^{\perp} & 0 \\ 0 & I\end{bmatrix}$,

$$S_1=(A+A_d)X+X(A+A_d)^T+(\varepsilon_1+\varepsilon_2+\varepsilon_3+2\tau\varepsilon_4+\tau\varepsilon_5+2\varepsilon_7^{-1})XN^TNX+\varepsilon_4^{-1}\tau I$$

$$S_2=Y(A+A_d)+(A+A_d)^TY+(\varepsilon_1+\varepsilon_2+\varepsilon_3+2\tau\varepsilon_4+\tau\varepsilon_5+2\varepsilon_7^{-1})YN^TNY+\varepsilon_4^{-1}\tau I$$

$$Z_1=P(\hat{A}+\hat{A}_d)+(\hat{A}+\hat{A}_d)^TP+(\varepsilon_1+\varepsilon_2+\varepsilon_3+2\tau\varepsilon_4+\tau\varepsilon_5+2\varepsilon_7^{-1})\bar{N}^T\bar{N}+\varepsilon_4^{-1}\tau I,$$

$Z_2=-(\varepsilon_2^{-1}-\varepsilon_7^{-1})I$, $Z_3=-\tau(I/2-\varepsilon_4\bar{M}_1\bar{M}_1^T)$, $Z_4=(-1+\varepsilon_8^{-1})I$, $Z_5=-0.5\gamma^2I+\bar{B}^T\bar{B}$, $Z_6=-\varepsilon_1^{-1}I$, $Z_7=-\varepsilon_3^{-1}I$, $Z_8=-\tau(I-\varepsilon_5\bar{M}_2\bar{M}_2^T)$, $Z_9=-3I$, $Z_{10}=-3(\varepsilon_6I-\bar{N}\bar{N}^T)$, $Z_{11}=-3\varepsilon_6I$.

Proof: From theorem 1, set of filter Θ satisfying $\|T_{we}\|_\infty<\gamma$ is nonempty if there are parameters $\varepsilon_i\,(i=1,2,\cdots,8)$ and a positive matrix P satisfying (16). Notice that (16) can be rewritten as

$$\Sigma=\Gamma\Theta\pi+(\Gamma\Theta\pi)^T+\phi+\bar{\phi}<0 \quad (21)$$

where

$$\Gamma=\begin{bmatrix}(P\bar{E}_1)^T & 0 & 0 & \bar{E}_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T,$$
$$\pi=\begin{bmatrix}\hat{C} & \bar{M}_3 & 0 & 0 & \bar{D} & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\phi=\begin{bmatrix} Z_1 & 0 & \hat{A} & \bar{L}^T & P\bar{B} & P\bar{M}_1 & P\bar{M}_3 & \hat{A}_d & P\hat{A}_d & P\hat{A}_d\bar{N}^T & P\bar{M}_2 \\ 0 & Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{A}^T & 0 & Z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{L} & 0 & 0 & Z_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (P\bar{B})^T & 0 & 0 & 0 & Z_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ (P\bar{M}_1)^T & 0 & 0 & 0 & 0 & Z_6 & 0 & 0 & 0 & 0 & 0 \\ (P\bar{M}_3)^T & 0 & 0 & 0 & 0 & 0 & Z_7 & 0 & 0 & 0 & 0 \\ \hat{A}_d^T & 0 & 0 & 0 & 0 & 0 & 0 & Z_8 & 0 & 0 & 0 \\ (P\hat{A}_d)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_9 & 0 & 0 \\ \bar{N}(P\hat{A}_d)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{10} & 0 \\ (P\bar{M}_2)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{11} \end{bmatrix}$$

$$\bar{\phi}=\begin{bmatrix} \bar{Z}_1 & 0 & (\bar{E}_1\Theta\hat{C})^T & 0 & 0 & 0 & 0 \\ 0 & \bar{Z}_2 & 0 & -(\bar{E}_2\Theta\bar{M}_3)^T & 0 & & \\ \bar{E}_1\Theta\hat{C} & 0 & \bar{Z}_3 & 0 & 0 & \ddots & \\ 0 & -\bar{E}_2\Theta\bar{M}_3 & 0 & \bar{Z}_4 & -\bar{E}_2\Theta\bar{D} & & \\ 0 & 0 & 0 & -(\bar{E}_2\Theta\bar{D})^T & \bar{Z}_5 & 0 & 0 \\ 0 & & & & 0 & & \\ & & \ddots & & & \ddots & \\ 0 & & & & 0 & & 0 \end{bmatrix}$$

where $\bar{Z}_1=-\varepsilon_4^{-1}\tau I$, $\bar{Z}_2=-\varepsilon_7^{-1}I$,

$$\bar{Z}_3 = -\tau/2\, I + \varepsilon_4 \tau \bar{E}_1 \Theta \bar{M}_3 (\bar{E}_1 \Theta \bar{M}_3)^T ,$$

$$\bar{Z}_4 = -I - \varepsilon_8^{-1} I + 2\varepsilon_7 \bar{E}_2 \Theta \bar{M}_3 (\bar{E}_2 \Theta \bar{M}_3)^T ,$$

$$\bar{Z}_5 = -\frac{1}{2}\gamma^2 I + \bar{B}(\bar{E}_1 \Theta \bar{D})^T + (\bar{E}_1 \Theta \bar{D})\bar{B}^T + 2(\bar{E}_2 \Theta \bar{D})^T (\bar{E}_2 \Theta \bar{D})$$

$$\tilde{\phi} = \begin{bmatrix} \tilde{Z}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{Z}_2 & -(\bar{E}_1\Theta\bar{M}_3)^T & -(\bar{E}_2\Theta\bar{M}_3)^T & 0 & & \\ 0 & -\bar{E}_1\Theta\bar{M}_3 & \tilde{Z}_3 & 0 & -\bar{E}_1\Theta\bar{D} & \ddots & \\ 0 & -\bar{E}_2\Theta\bar{M}_3 & 0 & \tilde{Z}_4 & -\bar{E}_2\Theta\bar{D} & & \\ 0 & 0 & -(\bar{E}_1\Theta\bar{D})^T & -(\bar{E}_2\Theta\bar{D})^T & \tilde{Z}_5 & 0 & 0 \\ 0 & & & & 0 & & \\ & & \ddots & & & \ddots & \\ 0 & & & & 0 & & 0 \end{bmatrix}$$

where $\tilde{Z}_1 = -\varepsilon_4^{-1}\tau I$, $\tilde{Z}_2 = -\varepsilon_7^{-1} I$,

$\tilde{Z}_3 = -\varepsilon_4 \tau (\bar{E}_1 \Theta \hat{C})(\bar{E}_1 \Theta \hat{C})^T$,

$\tilde{Z}_4 = -\varepsilon_8^{-1} I - \varepsilon_7 (\bar{E}_2 \Theta \bar{M}_3)(\bar{E}_2 \Theta \bar{M}_3)^T$,

$\tilde{Z}_5 = -\varepsilon_8 (\bar{E}_2 \Theta \bar{D})^T (\bar{E}_2 \Theta \bar{D})$.

Owning to $\bar{Z}_5 < 0$, then must exist a $1 > \varepsilon_8 > 0$ such that

$$-\frac{1}{2}\gamma^2 I + \bar{B}(\bar{E}_1 \Theta \bar{D})^T + (\bar{E}_1 \Theta \bar{D})\bar{B}^T + 2(\bar{E}_2 \Theta \bar{D})^T \times (\bar{E}_2 \Theta \bar{D}) < -\varepsilon_8 (\bar{E}_2 \Theta \bar{D})^T (\bar{E}_2 \Theta \bar{D})$$

Choose suitable $\varepsilon_7 > 0$ such that

$$I - \varepsilon_7 [\bar{E}_2 \Theta \bar{M}_3 (\bar{E}_2 \Theta \bar{M}_3)^T + \bar{E}_2 \Theta \hat{C} (\bar{E}_2 \Theta \hat{C})^T] > 0 ,$$

so

$$-I - \varepsilon_8^{-1} I + \varepsilon_7 \bar{E}_2 \Theta \bar{M}_3 (\bar{E}_2 \Theta \bar{M}_3)^T < -\varepsilon_7 (\bar{E}_2 \Theta \hat{C})(\bar{E}_2 \Theta \hat{C})^T$$

It can be testified that

$$\bar{\phi} < \tilde{\phi} \le 0 ,$$

Then $\Sigma < 0$, if

$$\Gamma \Theta \pi + (\Gamma \Theta \pi)^T + \phi < 0 .$$

We know that

$$\Gamma^{\perp} = \begin{bmatrix} \begin{bmatrix} P\bar{E}_1 \\ 0 \\ 0 \\ \bar{E}_2 \end{bmatrix}^{\perp} & \\ & I \end{bmatrix} ,$$

where

$$\begin{bmatrix} P\bar{E}_1 \\ 0 \\ 0 \\ \bar{E}_2 \end{bmatrix}^{\perp} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} P^{-1} & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} ,$$

$$\begin{bmatrix} \hat{C}^T \\ \bar{M}_3^T \\ 0 \\ 0 \\ \bar{D}^T \end{bmatrix}^{\perp} = \begin{bmatrix} \begin{bmatrix} C^T \\ M_3^T \\ D^T \end{bmatrix}^{\perp} & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}$$

Let $P = \begin{bmatrix} Y & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, P^{-1} = \begin{bmatrix} X & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}$. $P > 0$ would lead to $Y - X = Y_{12} Y_{22}^T Y_{12}^T \ge 0$, therefore $0 < X \le Y$. According to Lemma 1, (19) and (20) follow immediately. □

Remark 1: If $rank(Y_{22}) = \hat{n} \le n$, this method is also suitable for the design of reduced-order filter.

Theorem 3: Correspond to a feasible matrix pair (X, Y), all $\hat{n}$ th –order filter Θ, which satisfying $\|T_{we}\| < \gamma$, are given by

$$\Theta = \begin{bmatrix} G & H \\ J & K \end{bmatrix} = -R^{-1}\Gamma^T T \pi^T \Lambda + \Omega^{1/2} \beta \Lambda^{1/2} \quad (22)$$

where $\Gamma = [\Gamma_1^T \quad \Gamma_2^T]^T, \pi = [\pi_1 \quad \pi_2]$,

$$\phi = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 + I \end{bmatrix}, \bar{\phi} = \begin{bmatrix} 0 & \varphi^T \\ \varphi & -I \end{bmatrix},$$

$$\varphi = \begin{bmatrix} P\bar{E}_1 \Theta \tilde{C} + (\bar{E}_1 \Theta \tilde{C})^T P & P\bar{E}_1 \Theta \bar{D} & (\bar{E}_2 \Theta \tilde{C})^T \\ 0 & 0 & 0 \\ (\varepsilon \bar{E}_1 \Theta \bar{M}_2)^T P & 0 & (\varepsilon \bar{E}_2 \Theta \bar{M}_2)^T \\ 0 & 0 & 0 \end{bmatrix}$$

and T, R and β are free parameters subject to

$$T = (\Gamma R^{-1} \Gamma^T - \phi)^{-1} > 0 , \quad R > 0 , \quad \|\beta\| < 1$$

and Ω and Λ are defined by

$$\Omega = R^{-1} - R^{-1} \Gamma^T (T - T\pi^T \Lambda \pi T) \Gamma R^{-1}$$

$$\Lambda = (\pi T \pi^T)^{-1} .$$

Remark 2: The upper bound on the H_∞ performance may be conservative, and we can reduce the conservative upper bound to obtain the optimal upper bound by solving optimization problems.

$$\min_{\varepsilon_i (i=1,\cdots 8), 0 < X \le Y} \gamma \quad (23)$$

4. EXAMPLE

In this section, a numerical example is presented to demonstrate the effectiveness of the proposed approach. Consider the uncertain time-delay system (1) with parameters as follows:

$A = \begin{bmatrix} -11 & 1 \\ 0 & -1 \end{bmatrix}$, $A_d = \begin{bmatrix} -1 & -0.04 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.01 \end{bmatrix}$,

$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -0.1137 & -0.2226 \\ 0.1695 & -0.4110 \end{bmatrix}$,

$L = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $M_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $N = \begin{bmatrix} 1 & 0 \\ 0 & 0.34 \end{bmatrix}$,

$M_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}$, $M_3 = \begin{bmatrix} 0.1 & -0.05 \\ 0 & 0.2 \end{bmatrix}$, $f(t) = \sin t$,

$w(t)$ is zero mean Gaussian white noise process with covariance I and the H_∞ filtering problem with an attenuation level $\gamma = 0.6$. Using the theorem 1, we choose the filter parameters as follows

$\varepsilon_1 = 2$, $\varepsilon_2 = 0.3$, $\varepsilon_3 = 1$, $\varepsilon_4 = 0.5$, $\varepsilon_5 = 0.01$, $\varepsilon_6 = 2.3$, $\varepsilon_7 = 0.5$, $\varepsilon_8 = 0.8$

When $\tau = 80$, we obtain $\Theta_I = \begin{bmatrix} K_I & J_I \\ H_I & G_I \end{bmatrix}$, where

$$X = \begin{bmatrix} 3.9514 & 0.2886 \\ 0.2886 & 1.11480 \end{bmatrix} \qquad Y = \begin{bmatrix} 4.7333 & 0.2195 \\ 0.2195 & 5.1725 \end{bmatrix},$$

$$K_{\mathbf{I}} = \begin{bmatrix} 0.1194 & -0.0240 \\ -0.0226 & 0.0649 \end{bmatrix} \qquad J_{\mathbf{I}} = \begin{bmatrix} -0.0004 & 0.0112 \\ -0.0055 & -0.0009 \end{bmatrix}$$

$$H_{\mathbf{I}} = \begin{bmatrix} -0.0000 & 0.0000 \\ -0.0003 & 0.0110 \end{bmatrix} \quad G_{\mathbf{I}} = \begin{bmatrix} -1.000 & 0.0000 \\ -0.0337 & -1.200 \end{bmatrix}.$$

The estimated errors of signal z_1 and z_2 can be seen in following figures. We denote it as case I, which are caused by the filter $\Theta_{\mathbf{I}}$.

When $\tau = 0.8$, we get $\Theta_{\mathbf{II}} = \begin{bmatrix} K_{\mathbf{II}} & J_{\mathbf{II}} \\ H_{\mathbf{II}} & G_{\mathbf{II}} \end{bmatrix}$, where

$$X = \begin{bmatrix} 0.1219 & 0.0026 \\ 0.0026 & 0.4487 \end{bmatrix}, \quad Y = \begin{bmatrix} 18.3582 & 0.0636 \\ 0.0036 & 6.6654 \end{bmatrix},$$

$$K_{\mathbf{II}} = \begin{bmatrix} 0.2167 & -0.0831 \\ -0.0293 & 0.0642 \end{bmatrix}, \quad J_{\mathbf{II}} = \begin{bmatrix} 0.0063 & 0.1122 \\ 0.1122 & -0.0325 \end{bmatrix},$$

$$H_{\mathbf{II}} = \begin{bmatrix} -0.0039 & 0.0082 \\ -0.1102 & 0.0594 \end{bmatrix}, \quad G_{\mathbf{II}} = \begin{bmatrix} -0.9989 & -0.0023 \\ -0.1667 & -1.1360 \end{bmatrix}.$$

In order to compare the results with the ones of filter $\Theta_{\mathbf{I}}$. We consider the results of filter $\Theta_{\mathbf{II}}$ when $\tau = 80$. And estimated errors of signal z_1 and z_2 can also be seen in following figures.

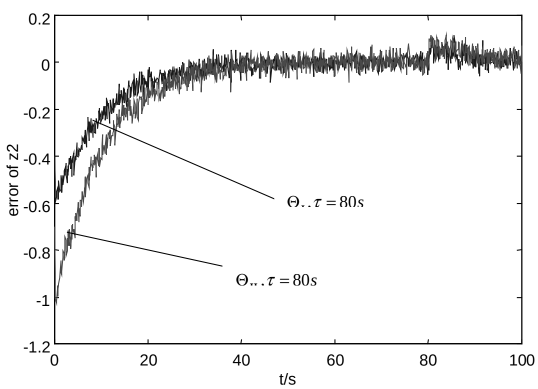

Fig. 1: filtering error e_1

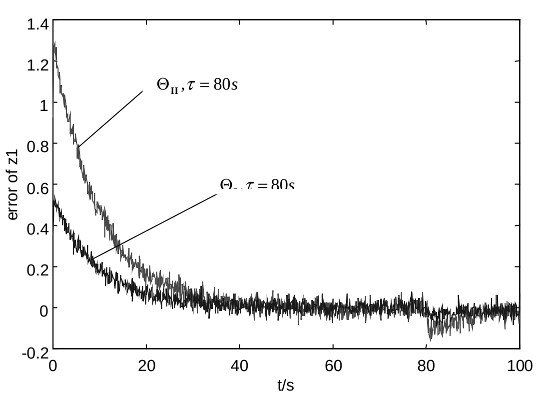

Fig. 2: filtering error e_2

These figures reflect the importance of delay-dependence. When we use the filter $\Theta_{\mathbf{II}}$, which is obtained under $\tau = 0.8$, to solve the filtering problem with $\tau = 80$, the filtering performance is worse than the one of $\Theta_{\mathbf{II}}$, which is obtained under $\tau = 80$. Therefore the delay-independent is conservative, and delay-dependent filter would be superior to it.

Using theorem 2 and theorem 3, we can also get reduced-order filter. For $\tau = 0.8$, the one-dimensional filter $\Theta_{\mathbf{III}}$ we obtain is as follows

$$K = \begin{bmatrix} 0.1192 & -0.0248 \\ -0.0188 & 0.0609 \end{bmatrix}, \quad J = \begin{bmatrix} 0.0057 \\ -0.0156 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.1111 & -0.0886 \end{bmatrix}, G = -0.3300$$

Substituting these filter parameters into (12) and (13), we find that the one-dimensional filter $\Theta_{\mathbf{III}}$ has good robust property and it meets the H_∞ norm constraint.

5. CONCLUSION

A method to design delay-dependent filter has been developed. The system we addressed is linear system with time vary parameter uncertainties. The method we proposed can avoid the conservative property of traditional time-independent method, and can cause high precision. A numerical example clearly demonstrated the effectiveness and the advantage of our approach.

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