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APPLICATION OF FREQUENCY-FOLLOWING SERVOCOMPENSATOR TO TRACKING CONTROL

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Abstract: A servocompensator with performance of frequency following is applied to a tracking control system that simulates disk drive tracking systems. In such systems, the misalignment of the disk causes the reference signal to include sinusoidal components. The applied servocompensator is characterized by using signals synchronized with the reference signal in calculating the control input by means of convolution integral. Since the transmission zeros of the closed-loop system are generated at the same locations as the unstable poles of the reference signal automatically, the pole-zero cancellation necessary for asymptotic regulation is achieved. The efficacy of the servocompensator was confirmed experimentally.

Keywords: tracking systems, frequency tracking, convolution integral, synchronization, servomechanisms.

1. INTRODUCTION

Many industrial control systems are subjected to sinusoidal disturbance/reference signals. Typical examples are magnetic bearings, vibration control systems and computer disk drives. The robust servomechanism controller can control these systems so that asymptotic regulation is achieved (Davison and Patel, 1988). It consists of two devices: a servo-compensator and a stabilizing compensator. The servocompensator includes an internal model of the disturbance/reference signals so that asymptotic regulation can be achieved even in the presence of small variations in system parameters. However, the regulation generally becomes imperfect if there is some discrepancy between the exogenous signals and their internal model. This discrepancy is caused by a change of frequency from its nominal value. Hence, the performance of frequency following is required for many practical controllers.

Mizuno and Araki (1996) have proposed to modify the servocompensator to follow the frequencies of disturbance/reference signals automatically without any complicated adaptive algorithm. In the modified compensator, the control input is calculated by means of convolution integral instead of state-space equation; in calculating the convolution, exogenous signals synchronized with the disturbance/reference signals are used as weighting functions. As a result, the transmission zeros of the closed-loop system are generated at the same locations as the unstable poles of the disturbance/reference signals.

The modified controller has been applied to systems subject to sinusoidal *disturbances*, and its performance of frequency following was certified experimentally (Mizuno and Negishi, 1998). However, it has not been applied to systems with sinusoidal *reference* signals although they are expected to constitute another application field.

In this research the modified controller is applied to a developed tracking control system that simulates disk drive tracking systems. In disk drive tracking systems, the misalignment of the disk causes the reference signal to include harmonic components synchronized with rotation.

This paper is organized as follows. First, the servocompensator using exogenous signals synchronized with disturbance/reference signals is briefly reviewed. Second, a basic model of the developed tracking control system is derived. Third, its control system is designed. Fourth, the efficacy of the designed controller is confirmed experimentally.

2. FREQUENCY-FOLLOWING SERVOCOMPENSATOR

2.1 A General Form of Servocompensator

Target plants are assumed to be described by the following equations:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{D}\boldsymbol{w}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{H}\boldsymbol{u}(t) + \boldsymbol{J}\boldsymbol{w}(t); \quad \boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_{ref}(t), \tag{2}$$

where $\boldsymbol{x}$ is the state, $\boldsymbol{u}$ is the input, $\boldsymbol{y}$ is the output that is to be regulated and $\boldsymbol{w}$ is a disturbance vector. The vector $\boldsymbol{e}$ is the error in the system, which is the difference between the output $\boldsymbol{y}$ and the specified reference input $\boldsymbol{y}_{ref}$. The elements of $\boldsymbol{w}$ and $\boldsymbol{y}_{ref}$ are sinusoidal signals with angular frequencies $\omega_k\,(k = 1, \cdots \nu)$.

A robust controller that stabilizes and regulates them is generally given by (Davison and Patel, 1988, Mizuno and Araki, 1996):

$$\boldsymbol{u}(t) = \boldsymbol{F}\hat{\boldsymbol{x}}(t) + \boldsymbol{G}\boldsymbol{\eta}(t), \tag{3}$$

where $\hat{\boldsymbol{x}}$ is the output of a stabilizing controller and $\boldsymbol{\eta}$ is the output of the servocompensator given by

$$\dot{\boldsymbol{\eta}}(t) = \text{block diag}(\boldsymbol{\Omega}_1, \ \cdots, \ \boldsymbol{\Omega}_\nu)\boldsymbol{\eta}(t) + \begin{bmatrix} \boldsymbol{0}_r \\ \boldsymbol{I}_r \\ \vdots \\ \boldsymbol{0}_r \\ \boldsymbol{I}_r \end{bmatrix} \boldsymbol{e}(t), \tag{4}$$

where

$$\boldsymbol{\Omega}_i = \begin{bmatrix} \boldsymbol{0}_r & \omega_i \boldsymbol{I}_r \\ -\omega_i \boldsymbol{I}_r & \boldsymbol{0}_r \end{bmatrix} \quad (i = 1, \cdots, \nu). \tag{5}$$

Equations (4) and (5) show well that the servo-compensator includes a model of sinusoidal signals.

2.2 Servocompensator Using Synchronous Signals

The internal model principle guarantees that the transmission zeros canceling the unstable poles of the disturbance/reference signals do not move in the presence of perturbations in plants so that output regulation is achieved (Francis and Wonham, 1975). For a change of frequency of disturbance/reference signals, however, the asymptotic regulation generally fails because the unstable poles move from the nominal values. In some applications, the performance of frequency following is strongly required.

Mizuno and Araki (1996) proposed to use exogenous signals synchronized with the disturbance/reference signals to realize such performance. The proposed servocompensator is obtained by applying the following formula of Laplace transformation:

$$L^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t-\tau) f_2(\tau)\mathrm{d}\tau \tag{6}$$

where

$$F_1(s) = L[f_1(t)], \quad F_2(s) = L[f_2(t)].$$

From (4) and (5), we get

$$\dot{\eta}_i^{(k)}(t) = \omega_k \eta_{r+i}^{(k)}(t) \tag{7}$$

$$\dot{\eta}_{r+i}^{(k)}(t) = -\omega_k \eta_i^{(k)}(t) + e_i(t) \tag{8}$$

$$(i = 1, \cdots, r; \quad k = 1, \cdots, \nu)$$

The block diagram of this part of the compensator is shown in **Fig.1**. Assuming the initial values are zero for simplicity, taking Laplace transforms in (7) and (8) leads to

$$H_i^{(k)}(s) = \frac{\omega_k}{s^2 + \omega_k^2} E_i(s) \tag{9}$$

$$H_{r+i}^{(k)}(s) = \frac{s}{s^2 + \omega_k^2} E_i(s) \tag{10}$$

where

$$H_*^{(k)}(s) = L[\eta_*^{(k)}], \quad E_i(s) = L[e_i(t)].$$

Applying the formula of convolution (6) to (9) and (10) gives:

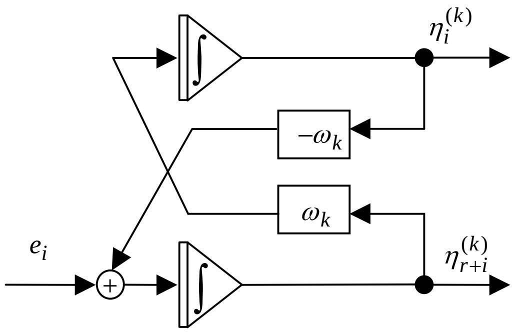


Fig.1 Block diagram of an internal-model compensator

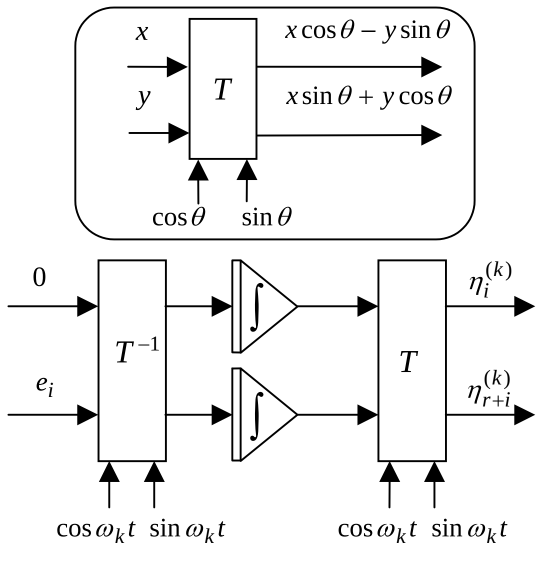


Fig.2 Block diagram of the modified compensator

$$\eta_i^{(k)}(t) = \int_0^t \sin\omega_k(t-\tau)\cdot e_i(\tau)\mathrm{d}\tau$$
$$= [\cos\omega_k t \quad \sin\omega_k t]\int_0^t \begin{bmatrix} -\sin\omega_k\tau \\ \cos\omega_k\tau \end{bmatrix} e_i(\tau)\mathrm{d}\tau \quad (11)$$

$$\eta_{r+i}^{(k)}(t) = \int_0^t \cos\omega_k(t-\tau)\cdot e_i(\tau)\mathrm{d}\tau$$
$$= [-\sin\omega_k t \quad \cos\omega_k t]\int_0^t \begin{bmatrix} -\sin\omega_k\tau \\ \cos\omega_k\tau \end{bmatrix} e_i(\tau)\mathrm{d}\tau \quad (12)$$

The block diagram of the part of the compensator described by (11) and (12) is shown in **Fig.2**. In this figure, the element T converts coordinates between the fixed reference system and a reference system rotating at an angular velocity of ω_k. When the signals synchronized with the disturbance/reference signal are used in the conversions, the critical parameters ω_k of the compensator equal the actual values automatically. Therefore, the property of asymptotic regulation is preserved even if the frequencies of the disturbance/reference signals vary from their nominal values.

3. TRACKING CONTROL SYSTEM

3.1 Target System

Figure 3 shows a developed apparatus that simulates disk drive tracking systems. In this apparatus, the head is controlled to keep the gap from the side face of the rotational disk constant by a voice coil motor (VCM). When some misalignment exists in the attachment of the disk, the target position of the head changes synchronously to rotation.

3.2 Basic Model

Figure 4 shows a simple model of the head drive mechanism. The head is assumed to be connected to the base through a spring and a damping element. The equation of motion is represented by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)\,, \quad (13)$$

where

- x : displacement of the head,
- m : mass of the head,
- c : damping constant,
- k : spring constant,
- F : driving force produced by the VCM.

The force is proportional to the current in coil:

$$F(t) = k_i i(t)\,, \quad (14)$$

where

- k_i : motor constant,
- i : coil current.

Since the displacement of the head is treated to be the output of the system, the dynamics is represented in a state-vector form as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\,, \quad (15)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t); \quad \boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_{ref}(t) \quad (16)$$

where

$$\boldsymbol{x}(t) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad \boldsymbol{u}(t) = i(t), \quad \boldsymbol{y}(t) = x\,,$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, \quad \boldsymbol{B}(t) = \begin{bmatrix} 0 \\ b_0 \end{bmatrix}, \quad \boldsymbol{C} = [1 \quad 0],$$

$$a_0 = \frac{k}{m}, \quad a_1 = \frac{c}{m}, \quad b_0 = \frac{k_i}{m}\,.$$

As mentioned above, the misalignment of the disk causes the reference signal to include harmonic components synchronized with rotation. In the following the fundamental component is treated. Assuming that rotational speed ω is constant, the reference signal can be represented as

$$y_{ref}(t) = \varepsilon \sin\omega t\,. \quad (17)$$

4. DESIGN OF THE CONTROLLER

4.1 Servocompensators

The control system is designed to reduce the tracking error e to zero at a specified frequency ω_1. For

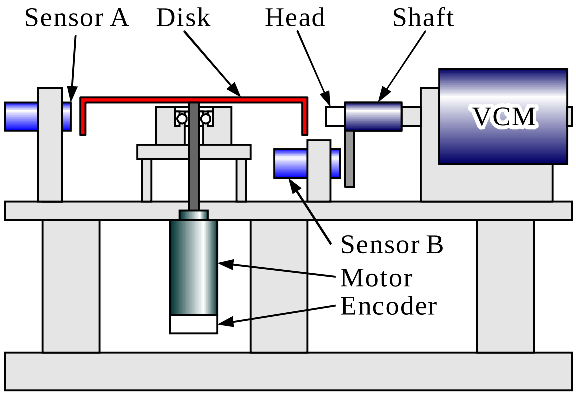


Fig.3 Developed experimental apparatus

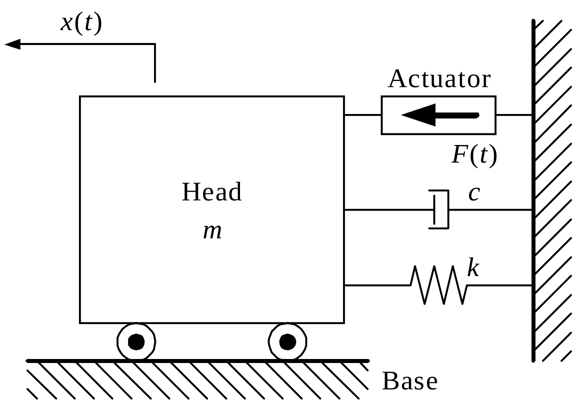


Fig.4 Basic model of the head drive mechansm

stabilization, state feedback is adopted here. The integral feedback is also introduced to reduce DC-component error to zero. As a result, the controller is given by

$$\boldsymbol{u}(t) = \boldsymbol{F}\boldsymbol{x}(t) + \tilde{\boldsymbol{G}}\tilde{\eta}(t)\,, \tag{18}$$

$$\dot{\tilde{\eta}} = \boldsymbol{E}\tilde{\eta}(t) + \boldsymbol{H}\boldsymbol{e}(t)\,, \tag{19}$$

where

$$\boldsymbol{F} = -\begin{bmatrix} p_d & p_v \end{bmatrix}, \quad \tilde{\boldsymbol{G}} = \begin{bmatrix} g_0 & g_1 & p_I \end{bmatrix},$$

$$\boldsymbol{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad \tilde{\eta}(t) = \begin{bmatrix} \eta_1 \\ \eta_2 \\ z \end{bmatrix},$$

$$\boldsymbol{E} = \begin{bmatrix} 0 & -\omega_1 & 0 \\ \omega_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix},$$

z: auxiliary variable for describing an integral action.

This controller is a combination of the conventional I-PD controller and a servocompensator with an internal model of sinusoidal signals described by (4) and (5).

According to the discussion in Section 2.2, the corresponding controller using exogenous signals with rotation is given by

$$u(t) = -(p_d x + p_v \dot{x}) + p_I \int_0^t e(\tau)d\tau$$

$$+\begin{bmatrix} g_0 & g_1 \end{bmatrix} \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix} \int_0^t \begin{bmatrix} -\sin\omega\tau \\ \cos\omega\tau \end{bmatrix} e(\tau)d\tau\,. \tag{20}$$

4.2 Pole Assignment

The stabilization of the closed-loop system is discussed in this section. The pole placement technique is applied to construct stable systems.

The Laplace transformation of (15) with all the initial conditions assumed to be zero leads to

$$X(s) = \frac{b_0}{s^2 + a_1 s + a_0} U(s)\,. \tag{21}$$

From (18) and (19), the dynamics of the controller can be represented by

$$U(s) = -(p_d + s p_v)X(s) + \left(\frac{g_1 s + g_0 \omega_1}{s^2 + \omega_1^2} + \frac{p_I}{s} \right) E(s) \tag{22}$$

Substituting (22) into (21) leads to

$$Y(s) = \frac{b_0\{(g_1 s + g_0\omega_1)s + p_I(s^2 + \omega_1^2)\}}{t_c(s)} Y_{ref}(s)\,, \tag{23}$$

.

where

$$t_c = s^5 + (a_1 + b_0 p_v)s^4 + (a_0 + b_0 p_d + \omega^2)s^3$$
$$+ \{\omega^2(a_1 + b_0 p_v) + b_0 g_1 + b_0 p_I\}s^2$$
$$+ \{\omega^2(a_0 + b_0 p_d) + b_0 g_0 \omega\}s + b_0 p_I \omega^2\,. \tag{24}$$

It is assumed that a characteristic polynomial specifying the locations of the close-loop poles is given by

$$t_d(s) = s^5 + c_4 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0\,. \tag{25}$$

It is selected for all the roots to lie in the left half-plane. Comparing (24) with (25) gives

$$p_d = \frac{c_3 - a_0 - \omega^2}{b_0}\,, \tag{26}$$

$$p_I = \frac{c_0}{\omega^2 b_0}\,, \tag{27}$$

$$p_v = \frac{c_4 - a_1}{b_0}\,, \tag{28}$$

$$g_0 = \frac{c_1 - \omega^2(a_0 + b_0 p_d)}{\omega b_0}\,, \tag{29}$$

$$g_1 = \frac{c_2 - b_0 p_I - \omega^2(a_1 + b_0 p_v)}{b_0}\,. \tag{30}$$

5. EXPERIMENT

5.1 Experimental Setup

The developed experimental apparatus was already

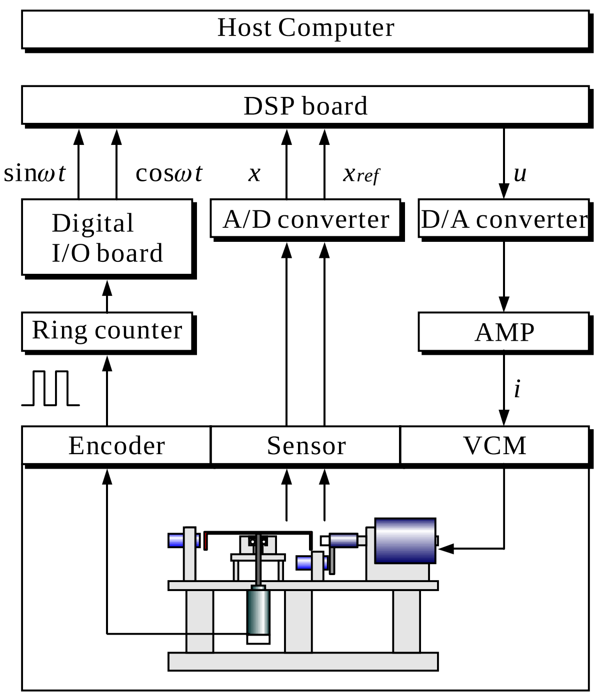


Fig.5 Experimental system

Table 1 Parameters of the experimental insrurment

Parameter	Value
a_0	0.0 [1/s^2]
a_1	2.0 [1/s]
b_0	96.0 [m/(As^2)]

shown in **Fig.3**. This apparatus has an adjusting mechanism of the misalignment of the disk for experimental study. An eddy-current displacement sensor (Sensor A) detects the side face of the rotational disk to produce a reference signal. Another sensor (Sensor B) detects the position of the head. Their difference is treated as the error signal. An encoder attached to the motor produces signals synchronized with rotation.

The outline of the experimental system is shown in **Fig.5**. The output signals of Sensor A and Sensor B are fed into a DSP-based controller through 12-bit A/D converters. The DSP is TMS320C30 from Texas Instruments.

A rotary encoder attached to the motor produces 500 pulses per revolution. These pulse signals are inputted to a divided-by-500 counter. The counted values are inputted into the controller through a digital input port. The sinusoidal signals used in calculating the convolution integral in (20) are generated based on these values.

The parameters of the apparatus are listed in **Table 1**.

5.2 Experimental Result

In designing the controller, the rotational angular frequency ω_1 is set to be $2\pi \times 10.0[1/s]$. The characteristic polynomial specifying the close-loop poles is selected as

$$t_d = (s+\alpha)(s^2 + 2\zeta_1\Omega_1 + \Omega_1^2)(s^2 + 2\zeta_2\Omega_2 + \Omega_2^2),$$

where

$$\alpha = \Omega_1 = \Omega_2 = 2\pi \times 20.0[1/s],$$

$$\zeta_1 = \zeta_2 = 0.30.$$

Figure 6 shows the amplitude of a frequency component of rotational frequency (synchronized fundamental component) of error signal at various rotational speeds in the cases where

(a) only I-PD control is used,

(b) servocompensator given by (18) and (19) (internal-model compensator) is activated.

(c) modified servocompensator given by (20) is activated.

When only I-PD control is used ((a)), the

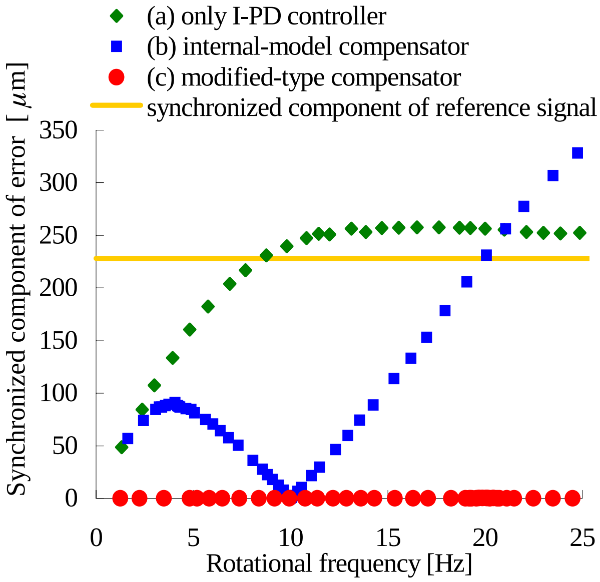
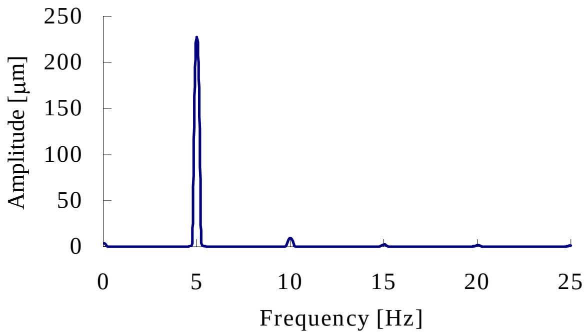
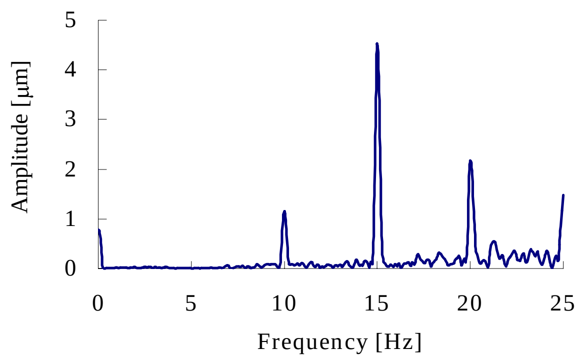


Fig.6 Amplitude of the fundamental component of the error signal at each rotational speed



(a) Reference signal



(b) Error signal

Fig.7 Spectral amplitudes of the reference and error signals.

synchronized component of error signal increases as the rotational speed increases up to about 10Hz. Above 11Hz, the amplitude of synchronized component of error signal is about 250μm. When the internal-model compensator is activated ((b)), the synchronized component of error signal is removed completely at the specified rotational frequency $\omega_1 = 2\pi \times 10.0[1/s]$. However, the synchronized component of error signal remains to some extent at the other rotational frequencies. In contrast, the modified compensator succeeds in removing the synchronized component of error signal at any rotational frequency ((c)).

Figure 7 shows spectral amplitude of the reference and error signals. The rotational frequency is $2\pi \times 5.0[1/s]$. The actual reference signal includes components of N times rotational frequency for $N = 2,3,4,\cdots$. These components remain in the error signal almost as they are without the compensation. They will be also reduced by augmenting the compensator for these higher-frequency components.

6. CONCLUSIONS

The servocompensator with performance of frequency following was applied to the developed tracking control system that simulates disk drive tracking systems. The frequency-following performance was realized with the modified servocompensator that uses signals synchronized with rotation in calculating the control input by means of convolution integral. The efficacy of the modified servocompensator was confirmed experimentally in the developed setup.

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