15th Triennial World Congress, Barcelona, Spain

ADJOINT-TYPE ITERATIVE LEARNING CONTROL FOR A SINGLE-LINK FLEXIBLE ARM

Koji Kinoshita * **Takuya Sogo** * **Norihiko Adachi** *

* *Graduate School of Informatics, Kyoto University*
Yoshida-Honmachi, Sakyo ward, Kyoto city, Kyoto 606-8501, JAPAN
(E-mail: kinosita@sys.i.kyoto-u.ac.jp)

Abstract: Iterative learning control (ILC) obtains a desired input that exactly generates the desired output through repetitions of the similar tasks. In this paper, an experiment of the adjoint-type ILC based on the gradient method is carried out by using a single-link flexible arm. Experimental results showed that, even if the single-link flexible arm is modeled by a simple method and has some uncertainties, exact output tracking is achieved. Moreover, pre-actuation, which is remarkable aspect of the adjoint-type ILC, is observed.

Keywords: Learning control, Iterative methods, Flexible arms, Non-minimum phase systems, Inverse system, Gradient methods

1. INTRODUCTION

Robot manipulators are often required to achieve exact tracking for a given desired output $y_d(t)$. However, it is difficult for conventional feedback control to achieve such a requirement. One of the methods for achieving exact tracking is feedforward control based on the classical inversion (Silverman, 1970). With this method, a desired input that exactly generates the desired output is easily constructed. However, this method has two difficulties. First, it requires completely accurate information about the systems to be controlled. Second, for non-minimum phase systems, the desired input increases exponentially with the evolution of time because of the unstable zeros of the systems.

In order to overcome the first problem, Arimoto et al. proposed iterative learning control (ILC)(Arimoto, *et al.*, 1984). ILC enables finding a desired input through repetitions of the same tasks. Even if the system has some uncertainties, it is possible to achieve exact tracking. However, if this type of ILC is applied to a non-minimum phase system, the magnitude of the input sequence becomes too large because this ILC is closely related to the classical inversion.

On the other hand, it is clarified that the adjoint-type ILC, which is based on the gradient method, obtains the desired input defined by Stable Inversion (Devasia, *et al.*, 1996), which constructs a bounded desired input achieving exact tracking for non-minimum phase systems (Kinosita, *et al.*, 2000). ILC can obtain the desired input for uncertain systems and Stable Inversion can obtain the bounded desired input for non-minimum phase systems. This implies the effectiveness of the adjoint-type ILC for uncertain non-minimum phase systems.

The above relationship between the adjoint-type ILC and Stable Inversion was clarified for finite dimensional systems; however, this relationship suggests that the adjoint-type ILC can achieve the good output tracking for a flexible arm, which is a non-minimum phase system and an infinite dimensional system. In this paper, an experiment of the adjoint-type ILC is carried out for

Table 1. Physical parameters

Physical parameters	Symbol	Numerical value
Natural frequency	f_c	1.8Hz
Length	L	4.50×10^{-1}m
Link mass	M_l	6.00×10^{-2}kg
Link rigid body inertia	J_l	4.05×10^{-3}Kgm^2
Payload mass	M_p	5.00×10^{-2}kg
Payload inertia	J_p	1.01×10^{-2}kgm^2
Hub inertia	J_{hub}	2.00×10^{-3}kgm^2
Armature resistance	R	2.60Ω
Motor torque constant	Km	7.67×10^{-3}Nm/A
Gear ratio	Kg	1:70

a single-link flexible arm. According to (Cannon and Schmitz, 1984), the single-link flexible arm's transfer function from the actuator to its tip position can be written as

$$\mathcal{S}(s) = \sum_{i=0}^{\infty} \frac{k_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}. \quad (1)$$

Many researchers have proposed a controller to the single-link flexible arm by using an approximate finite dimensional model constructed from this infinite dimensional model (Cannon and Schmitz, 1984; Krishnan and Vidyasagar, 1998). In (Cheng, *et al.*, 1993), the learning control for the flexible arm is carried out by the modal truncated model. In this experiment, however, a simple model of the single-link flexible arm is used.

The objective of this paper is to show experimentally that it is possible to achieve good output tracking by only using a simple finite dimensional model and to consider why the good output tracking is achieved.

The remainder of this paper is organized as follows. In Section 2, a simple model of a single-link flexible arm is derived. In Section 3, the adjoint-type iterative learning controller based on this simple model is designed. In Section 4, the experimental set-up and results are shown. Finally, Section 5 concludes this paper.

2. SIMPLE MODEL FOR A SINGLE-LINK FLEXIBLE ARM

The single-link flexible arm used in this experiment is shown schematically in Fig. 1, where

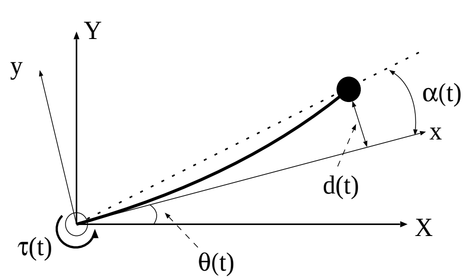

Fig. 1. The single-link flexible arm

θ, α and d denote the hub angle, the angular deflection of the tip and the deflection of the tip, respectively. The physical parameters of the experimental flexible arm are shown in Table. 1. It is assumed that the deflection of the arm is small and the arm moves and vibrates only in the horizontal direction. In this paper, a mathematical model for Fig. 1 is derived by using the following simple approach.

First, the natural frequency f_c of the link with the base clamped is obtained. Since the angular deflection of the tip is given by

$$\ddot{\alpha} = -(2\pi f_c)^2\alpha, \quad (2)$$

the stiffness of the link can be estimated as follows:

$$K_{stiff} = (2\pi f_c)^2(J_l + J_p) \quad (3)$$

where $J_l = M_l L^2/3$ and $J_p = M_p L^2$ are link rigid body inertia and payload inertia, respectively. Second, the kinetic and potential energies of this system are given by

$$KE_{hub} = \frac{1}{2}J_{hub}\dot{\theta}^2 \quad (4)$$

$$KE_{load} = \frac{1}{2}J_{load}(\dot{\theta} + \dot{\alpha})^2 \quad (5)$$

$$PE = \frac{1}{2}K_{stiff}\alpha^2 \quad (6)$$

where $J_{load} = J_l + J_p$ is the total load inertia. The dynamic equations of this system are formed by using the Lagrange equation

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i, \quad (i = 1, 2), \quad (7)$$

where the Lagrangian $L = KE_{hub} + KE_{load} - PE$, generalized coordinate $q = [\theta,\ \alpha]^{\mathrm{T}}$, and generalized force $Q = [\tau,\ 0]^{\mathrm{T}}$. The following dynamic equations are obtained by solving this Lagrange equation.

$$(J_{hub} + J_{load})\ddot{\theta} + J_{load}\ddot{\alpha} = \tau \quad (8)$$

$$J_{load}\ddot{\theta} + J_{load}\ddot{\alpha} + K_{stiff}\alpha = 0. \quad (9)$$

The torque is generated by a DC motor with the following equation

$$\tau = \frac{K_m K_g}{R}V_{in} - \frac{K_m^2 K_g^2}{R}\dot{\theta}. \quad (10)$$

The angular deflection α is translated to the deflection d

$$d = \alpha L, \quad for\ d \ll L. \quad (11)$$

The tip position which is the output of this system can be represented as

$$y(t) = L\theta(t) + d(t). \quad (12)$$

Thus, the state space model can be obtained as

$$\begin{aligned}\dot{x}(t) &= \hat{A}x(t) + bV_{in}(t)\\ y(t) &= cx(t)\end{aligned} \quad (13)$$

where $x = [\theta, d, \dot{\theta}, \dot{d}]^{\mathrm{T}}$,

$$\hat{A} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \dfrac{K_{stiff}}{J_{hub}L} & -\dfrac{K_m^2K_g^2}{RJ_{hub}} & 0\\ 0 & -\dfrac{K_{stiff}(J_{load}+J_{hub})}{J_{hub}J_{load}} & \dfrac{LK_m^2K_g^2}{RJ_{hub}} & 0\end{bmatrix}$$

$$b = \begin{bmatrix} 0 & 0 & \dfrac{K_mK_g}{RJ_{hub}} & -\dfrac{LK_mK_g}{RJ_{hub}}\end{bmatrix}^{\mathrm{T}}$$

$$c = \begin{bmatrix} L & 1 & 0 & 0\end{bmatrix}.$$

One pole of this simple model is on the imaginary axis. Thus, the following state feedback controller is applied.

$$V_{in}(t) = -Kx(t) + u(t) \quad (14)$$

where $u(t)$ is the feed-forward input generated by the adjoint-type ILC. The closed loop model of this flexible arm is

$$\begin{aligned}\dot{x}(t) &= Ax(t) + bu(t)\\ y(t) &= cx(t)\end{aligned} \quad (15)$$

where $A = \hat{A} - bK$.

This approach is gross modeling, however, it is possible to achieve exact output tracking by the adjoint-type ILC as shown in Section 4.

3. ADJOINT-TYPE ITERATIVE LEARNING CONTROL

In this section, an iterative learning controller is designed for the simple model (15).

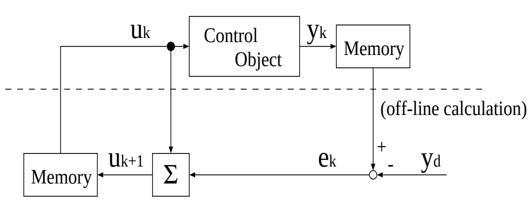

Fig. 2. The basic structure of ILC

ILC enables finding a desired input that exactly generates the desired output through repetition of trials on a finite time interval $[T_0, T_f]$. The basic structure of ILC is shown in Fig. 2. A control input in the next trial is defined by the current control input and the error signal, that is,

$$u_{k+1} = \Sigma(u_k, e_k) \quad (16)$$

where index k is a trial number; u_k, y_k, and $e_k \triangleq y_k - y_d$ are input, output and error signal on the k-th trial, respectively. $\Sigma(\cdot,\cdot)$ is the operator such that $\Sigma : \mathcal{U}\times\mathcal{E} \to \mathcal{U}$ where $\mathcal{U}$ and $\mathcal{E}$ are input and error function space, respectively.

The problem of ILC is to design an update law (16) using partial information about the system. There are two different design methods. One of them is forward-time updating (Arimoto and Miyazaki, 1984), and the other is backward-time updating (Yamakita and Furuta, 1991; Kinosita and Adachi, 2000). In this paper, the adjoint-type ILC with a backward-time update law is employed.

Consider the following linear time-invariant system,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + bu(t), \quad x(T_0) = 0\\ y(t) &= cx(t)\end{aligned} \quad (17)$$

where $x \in R^n$, $u \in R$ and $y \in R$ are the state, input, and output, respectively. A, b and c are a matrix or vectors with appropriate dimensions. The input-output map of (17) is denoted by

$$\begin{aligned}y(t) &= \int_{T_0}^{t} ce^{A(t-\tau)}bu(\tau)d\tau\\ &= [\hat{S}u](t).\end{aligned} \quad (18)$$

The update law of ILC based on the adjoint system is defined by

$$\begin{aligned}u_{k+1}(t) &= u_k(t) - \gamma\eta_k(t), \quad t \in [T_0, T_f]\\ u_0 &= 0\end{aligned} \quad (19)$$

where $\eta_k = \hat{S}^* e_k$, and $\hat{S}^*$ denotes the adjoint operator of $\hat{S}$ and γ is a positive constant. The adjoint operator S^* is denoted as follows:

$$\hat{S}^* e(\tau) = \int_{\tau}^{T_f} b^{\mathrm{T}} e^{A^{\mathrm{T}}(\tau - t)} c^{\mathrm{T}} e(t)dt. \quad (20)$$

Moreover, the adjoint operator $\hat{S}^*$ expresses the input-output map of the adjoint system

$$\begin{aligned}\dot{p}(t) &= -A^{\mathrm{T}}p(t) - c^{\mathrm{T}}e(t), \quad p(T_f) = 0\\ \eta(t) &= b^{\mathrm{T}}p(t).\end{aligned} \quad (21)$$

Note that the adjoint system (21) must be calculated from T_f to T_0 (backward-time). Since the updating of ILC is carried out off-line, it is possible to calculate (21).

Suppose that the simple model $\hat{S}$ is related to a input-output map of control object $\mathcal{S}$ by the following equation,

$$\mathcal{S} = \hat{\mathcal{S}}\mathcal{U}, \tag{22}$$

where $\mathcal{U}$ is an unknown or unmodeled part of the control object. If $\mathcal{S}$ is expressed by the finite dimensional model (e.g. , rigid link arms), convergence conditions for $\mathcal{U}$ are obtained in (Kinoshita, *et al.*, to be published). However, $\mathcal{S}$ is expressed by the infinite dimensional model for the flexible arm. It is difficult to obtain the convergence condition for $\mathcal{U}$. Thus, in the next section, it is shown experimentally that the exact output tracking for the flexible arm is achieved by the simple model (15).

4. EXPERIMENTAL RESULTS

The experimental set-up is shown in Fig. 3. The tip deflection is measured by a light source attached on the tip and an optical sensor mounted on the rotating base. The rotating angle of the hub is measured by a potentiometer .

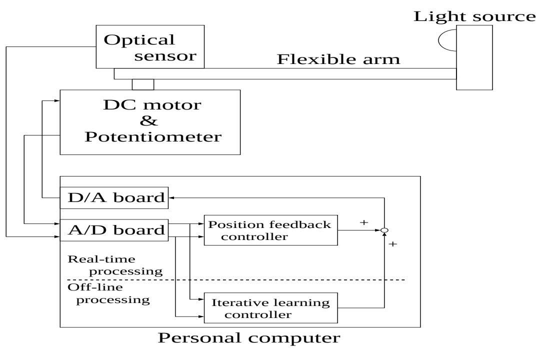

Fig. 3. Experimental setup

Poles and zeros of the truncated model by the modal expansion and the simple model of the flexible arm the flexible arm are shown in Table. 2.

Table 2. Poles and zeros of truncated and simple model

Flexible arm		Simple model	
pole	zero	pole	zero
0	-83.1	0	$--$
-5.37	93.0	-26.5	
$-0.118 \pm 11.8j$	-326	$-14.5 \pm 7.63j$	
$-0.275 \pm 27.5j$	263		

In this experiment, the feedback gain is chosen as $K = \begin{bmatrix} 2, & -19.5, & 0, & 0 \end{bmatrix}$. The adjoint system is constructed by a simple model and this feedback gain. In order to implement the scheme on the personal computer, a discretised adjoint system with sampling period $Ts = 10$ms is obtained as follows:

$$p((n-1)Ts) = -A_d^{\mathrm{T}} p(nTs) - c_d^{\mathrm{T}} e(nTs) \tag{23}$$
$$\eta(nTs) = b_d^{\mathrm{T}} p(nTs), \quad p(T_f) = 0$$

where

$$A_d = \begin{bmatrix} 0.991 & 0.141 & 7.68\times10^{-3} & 4.94\times10^{-4} \\ 3.84\times10^{-3} & 0.933 & 1.04\times10^{-3} & 9.77\times10^{-3} \\ -1.54 & 25.4 & 0.578 & 0.141 \\ 0.692 & -12.2 & 0.190 & 0.933 \end{bmatrix}$$
$$b_d = \begin{bmatrix} 4.27\times10^{-3} & -1.92\times10^{-3} & 0.770 & -0.346 \end{bmatrix}^{\mathrm{T}}$$
$$c_d = \begin{bmatrix} 0.45 & 1 & 0 & 0 \end{bmatrix}$$

The time interval of the trial is $[T_0, T_f] = [0, 10]$. The desired output is

$$y_d(t) = \begin{cases} \dfrac{L\pi}{4}(1 - \cos\pi(t-4)), & 4 \le t \le 6 \\ 0 & otherwise \end{cases} \tag{24}$$

The desired output and outputs at the 10th, 20th, and 50th trials are shown in Fig. 4. The feed-forward inputs at the 10th, 20th, and 50th trials are shown in Fig. 5. According to theory of ILC, accurate physical parameter values are not required for designing iterative learning controllers. In order to verify this feature, a weight (mass $= 3.0 \times 10^{-2}$kg) is attached on the tip. The desired output and outputs at the 10th, 20th, and 50th trials are shown in Fig. 6. The feed-forward inputs at the 10th, 20th, and 50th trials are shown in Fig. 7.

Fig. 4 and Fig. 6 show that the good output tracking is achieved after the 50th trial. From Fig. 6, the undershoot at 6sec is observed in the 10th and 20th trials because of the weight attached on the tip. However, this undershoot is eliminated in the 50th trial. From Fig. 5 and Fig. 7, the adjoint-type ILC obtains the pre-actuating input that arises before the desired output leaves 0. This phenomenon is a remarkable aspect of the adjoint-type ILC for non-minimum phase systems.

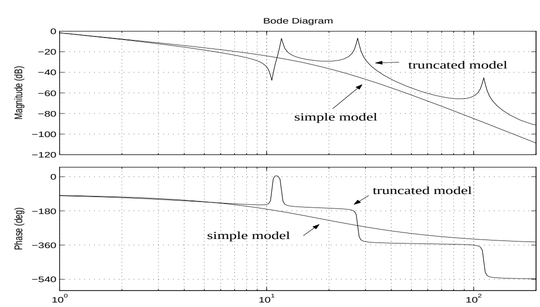

Fig. 8. Bode diagram of the truncated and simple model

The simple model approximates the flexible arm well on the low-frequency range (Fig. 8). Moreover, $|\hat{y}_d(j\omega)| = O(\omega^{-2})$, where $\hat{y}_d$ is the Fourier transform of the desired output (Fig. 9). This is why the good output tracking is achieved by this simple model.

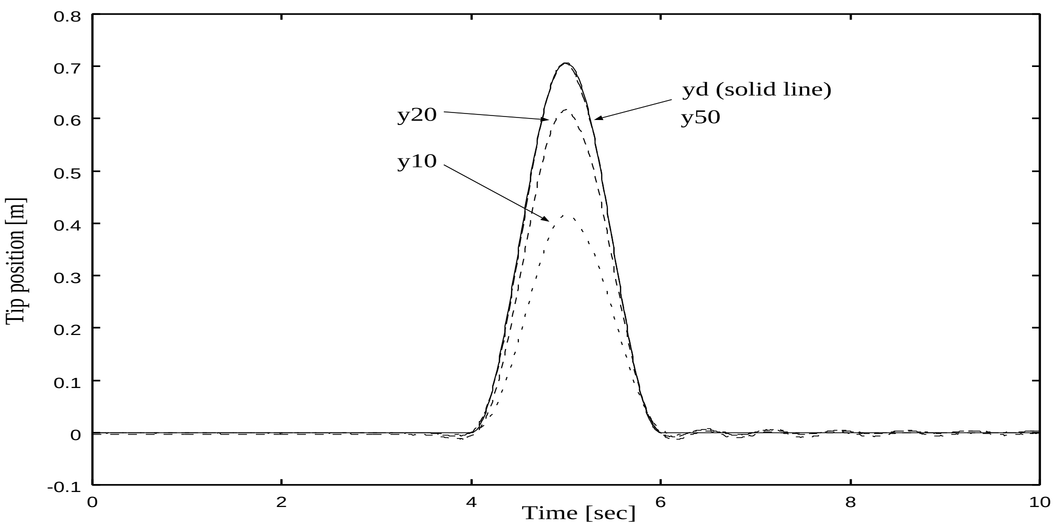

Fig. 4. The output and the desired output

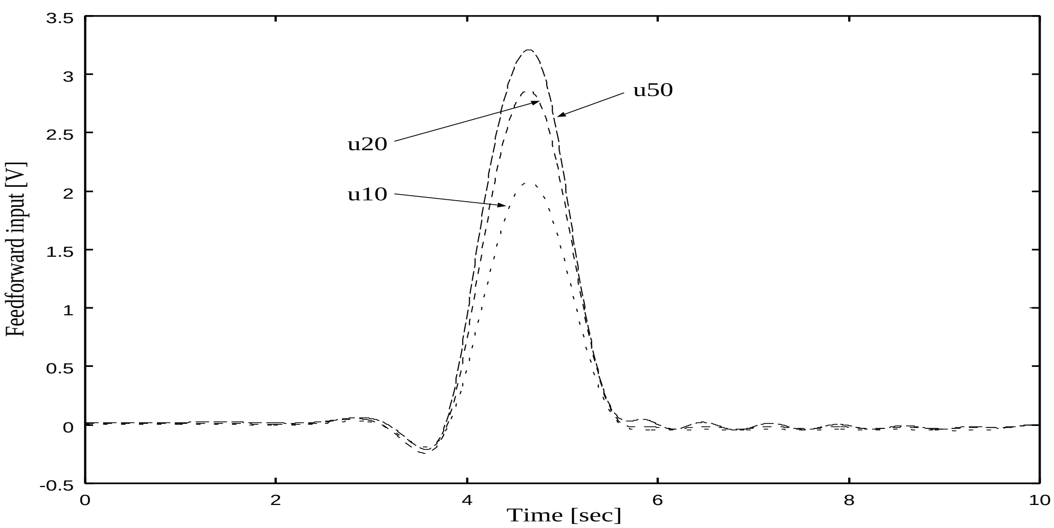

Fig. 5. The feed-forward input

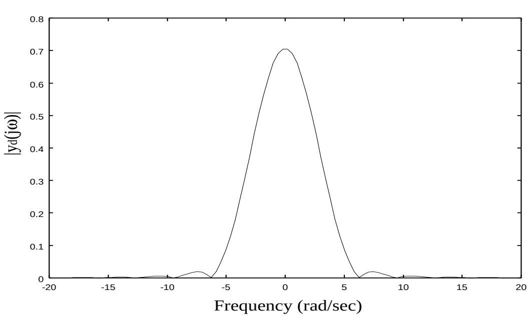

Fig. 9. Magnitude spectrum of the desired output

5. CONCLUSION

In this paper, experimental results of the adjoint-type ILC for a single-link flexible arm are presented. Since flexible arm is an infinite dimensional system and has some uncertain parameters, complete modeling is impossible. However, the adjoint-type ILC can achieve good output tracking by using only a simple model of the flexible arm. Moreover, it was shown that the adjoint-type ILC realizes pre-actuation. This phenomenon implies the possibility of good output tracking for non-minimum phase systems by a bounded input. In (Ghosh and Paden, 2001), ILC using Stable Inversion was discussed for non-minimum phase systems. The update law of this method uses Stable Inversion directly as follows:

$$u_{k+1} = u_k + S_{SI}^{-1}\mathcal{P}r(d/dt)(y_k - y_d) \quad (25)$$

where S_{SI}^{-1} denotes Stable Inversion of the system S and $\mathcal{P}r$ is a stable polynomial of order r. This method is easily expected to obtain the pre-actuation input. On the other hand, the update law (19) does not directly use Stable Inversion. However, this update law also obtains the pre-actuation input. Moreover, the adjoint-type ILC does not require the derivative of output signals.

Convergence conditions of the adjoint-type ILC are shown for linear finite dimensional systems (Kinoshita and Adachi, to be published). Our fu-

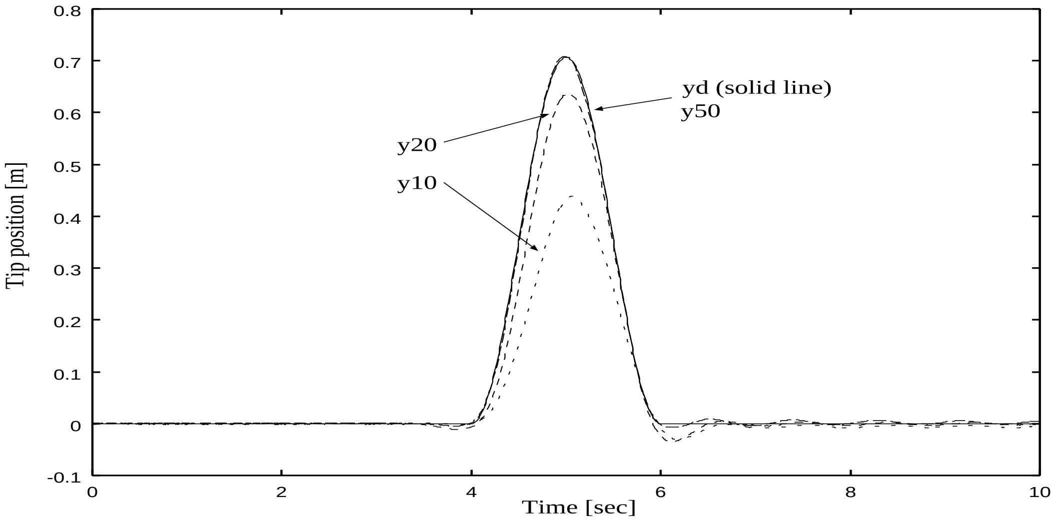

Fig. 6. The output and the desired output for additional weight

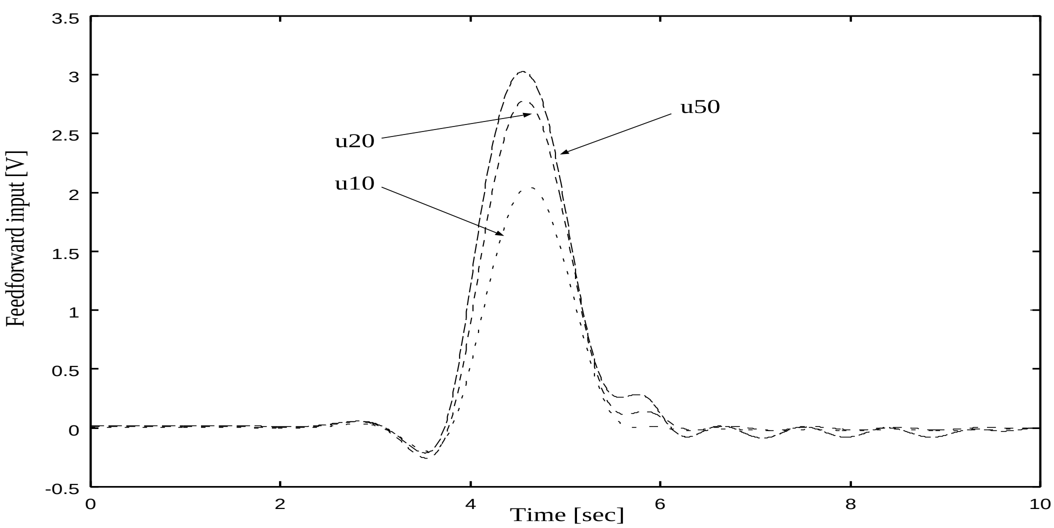

Fig. 7. The feed-forward input for existing weight

ture work will focus on extending the convergence conditions to infinite dimensional systems.

6. REFERENCES

Arimoto, S., Kawamura S. and F. Miyazaki (1984). Bettering operation of robots by learning. *J. of Robotic System* **1**, 123–140.

Cannon, R. H. and E. Schmitz (1984). Initial experiments on the end-point control of flexible one-link robot. *Int. J. Robot. Res.* **3**(3), 62–75.

Cheng, C., Wen J. T. and D. Hughes (1993). Experimental results of a learning controller applied to tip tracking of a flexible beam. In: *Proc. American Control Conference*. San Francisco, Callfornia. pp. 1548–1553.

Devasia, S., Chen D. and B. Paden (1996). Nonlinear inversion-based output tracking. *IEEE Trans. Automat. Contor.* **41**(7), 930–942.

Ghosh, J. and B. Paden (2001). Iterative learning control for nonlinear nonminimum phase plants. *Trans. ASME, J. of Dyn. Syst., Meas., and Control* **123**(1), 21–30.

Kinoshita, K., Sogo T. and N. Adachi (to be published). Iterative learning control using adjoint systems and stable inversion. *Asian J. of Control.*

Kinosita, K., Sogo T. and N. Adachi (2000). Iterative learning control using noncausal updating for non-minimum phase systems. In: *The Third Asian Control Conference*. Shanghai, China. pp. 1548–1553.

Krishnan, H. and M. Vidyasagar (1998). Control of single-link flexible beam using hankel norm-based reduced order model. *IEE Proc. Control Theory and Appl.* **145**(2), 151–158.

Silverman, L. M. (1970). Inversion of multivariable linear systems. *IEEE Trans. on Aoutomat. Contr.* **14**(3), 270–276.

Yamakita, M. and K. Furuta (1991). Iterative generation of virtual reference for a manipulator. *Robotica* **9**, 71–80.