15th Triennial World Congress, Barcelona, Spain

STABILITY PRESERVING TRANSITION FROM DERIVATIVE FEEDBACK TO ITS DIFFERENCE COUNTERPARTS

H. Kokame [1] **T. Mori** [2]

1) Dept. of Electrical & Electronic Systems, Osaka Prefecture University
kokame@ecs.ees.osakafu-u.ac.jp
2) Dept. of Electronics & Information Science, Kyoto Institute of Technology
mori@dj.kit.ac.jp

Abstract: The present paper disusses the preservation of the closed-loop stability when a derivative feedback is replaced by its difference counterparts. A sufficient condition for such stability preservation is provided. Applying this condition leads to a favorable result that the stability preservation is always true for single-input systems. It is pointed out however that multi-output systems do not allow such simplicity. *Copyright ©2002 IFAC*

Keywords: difference feedback, delay differential equation, derivative feedback, stabilization

1. INTRODUCTION

While the derivative feedback is the most fundamental technique in designing control systems, it is awkward to adopt the technique in the case of noisy measurement condition. If the derivatives are replaced by their difference approximation, noise effects might be somewhat alleviated, because the high frequency gain is not divergent unlike the derivative action. It is also expected (see *e.g.*, Kokame *et al.*, 2000b) that the difference action has a better performance with respect to the stability robustness. Further it is easy to implement on digital computers. With these points in mind, the present paper aims at disclosing the condition under which the closed-loop stability is preserved when replacing the derivative action by its difference counterparts.

The above-mentioned problem does not seem to have been studied in the literature, but a related problem has been assessed in a different context, yielding a favorable result that there exists a stabilizing difference feedback whenever the so-called odd number condition is inactive (Kokame *et al.*, 2000a, 2001). The existence was shown through finding a stabilizing derivative feedback, and then approximating it by a difference action. This means that for some derivative feedback law, the closed-loop stability is retained when transferring to its difference approximates. This favorable result has motivated us to study a general problem to see when such a stability preserving transition is guaranteed.

By analysing the characteristic function of a relevant delay differenctial equation, we will provide a sufficient condition, which is very close to necessary one. Applying this condition yields the most desirable result that the stability preservation is always true for single-input systems. It is pointed out however that multi-output systems do not allow such simplicity. In the following, the determinant of a matrix $X \in C^{n\times n}$ is denoted by det $[X]$, and its eigenvalues are by $\lambda_i(X), i = 1, \ldots, n$. The spectral norm of a matrix X is expressed by $||X||$.

2. STABILITY FOR DIFFERENCE FEEDBACK

In the section, we would like to disclose a relation between the derivative feedback and difference feedback. Let us apply a derivative feedback $u(t) = -K\dot{x}(t) + v(t)$ to the provided unstable linear system,

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x \in R^n$ and $u \in R^m$ respectively denote the state vector and input vector. In what follows, we assume det $[I + BK] \neq 0$ in order that the closed-loop system is well defined. Supposing that the close-loop system is asymptotically stable, *i.e.*, $H = (I + BK)^{-1}A$ is Hurwitz, we are interested in whether the closed-loop stability is inherited by the difference feedback approximation,

$$u(t) = -\frac{1}{T}K(x(t) - x(t-T)) + v(t), \tag{2}$$

where $v(t)$ is the reference input. In other words, we aim at finding a condition which guarantees the asymptotic stability of the closed-loop system,

$$\dot{x}(t) = Ax(t) - BK\frac{1}{T}(x(t) - x(t-T)) + Bv(t), \tag{3}$$

with H being Hurwitz. The following stability analysis is based on the characteristic function of the closed-loop system (Hale, 1977),

$$f(s) = \det[sI - A + BK(1 - e^{-sT})/T]. \tag{4}$$

Noting $f_0(s) = \det[s(I+BK)-A] = \det[I+BK]\det[sI-H]$ does not vanish in the closed right half-plane, it is convenient to normalize $f(s)$ as $F(s) = f(s)/f_0(s)$. Now the closed-loop stability of the difference feedback (2) is equivalent to $F(s)$ vanishing nowhere in the closed right half-plane. A simple calculation yields the equality

$$F(s) = \det[I - \rho(sT)\Gamma(s)], \tag{5}$$

where

$$\rho(s) = 1 - \frac{1-e^{-s}}{s}, \tag{6}$$

$$\Gamma(s) = sK(sI-H)^{-1}(I+BK)^{-1}B. \tag{7}$$

See Kokame *et al.*, (2000b) for a derivation of (5). It is noticed that $\Gamma(s)$ is complementary sensitivity matrix for the input v, i.e., the tranfer matrix from v to $K\dot{x}$ in the derivative feedback configuration. Thus it is proper and stable.

The function $F(s)$ explains how the time-delay T affects on the closed-loop stability. By observing the behavior of the function at the infinity of the complex plane, we have the following main result.

Theorem 1 Suppose that the derivative feedback $u(t) = -K\dot{x}(t)$ stabilizes the linear system (1). Then if $G(s) = \det[I - \rho(s)\Gamma(\infty)]$ has no zeros in the closed right half-plane, the difference feedback (2) stabilizes the same system for sufficiently small T.

Proof: It is enough to show that $F(s)$ has no zeros in the closed right half-plane for small T. In the first place, notice that $G(s)$ approaches a nonzero value when s tends to the infinity in the closed right half-plane. That is, since $\lim_{s\to\infty, \text{Re } s\geq 0} \rho(s) = 1$, we have

$$\lim_{s\to\infty, \text{Re } s\geq 0} G(s) = 1/\det[I+KB] \neq 0.$$

Taking this together with the fact that G is an analytic function into account, we know that the perturbed function $G_\Gamma(s) = \det[I - \rho(s)\Gamma]$, $||\Gamma - \Gamma(\infty)|| < \epsilon$, also have no zeros in the closed right half-plane, if $\epsilon > 0$ is chosen to be sufficiently small. Further we can find a large $\bar{\omega} > 0$ such that $||\Gamma(s) - \Gamma(\infty)|| < \epsilon$, for all $s \in D = \{s : |s| \geq \bar{\omega}, \text{Re } s \geq 0\}$ (Boyd and Desoer, 1985). Combining the above two observations leads to a conclusion that for an arbitrary $z \in D$, $\det[I - \rho(s)\Gamma(z)]$ has no zero in the closed right half-plane, hence so is the function $\det[I - \rho(sT)\Gamma(z)]$. Taking $s = z$, we know that $F(z) = \det[I - \rho(zT)\Gamma(z)]$ does not vanish in the region D for all $T > 0$.

The proof is completed if we show $F(s)$ has no zeros in the bounded region $\bar{D} = \{s : \text{Re } s \geq 0, |s| \leq \bar{\omega}\}$ for sufficiently small T. Notice that since $\Gamma(s)$ is a stable rational function, its H_∞-norm $||\Gamma(s)||_\infty$ is finite, and $||\Gamma(s)|| \leq ||\Gamma(s)||_\infty, \text{Re } s \geq 0$. Let $\epsilon > 0$ be a small constant satisfying $\epsilon < 1/||\Gamma(s)||_\infty$. Then since $\lim_{s\to 0} \rho(s) = 0$, we can find a small $T_0 > 0$ such that $|\rho(sT)| < \epsilon$ holds for $s \in \bar{D}$ and $T < T_0$. Therefore for $T < T_0$, we have

$$F(s) \geq 1 - |\rho(sT)|\,||\Gamma(s)|| > 1 - \epsilon||\Gamma(s)||_\infty > 0, \quad s \in \bar{D},$$

which validates the desired property that $F(s)$ has no zeros in $s \in \bar{D}$. Q.E.D.

The condition of Theorem 1 is a sufficient condition for the stability brought by a derivative feedback being inherited in the corresponding difference feedback. However the condition is very close to a necessary condition. In fact, the following holds.

Theorem 2 Suppose that the derivative feedback $u(t) = -K\dot{x}(t)$ stabilizes the linear system (1). If the function $G(s) = \det[I - \rho(s)\Gamma(\infty)]$ has a zero in the open right half-plane, then the difference feedback $u(t) = -\frac{1}{T}K(x(t) - x(t-T))$ can not stabilize for all sufficiently small T.

Proof: Suppose that $G(s_0) = 0$ for Re $s_0 > 0$. There exists a small disk $D = \{s : |s - s_0| \leq \epsilon\}, \epsilon < |s_0|$, such that $G(s) \neq 0$ on the boundary of D, ∂D. Consider a function $F(s/T) = \det[I - \rho(s)\Gamma(s/T)]$. Noting that both $G(s)$ and $F(s/T)$ are analytic on D, we know that as T tends to 0, $F(s/T)$ converges to $G(s)$ uniformly on ∂D. Then the continuity argument based on Rouché's theorem ensures that $F(s/T)$ has a zero in the interior of D if T is small enough. This implies that $F(s)$ has a zero in the interior of the enlarged disk $D_T = \{s : |s - s_0/T| \leq \epsilon/T\}$. Q.E.D.

Notice that if $G(s)$ has a zero on the imaginary axis, and has no other zeros in the open right half-plane, the desired inheritance of the closed-loop stability may sometimes be true, but sometimes be false.

A detailed analysis shows that the interested condition of Theorem 1 can be described directly in terms of the eigenvalues of $\Gamma(\infty)$, or those of KB.

Theorem 3 The function $G(s) = \det[I - \rho(s)\Gamma(\infty)]$ has no zeros in the closed right half-plane if and only if for every nonzero $\gamma_i = \lambda_i(\Gamma(\infty))$, its inverse $1/\gamma_i$ is outside Ω, where Ω is the closed region bounded by the curve $\{\rho(j\omega) : -2\pi \leq \omega \leq 2\pi\}$ (See Fig. 1).

Further $G(s)$ has a zero in the open right half-plane if and only if there exists a nonzero γ_i for which $1/\gamma_i \in \Omega$.

Proof: We show the proof of the former part alone, since the latter is similar. Rewrite $G(s)$ as follows:

$$G(s) = \prod_{i=1}^{m}(1 - \rho(s)\gamma_i) = \prod_{\gamma_i\in\Lambda}\gamma_i(\frac{1}{\gamma_i} - \rho(s)),$$

where Λ is the set of nonzero eigenvalues of $\Gamma(\infty)$.

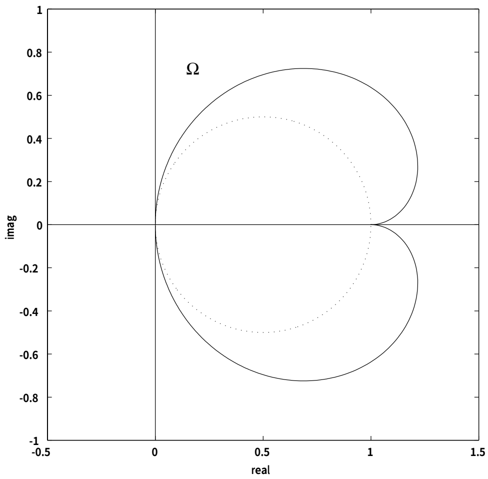

Fig. 1. Region Ω, the inside of the solid line.

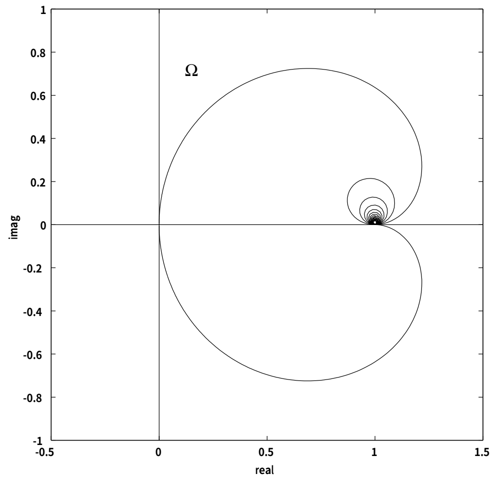

Fig. 2. Vector plot of ρ.

In the first place, assume that $G(s)$ has no zeros in the closed right half-plane. Then G does not vanish on the imaginary axis. Recall that under such non-vanishing condition, the argument principle assures that the number of the right half-plane zeros of G, denoted by Z, is equal to

$$Z = \sum_{\gamma_i \in \Lambda} n(\frac{1}{\gamma_i};\ \rho),$$

where n denotes the number of clockwise encirclements of the vector $\rho(j\omega)$ with respect to the point $1/\gamma_i$, when ω increases from $-\infty$ to $+\infty$. As shown in Fig. 2, when ω increases from 0, the vector ρ moves off the origin into the first quadrant and reach the real axis at 1 when $\omega = 2\pi$. For $\omega > 2\pi$, it stays inside Ω. Observe also that the segment of the vector locus, $R_k = \{\rho(j\omega) :\ 2k\pi < \omega \leq 2(k+1)\pi\}$, is encircled by the former one R_{k-1}. Thus $Z = 0$ implies and is implied by the condition that for any $\gamma_i \in \Lambda$, $1/\gamma_i$ is outside of Ω. The converse is now immediate. Q.E.D.

By noting $\Gamma(\infty) = (I + KB)^{-1}KB$, the eigenvalues $\gamma_i = \lambda_i(\Gamma(\infty))$ and $\mu_i = \lambda_i(KB)$ are related as $1/\gamma_i = 1 + 1/\mu_i$ for nonzero γ_i. Thus Theorem 3 may be rephrased in terms of μ_i.

Corollary 4 The function $G(s) = \det[I - \rho(s)\Gamma(\infty)]$ has no zeros in the closed right half-plane if and only if for every nonzero $\mu_i = \lambda_i(KB)$, its inverse $1/\mu_i$ is outside Ω_-, where Ω_- is the closed region obtained by shifting Ω in the negative direction by 1.
Further $G(s)$ has a zero in the open right half-plane if and only if there exists a nonzero μ_i for which $1/\mu_i \in \Omega_-$.

In the case where $G(\infty) = \det[I - \Gamma(\infty)] < 0$, or equivalently if there exists at least one real eigenvalue γ_i such that $1 - \gamma_i < 0$, then $0 < 1/\gamma_i < 1$, hence $1/\gamma_i \in \Omega$. Thus we have a simple consequence that if $\det[I - \Gamma(\infty)] < 0$, the difference feedback can not stabilize for small T.
Notice that since $I - \Gamma(\infty) = (I + KB)^{-1}$, the condition is equal to $\det[I + KB] < 0$.
The above observation can be generalized by noting that the interior of the forbidden domain Ω entirely includes the disk $D_u = \{s :\ |s - 1/2| \leq 1/2\}$, except $s = 0$ and 1. We omit the proof, for it is straightforward. In Fig. 1, the boundary of the disk is plotted by the dotted curve.

Lemma 5 Suppose that the derivative feedback $u(t) = -K\dot{x}(t)$ stabilizes the linear system (1). If the matrix $\Gamma(\infty) - I$ is not Hurwitz, or equivalently if $-(I+KB)$ is not Hurwitz, then the difference feedback $u(t) = -\frac{1}{T}K(x(t) - x(t-T))$ can not stabilize for all sufficiently small T.
Proof: The condition of the lemma signifies that Re $\gamma_i - 1 \geq 0$ for some $\gamma_i = \lambda_i(\Gamma(\infty))$. Obviously $1/\gamma_i$ belongs to D_u. Further recalling the assumption $\det[I + KB] \neq 0$, we know $\gamma_i \neq 1$, and hence $1/\gamma_i$ belongs to the interior of Ω. Thus the latter part of Theorem 3 combined with Theorem 2 ensures the conclusion. Q.E.D.

It is warned from Lemma 5 that to obtain a successful difference feedback, it is necessary to choose a derivative feedback gain K so that $I + KB$ becomes an anti-Hurwitz matrix, that is, it must have all the eigenvalues in the open right half-plane.

Obviously the condition $\det[I + KB] < 0$ contradicts the anti-Hurwitzness. Conversely if the anti-Hurwitzness condition is fulfilled, and if the derivative feedback is stabilizing, *i.e.*, H is Hurwitz, it follows from $A = (I + BK)H$ that $\det[-A] > 0$. That is, $\det[-A] > 0$ is an essential restriction in order to have a stabilizing difference feedback for small T.

At this point, it is emphasized that the requirement det $[-A] > 0$ is quite natural. In fact it is known that if det $[-A] \leq 0$, the delay differential equation,

$$\dot{x}(t) = Ax(t) + H(x(t) - x(t-T)),$$

can not be asymptotically stable for any H, and for any choice of the delay time T. The condition is often called the *odd-number condition*, as it means that the open loop system has an odd number of real positive eigenvalues or at least one eigenvalue at the origin. See Ushio (1996) for discrete-time systems, and Nakajima (1997) for continuous-time systems. Their results are motivated from the delayed feedback control proposed by Pyragas (1992).

We close this section by putting a simple result about the stability preservation of the inverse direction, *i.e.*, from difference feedback to the derivative feedback.

Proposition 6 Suppose that det $[I + KB] \neq 0$, and for all sufficiently small $T > 0$, the difference feedback $u(t) = -\frac{1}{T}K(x(t) - x(t-T))$ stabilize the linear system (1). More restrictively if the characteristic function $f(s)$ of (4) has no zeros in the region $H_\epsilon = \{s : \text{Re } s > -\epsilon\}$ for some $\epsilon > 0$, then the corresponding derivative feedback $u(t) = -K\dot{x}(t)$ also stabilizes the system (1).

Proof: Assume to the contrary that det $[s_0 I - H] = 0$, Re $s_0 \geq 0$. We can find a small disk $D = \{s : |s - s_0| \leq \eta\}, \eta < \epsilon$, such that det $[sI - H] \neq 0$ on the boundary of D. On the other hand, as T tends to zero, $f(s)$ tends to $g(s) =$ det $[I + KB]$det $[sI - H]$ for every s. Thus for all sufficiently small T, $f(s)$ must have the same number of roots inside D as det $[sI - H]$ has. Noting $D \subset H_\epsilon$ leads to that $f(s)$ has a root in H_ϵ, which contradicts the assumption. Q.E.D.

3. APPLICATION OF THE CRITERION

We first give a simple example in which the derivative feedback stabilizes, whereas its difference approximation can not for all small T. Such ill-posedness may be compared to known instability caused by introducing small time-delays in the feedback loop (see *e.g.,* Datko 1988; Louisell 1995; Longemann and Townley 1996).

Example 1: Consider the following third order system,

$$\dot{x}(t) = \begin{bmatrix} -1 & 1 & 0 \\ -3 & 1 & 3 \\ -2 & 1 & 1 \end{bmatrix} x(t) + u(t), \qquad (8)$$
$$y(t) = [1\ 0\ 0]x(t), \quad x, u \in R^3.$$

This system has duplicated unstable poles on 1, and it can be stabilized by the derivative feedback $u = -\begin{bmatrix} -2 & 0 & 0 \\ -1 & 1 & -1 \\ 4 & 0 & -4 \end{bmatrix} \dot{x}$. The poles of the closed-loop system

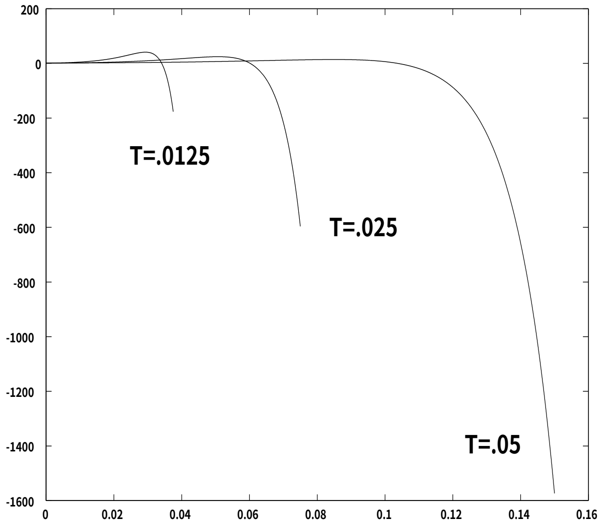

Fig. 3. Initial response of y for some T.

are -0.1255 and $-0.0206 \pm j1.1523$. However the eigenvalue inverses for $KB = K$ are $-0.5, -0.25$ and 1. Notice that the former two belongs to Ω_-. Thus the corresponding difference feedback will fail to stabilize. Computer simulation was made assuming the initial condition $x(t) = 0, -T \leq t < 0$, and $x(0) = [1\ 0\ 0]^T$. Fig. 3 shows the initial responses of the closed-loop system which incorporates the difference feedback for several T. Though every response is diverging, it should be noticed that the divergence rate is more rapid for smaller T.

As compared with multi-input systems, we can enjoy a simple result for the case of a single-input system,

$$\dot{x}(t) = Ax(t) + bu(t). \qquad (9)$$

Letting the control input be $u = -k^T\dot{x}$, the test matrix KB becomes a scalar quantity, $\mu = k^T b$. If $1 + k^T b > 0$, either $1/\mu > 0$ or $1/\mu < -1$ holds except for the case $\mu = 0$. Thus from Corollary 4, we have the following proposition.

Proposition 7 Suppose that the derivative feedback $u = -k^T\dot{x}$, $1 + k^T b > 0$, stabilizes the system (9). Then the difference feedback $u(t) = -\frac{1}{T}k^T(x(t) - x(t-T))$ also stabilizes for sufficiently small T.

In the previous section, the anti-Hurwitzness condition was explained as a necessary condition to obtain a stabilizing difference feedback with small T. It is to be noticed that for single-input systems, the anti-Hurwitzness condition acts also as a sufficient condition.

Further if we put the condition det $[-A] > 0$ explicitly, a derivative feedback stabilizing (9) requires $1 + k^T b > 0$. Thus Proposition 7 may be restated in a different form.

Theorem 8 Suppose that the linear system (9) satisfies the requisite condition $\det[-A] > 0$. Then for every stabilizing derivative feedback $u = -k^T\dot{x}$, its difference approximation, $u(t) = -\frac{1}{T}k^T(x(t) - x(t - T))$, also stabilizes for sufficiently small T.

If the linear system (9) with $\det[-A] > 0$ is controllable, any pole allocation may be realized by the derivative feedback except for the origin. Especially any pole allocation in the open left half-plane is possible. Taking this point into account, one may safely say that any stable pole allocation can be attained approximately by a difference feedback. This is clearly an advanced result of Theorem 2 of Kokame *et al.*, (2000a).

When the controllability assumption remains, the favorable property of Theorem 8 may be extended easily to a multi-input system (1), if A is cyclic. In fact, if (A, B) is controllable with a cyclic A, we can find a vector $w \in R^m$ for which (A, Bw) is controllable (see Wonham, 1967). Thus from Theorem 8, any stable pole allocation might be realized approximately by a difference feedback $u(t) = -wk^T((x(t) - x(t-T))$. However if A is not cyclic, the circumstance is quite different.

Example 2: Consider a simple second-order system,

$$\dot{x}(t) = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix} x(t) + Bu(t). \tag{10}$$

Note however that $A = \mu I$ is not cyclic. If $\mu > 0$, then the requisite condition $\det[-A] > 0$ is satisfied. On the other hand, the closed-loop stability under $u = -K\dot{x}$ means $H = \mu(I + BK)^{-1}$ is Hurwitz. That is, $I + BK$ is Hurwitz, which completely contradicts the anti-Hurwitzness condition. Thus the second order system (10) can not be stabilized by any difference approximation for small T.

In the rest of the section, we proceed to get some light for the case of the multi-input systems. Assume that the system (1) is controllable, and the matrix B has full column rank. For simplicity we may assume that it is given in Luengerger's canonical form of the second kind (Luenberger, 1967), where A and B are assumed to be $m \times m$ block matrix $[A_{ij}]$ and $m \times 1$ block matrix $[B_i]$, where $A_{ij} \in R^{\sigma_i \times \sigma_j}$ and $B_i \in R^{\sigma_i \times m}$ are given as follows:

$$A_{ii} = \begin{bmatrix} 0 & 1 & 0 & & \cdots & 0 \\ 0 & 0 & 1 & 0 & & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & & & & 0 & 1 \\ -a_0^{ii} & -a_1^{ii} & \cdots & & & -a_{\sigma_i-1}^{ii} \end{bmatrix}, \tag{11}$$

$$A_{ij} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \\ -a_0^{ij} & \cdots & -a_{\sigma_j-1}^{ij} \end{bmatrix}, \quad i \neq j, \tag{12}$$

$$B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & \cdots & 1 & b_{i+1}^i & \cdots & b_m^i \end{bmatrix}, \tag{13}$$

where $\sigma_i, i = 1, \ldots, m$, are controllability indices, and $\sum_{j=1}^m \sigma_j = n$ due to the controllability assumption. The characteristic polynomial of the block matrix A is given by

$$f_o(s) = \det \begin{bmatrix} g_o^{11}(s) & \cdots & g_o^{1m}(s) \\ \vdots & & \vdots \\ g_o^{m1}(s) & & g_o^{mm}(s) \end{bmatrix}, \tag{14}$$
$$g_o^{ij}(s) = \delta_{ij}s^{\sigma_j} + a_{\sigma_j-1}^{ij}s^{\sigma_j-1} + \cdots + a_1^{ij}s + a_0^{ij},$$

where $\delta_{ij} = 1$ for $i = j$, and 0 elsewhere.
The above B can be decomposed as $B = \bar{B}L$, where $\bar{B} = [\bar{B}_i]$ is partitioned compatibly to B, with $\bar{b}_k^i = 0, k = i+1, \ldots, m$. The $m \times m$ matrix L ought to be a nonsingular upper triangular matrix. Without loss of generality, we devote ourselves to the system having a new control input:

$$\dot{x}(t) = Ax(t) + \bar{B}v(t). \tag{15}$$

Applying the derivative feedback $v = -\bar{K}\dot{x}$ leads to the following expression:

$$f_c(s) = \det[F_c(s)]/\det[I + G], \tag{16}$$
$$F_c(s) = \begin{bmatrix} g_c^{11}(s) & \cdots & g_c^{1m}(s) \\ \vdots & & \vdots \\ g_c^{m1}(s) & & g_c^{mm}(s) \end{bmatrix},$$
$$g_c^{ij}(s) = (\delta_{ij} + \kappa_{\sigma_j}^{ij})s^{\sigma_j} + (a_{\sigma_j-1}^{ij} + \kappa_{\sigma_j-1}^{ij})s^{\sigma_j-1} + \cdots + (a_1^{ij} + \kappa_1^{ij})s + a_0^{ij}, \tag{17}$$

where $G = \bar{K}\bar{B} = [\kappa_{\sigma_j}^{ij}]$. Note that the determinant matrix polynomial $F_c(s)$ has the column degrees, $\sigma_1, \ldots, \sigma_m$, and taking the coefficients of the column degree σ_j for the j-th column makes the matrix $I + \bar{K}\bar{B}$.

In contrast to the single-input case, the difference feedback obtained from a stabilizing derivative feedback does not always work well. Assuming the controllability, there may exist two derivative feedback laws that yield the same pole distribution, but their difference approximation works well for one case, and does not work for the other one. Finally we give such an example.

Example 3: Consider a third-order system in the Luenberger's canonical form.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 2 & -1 \\ -1 & -1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t). \quad (18)$$

This system has duplicate eigenvalues on 1, and a single one on -1. Consider the derivative feedback with gains $K_a = \begin{bmatrix} 4 & 1 & 0 \\ -1 & 0 & -0.5 \end{bmatrix}$, and $K_b = \begin{bmatrix} -2 & -2 & 0 \\ -10 & 0 & -2 \end{bmatrix}$. Both gains make the closed-loop system to have the same characteristic polynomial $s^3 + 3s^2 + 3s + 1$. Therefore pole configurations are coincident for the two gains. However, for the former, the eigenvalues for K_aB are 1 and -0.5, thus their inverses are outside Ω_-. Corollary 4 together with Theorem 1 now guarantees that a corresponding difference feedback works to stabilize. On the other hand, $I + K_bB = -I$, which is a Hurwitz matrix. From Lemma 5, we know that the corresponding difference feedback will fail to stabilize.

4. CONCLUDING REMARKS

The paper has considered the fundamental problem of whether the closed-loop stability is preserved when replacing the state derivative by its difference counterparts. A sufficient condition for such a stability preserving replacement being true has been provided. The condition is very close to necessary one. Applying this condition, we have shown a favorable result that the desired stability preservation is always true for single-input systems. Using some examples, it is pointed out however that multi-output systems do not allow such simplicity. This study was partly supported by Grant-in Aid(No. 13650494) for Scientific Research.

REFERENCES

BOYD, S. and C. A. DESOER (1985). Subharmonic functions and performance bounds on linear time-invariant feedback systems, *IMA Journal of Mathematical Control & Information*, **2**, 153-170.

DATKO, R. (1988). Not all feedback stabilized hyperbolic systems are robust with respect to small time delays in their feedbacks, *SIAM J. Contr. Optimization*, **26**, 697-713.

HALE, J. (1977). *Theory of Functional Differential Equations*, Springer, New York.

KOKAME, H. and T. MORI (1999). Delayed feedback control: a practical method for stabilization under large uncertainties, *Proc. Korea-Japan Joint Workshop on Robust and Predictive Control of Time Delay Systems*, 143-154, Seoul.

KOKAME, H., K. HIRATA, K. KONISHI, and T. MORI (2000a). State difference feedback can stabilize uncertain steady states, *Proc. American Control Conference*, 1370-1374, Chicago.

KOKAME, H., K. HIRATA, K. KONISHI, and T. MORI (2000b). Stabilization via the delayed feedback with a large delay, *Proc. 2nd IFAC Workshop on Linear Time Delay Systems*, 15-20, Ancona.

KOKAME, H., K. HIRATA, K. KONISHI, and T. MORI (2001). State difference feedback for stabilizing uncertain steady states of nonlinear systems, *Int. J. Contr.*, **74**, 537-546.

LONGEMANN, H. and S. TOWNLEY (1996). The effects of small delays in the feedback loop on the stability of neutral systems, *Syst. Contr. Lett.*, **27**, 267-274.

LOUISELL, J. (1995). Absolute stability in linear dely-differential systems: Ill-posedness and robustness, *IEEE Trans. Automat. Contr.*, **40**, 1288-1291.

LUENBERGER, D. G. (1967). Canonical forms for linear multivariable systems, *IEEE Trans. Automat. Contr.*, **12**, 290-293.

NAKAJIMA, H. (1997). On analytical properties of delayed feedback control of chaos, *Phys. Lett. A*, **232**, 207-210.

PYRAGAS, K. (1992) Continuous control of chaos by self-controlling feedback, *Phys. Lett. A*, **170**, 421-428.

USHIO, T. (1996). Limitation of delayed feedback control in nonlinear discrete-time systems, *IEEE Trans. Circuits Syst. I*, **43**, 815-816.

WONHAM, W. M. (1967). On pole assignment in multi-input controllable linear systems, *IEEE Trans. Automat. Contr.*, **12**, 660-665.