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MODEL CONFIDENCE FOR NONLINEAR SYSTEMS

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Abstract: This paper deals with defining measures of model closeness and establishing quantitative confidence bounds on nominal models. Confidence in a model is an indication of how uniquely identifiable the best fitting parameter values are from the data. These concepts are examined in both the linear and nonlinear regimes, with a practical example used to explore these propositions.

Keywords: Model Confidence, Nonlinear Modeling, Combustion Instability, System Identification

1. INTRODUCTION

Determining confidence in a control-orientated model aims to help answer the question, "Is the model believable?". This question prefaces those concerning stabilizing controller availability and controlled plant performance. In essence, it is highlighting the need to ask more questions about the model quality before proceeding with the control design paradigm.

Standard approaches to modeling seek to find models which minimize the error between the plant and the model via some measure, e.g. one-step-ahead prediction error. This nominal model which is chosen as 'best' according to the measure, is then assumed to produce the best closed loop control performance, via control design treating the model as if it were the real plant. Some telling examples show that a model well suited to prediction does not necessarily imply a high performance controlled plant (Schrama, 1992; Bitmead and Sala, 2000). Use of this "certainty equivalence" approach in practice relies upon the modeling errors being dealt with by the controller via robust control techniques, resulting in degradation in the nominal performance. This degradation is related to how uniquely identifiable the best-fit parameter values are from the data - this property shall be referred to as *model confidence*.

The larger modeling-for-control question is, "Does the nominal model plus the confidence in that model, allow manageable performance bounds on the controlled real plant?" Establishing confidence in a nominal model helps answer this question.

Systematic approaches to identifying nominal models and model errors has received a great deal of attention (Ljung, 1999). Recently there has been analysis, using "certainty equivalence", to establish properties of control designs where there is a direct algebraic relationship between the nominal model and the controller. For example Model Reference Control (MRC) (Gevers *et al.*, 1997),

$$C = \frac{T_0}{\hat{P}(1 - T_0)}, \tag{1}$$

where T_0 is the desired complementary sensitivity function, and Internal Model Control (IMC) (Bitmead and Dunstan, 2001),

$$C = \frac{G}{\hat{P}(1 + G)}, \tag{2}$$

where,

$$G = \hat{P}_a \left[\frac{\lambda_i}{s + \lambda_i}\right]^n ,$$

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with $\hat{P}_a$ the all-pass transfer function of the model and n the relative degree of the model.

These techniques which have a simple mapping from $\hat{P}$ to C reveal much about how the controller design is constrained by nominal model properties. In both cases, although the mapping is simple, and the insight gained solid, the analysis is rather difficult.

It is research in these areas of the mapping between modeling and control that aim to show that successful controllers can be design based on low order models that capture only the essential characteristics of the plant for control. This paper aims to highlight that the nominal model measures and confidence are core in this arena, with a nonlinear application being be used as an example.

Section 2 is a discussion of the relations between the experimental data, the nominal model and the confidence. This shall begin with the linear case, followed by the nonlinear case.

Section 3 addresses the problem which motivates our research in this area, namely modeling for control of a jet turbine instability. We shall describe the some typical nominal model measures and attempt to draw conclusions, if possible, about the model confidence.

Finally some concluding remarks outline the direction for the forthcoming research.

2. NOMINAL MODELS, MEASURES AND CONFIDENCE

2.1 Linear Case

Consider input/output data from a linear plant,

$$y_t = P(\theta, z)\, u_t, \qquad t = 1, ..., N \tag{3}$$

where $P(\theta, z) = \frac{b(z)}{a(z)}$ whose polynomials are defined

$$a(z) = 1 + a_1 z^{-1} + ... + a_{na} z^{-na}$$
$$b(z) = b_1 z^{-1} + ... + b_{nb} z^{-nb}$$

and N is the number of data samples.
Equation (3) may be written as

$$y_t = \phi_t^T \theta \tag{5}$$

with

$$\phi_t^T = \begin{bmatrix} -y_{t-1} & -y_{t-2} & \cdots - y_{t-na} & u_{t-1} & \cdots & u_{t-nb} \end{bmatrix}$$
$$\theta = \begin{bmatrix} a_1 & a_2 & \cdots & a_{na} & b_1 & b_2 & \cdots & b_{nb} \end{bmatrix}^T,$$

or, alternatively,

$$0 = \Delta_t^T\, \Theta \tag{7}$$

with

$$\Delta_t^T = \begin{bmatrix} -y_t & \phi_t^T \end{bmatrix}$$
$$\Theta = \begin{bmatrix} 1 \\ \theta \end{bmatrix}.$$

Composing the regressor matrix, Υ_t, as

$$\Upsilon_t = \begin{bmatrix} \Delta_t & \Delta_{t-1} \cdots \Delta_1 \end{bmatrix}$$
$$= \begin{bmatrix} -y_t & -y_{t-1} & \cdots & -y_1 \\ \vdots & & & \\ -y_{t-na} & -y_{t-na-1} & \cdots & -y_{1-na} \\ u_{t-1} & u_{t-2} & \cdots & u_0 \\ \vdots & & & \\ u_{t-nb} & u_{t-nb-1} & \cdots & u_{1-nb} \end{bmatrix} \tag{9}$$

The modeling task is to find the non-trivial solution to,

$$0 = \Upsilon_N^T\, \Theta. \tag{10}$$

A unique exactly zeroing solution to (10) exists if and only if Υ_N has a single null vector with a leading one. Since Υ_N is an $(n_a + n_b + 1) \times N$ matrix with $N > (n_a + n_b + 1)$, one could equally consider the null space properties of $\Upsilon_N \Upsilon_N^T$. If the null space of $\Upsilon_N \Upsilon_N^T$ has dimension greater than one, then the zeroing solution is not unique. Such a circumstance is indicative of poor data excitation or the overparametrization of the model structure.

The more usual problem is that $rank(\Upsilon_N) = (n_a + n_b + 1)$ because of noise present in the data. One then seeks the value of Θ which solves,

$$\min_{\Theta} \left\| \Upsilon_N^T\, \Theta \right\|_2^2 = \min_{\Theta} \Theta^T \Upsilon_N \Upsilon_N^T \Theta. \tag{11}$$

This is the normal least squares ARX model prediction error criterion,

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} [y_t - \phi_t^T \theta]^2 = \frac{1}{N} \Theta^T \Upsilon_N \Upsilon_N^T \Theta. \tag{12}$$

The solution to the minimization problem (11) is $\hat{\Theta}$, equal to the eigenvector of $\Upsilon_N \Upsilon_N^T$ (corresponding to the least eigenvalue) scaled by it's leading term. This is clearly identical to the regular least squares solution vector, θ, extended by one. The resulting parameter estimate, $\hat{\theta}$, is then given by,

$$\hat{\theta} = \theta|_{\frac{\partial V_N(\theta)}{\partial \theta} = 0} \quad . \tag{13}$$

A notion of parameter estimate confidence is introduced by considering the extent to which $\hat{\theta}$ is sensitive to small changes in the data, $\Upsilon_N \Upsilon_N^T$.

The eigenvector corresponding to the smallest eigenvalue of a perturbed $\Upsilon_N \Upsilon_N^T$ will remain lightly perturbed provided the next smallest eigenvalue is sufficiently far away. Confidence can then be tied to the eigenvalue properties of $\Upsilon_N \Upsilon_N^T$.

Other experimental data samples, if available, provide the statistical independence required to test the model confidence, and as such are an important step in the model validation process.

Hence in the linear systems framework using the least squares criterion, the nominal model and the parameter robustness margins defining confidence, can be evaluated analytically.

2.2 Nonlinear Case

The first step as in the linear case is to define a suitable model fitting measure, $V_N(\theta)$. The difficulty now is in capturing what is believed to be important for the model's purpose. If the model is for controller design, then one should consider, "What are the important characteristics in the data to capture?", e.g. dominant spectral features, the system energy regions, or the time domain prediction error. Closely coupled is the choice of open loop, $V_N^{OL}(\theta)$, or closed loop $V_N^{CL}(\theta)$ measures. Which raises the question, "Does a highly confident nonlinear open loop model have any definite bounds in the closed loop?". This task of defining a measure, although the designer's choice, will ultimately affect the performance of the model's objective.

The definition of parameter estimate, $\hat{\theta}$, is still (13), i.e. that which minimizes the chosen measure w.r.t. variation in parameters.

However the notion of confidence in nonlinear systems is introduced in a slightly different way. Consider the Taylor series expansion of $V_N(\theta)$ about the minimizing solution, $\hat{\theta}$,

$$V_N(\hat{\theta}+d\theta) = V_N(\hat{\theta}) + \left.\frac{\partial V_N(\theta)}{\partial \theta}\right|_{\hat{\theta}} d\theta + \frac{1}{2} d\theta^T \left.\frac{\partial^2 V_N(\theta)}{\partial \theta^2}\right|_{\hat{\theta}} d\theta + \mathcal{O}(|d\theta|^3) \tag{14}$$

The minimizing solution, $\hat{\theta}$, was chosen based upon $\frac{\partial V_N(\theta)}{\partial \theta} = 0$, so the second term is zero. The variation in $V_N(\theta)$ in response to small variations in $\hat{\theta}$ is given by the third term. We shall refer to $\left.\frac{\partial^2 V_N(\theta)}{\partial \theta^2}\right|_{\hat{\theta}}$ as the sensitivity matrix. Provided the solution is minimal and unique the sensitivity matrix will be positive definite. Confidence in nominal parameter values is connected to this sensitivity matrix. Eigenvalue and eigenvector properties of the sensitivity matrix convey information about directions and magnitudes of observed variations in V_N for variations in $\hat{\theta}$. Large eigenvalues would appear to inspire high confidence as they indicate a strong sensitivity of $V_N(\theta)$ to parameters in the direction of the corresponding eigenvectors. Low eigenvalues, indicating low sensitivity of V_n in the corresponding direction, have a reduced sensitivity of the value of V_n to the specific value of θ, and thus inspire less confidence in the estimated measure-minimizing value. In this sense, $\lambda_{\min}\left(\left.\frac{\partial^2 V_N(\theta)}{\partial \theta^2}\right|_{\hat{\theta}}\right)$ represents the limit of confidence.

However, one must be careful with this interpretation. The eigenvalue and eigenvector properties will be affected by scaling of parameters (e.g. τ could be measured in sec or msec). One needs to link the variation of V_N to the permissible variations of the components of θ. Confidence should take into consideration allowable parameter ranges (e.g. physically f may be allowed to vary by ±10Hz, and τ by only ±0.0005 sec - a factor of order four in variation). Equally, in developing a formal notion of confidence one might also consider including a comparative measure of the minimal value of V_N and the size of local variations.

Referring back to the linear system case, with normal least squares ARX model prediction error measure, and using the minimum eigenvalue of the sensitivity matrix as a measure of model confidence, Γ, would yield:

$$\Gamma = \lambda_{min}\left(\frac{2}{N}\sum_{t=1}^{N}\phi_t\phi_t^T\right).$$

Thus Γ is determined explicitly by the quality of the experimental data used for the parametrization, i.e. the experiment design issues, as was concluded previously. One could equally well look at the eigenvalues of $\Upsilon\Upsilon^T$ as discussed before. The first eigenvalue, $\lambda_1(\Upsilon\Upsilon^T)$, should be zero or close to it. The remaining eigenvalues $(\lambda_2(\Upsilon\Upsilon^T), ..., \lambda_{n+1}(\Upsilon\Upsilon^T))$ should be comparably large in magnitude.

Hence this more general view of confidence being related to the sensitivity matrix, applied the linear case with least squares measure, reduces to the same result proposed in the linear case.

In the nonlinear case however there is no obvious analytical link between Γ and the data quality. So it is proposed to calculate the sensitivity matrix using empirical derivatives of simulation results. Employing centralized difference equations as follows,

$$\begin{aligned}
\left.\frac{\partial V_N(\theta)}{\partial \theta_i}\right|_{\hat{\theta}} &= \frac{1}{2}\left[\frac{V_N(\hat{\theta}+\delta\theta_i) - V_N(\hat{\theta}-\delta\theta_i)}{\delta\theta_i}\right] \\
\left.\frac{\partial^2 V_N(\theta)}{\partial \theta_j \theta_i}\right|_{\hat{\theta}} &= \frac{1}{2\delta\theta_j}\left[\frac{\partial V_N(\hat{\theta}+\delta\theta_j)}{\partial \theta_i} - \frac{\partial V_N(\hat{\theta}-\delta\theta_j)}{\partial \theta_i}\right] \\
&= \frac{V_N(\hat{\theta}+\delta\theta_j+\delta\theta_i)}{4\delta\theta_j\,\delta\theta_i} - \frac{V_N(\hat{\theta}+\delta\theta_j-\delta\theta_i)}{4\delta\theta_j\,\delta\theta_i} \\
&\quad - \frac{V_N(\hat{\theta}-\delta\theta_j+\delta\theta_i)}{4\delta\theta_j\,\delta\theta_i} + \frac{V_N(\hat{\theta}-\delta\theta_j-\delta\theta_i)}{4\delta\theta_j\,\delta\theta_i}
\end{aligned} \tag{16}$$

where $i, j = 1, ..., n$, and n is the number of free parameters in θ.

Providing we are careful about the size of δ_i and δ_j, (16) should result in an empirical value of the sensitivity matrix, which can be used in making quantitative judgements of the confidence in a parameter set.

Attempting to check these confidence statements using a validation set of data raises another broad question, "How do we perform a validation test on the confidence given to a particular $\hat{\theta}$ for a nonlinear model?". The intention is not to attempt answering this question here, merely to highlight its importance in the overall procedure.

3. PRACTICAL APPLICATION

The underlying motivation for this paper, is a practical problem occurring in combustion chambers of jet engines. Operating jet engines at low fuel-to-air ratios can produce medium-range(100-1000Hz) instabilities, believed to be from the nonlinear interaction between chamber pressure, p_t, and the heat release rate, q_t.

The objective is to ameliorate these instabilities using a control design from a low complexity model. So far this has been a problem of fitting a nonlinear model with data from experiments. The question sought to resolve now, is whether there is good confidence in the identified model.

The experimental data is shown the time domain in Figure 1 and the frequency domain in Figure 2. The

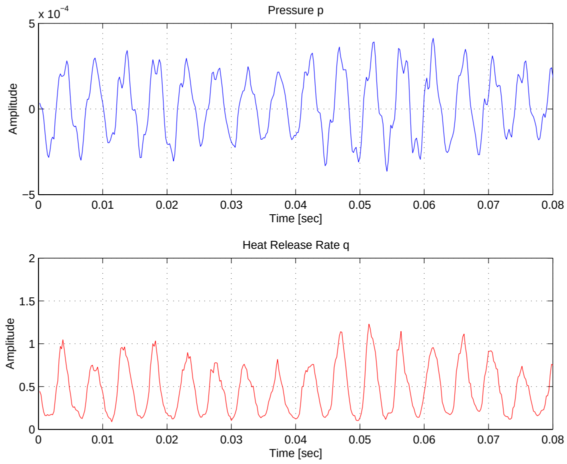

Fig. 1. An 80ms time segment of the p_t and $\bar{q}_t$.

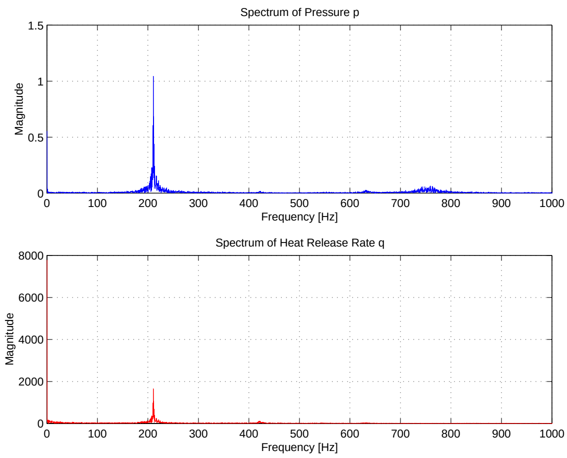

Fig. 2. Frequency plots of p_t and $\bar{q}_t$.

data, p_t and q_t, are characterized by sustained oscillations. It is assumed there is no external excitation (other than a small additive noise component), so the system must be limit-cycling, which implies non-linear dynamics are present. The data are almost periodic, with sinusoids at 210Hz and 740Hz plus harmonics of 210Hz. The $\bar{q}_t$ signal is deficient of high frequency components due to the q_t sensor exhibiting a low pass filtering effect.

The model structure, provided by a priori physical reasoning (Peracchio and Proscia, 1998), is shown in Figure 3. The model relates p_t and q_t in a closed

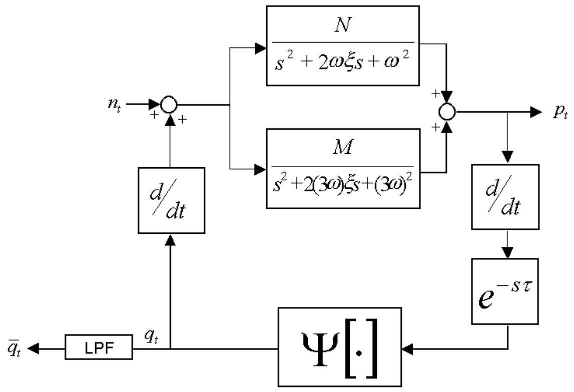

Fig. 3. The proposed model.

loop and consists of (clockwise from top): two coupled oscillators $(N, M, \omega = 2\pi f, \xi)$, differentiator, time delay (τ), negative saturation nonlinearity $(\Psi[\cdot])$, differentiator and an additive noise source (n_t).

An in depth data analysis was performed by Murray *et al.* (1998), while the fitting of the model parameters using the experimental data was shown by Savaresi *et al.* (2001). These parameter values shall be used as initial values and are shown in Table 3.

Table 1. Model parameters of Figure 3.

Par. No.	Parameter θ	Initial Value	Par. Est. $\hat{\theta}_{OL}$	Par. Est. $\hat{\theta}_{CL}$
1	M	1.00	1.00	1.00
2	N	1.00	0.43	0.80
3	f	206	211	202
4	ξ	0.20	0.19	0.18
5	τ	3.47	3.44	3.44
6	slope$(\Psi[\cdot])$	-0.56	-0.91	-0.77
7	min$(\Psi[\cdot])$	0.11	0.10	0.60
8	pwr(n_t)	1.00	-	1.00

3.1 Defining Measures

A measure, $V(\theta)$, must be chosen that allows a minimizing parameter vector, $\hat{\theta}$, to be found. Many measures are available which compare various time domain statistical or spectral properties. To narrow these options, issues affecting control design should be at the forefront of the measure choice. The aim of the control design is to ameliorate the 210Hz oscillation, while not creating similar oscillations in the local vicinity. Capturing the two dominant spectral components (210Hz and 740Hz) in both frequency and magnitude appears a reasonable target to begin with. It does not appear that close matching of the noisy low amplitude spectral components is needed, and so the measure should not be penalized for excluding these. However the noise power, n_t, is included in the parameter vector, as it was shown in Dunstan *et al.* (2001) that this directly affects the simultaneous existence the 210Hz and 740Hz components.

Two candidate measures are proposed:
(1) An open loop prediction error measure using least squares.
(2) A closed loop simulation measure using a spectral matching measure.

3.2 Open Loop Prediction Measure

This measure involves breaking the loop at p_t as shown in Figure 4. The experimental pressure data, p_t, is used as the input to the model and the predicted output is $\hat{p}_t$. The external noise source, n_t, is not needed

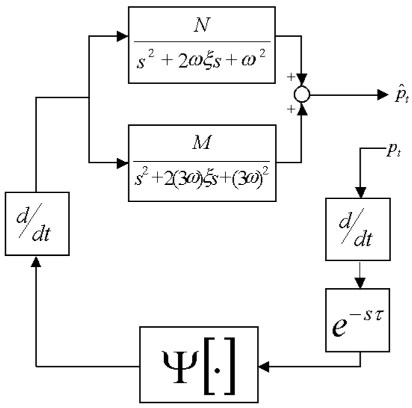

Fig. 4. Open loop prediction error measure.

as the input, p_t, already possesses noise. Incorporating n_t could only worsen the prediction error and so will not be identified.

The two signals are compared using a least squares approach:

$$V_N^{OL}(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left[p_t - \hat{p}_t\right]^2 \tag{17}$$

The minimum measure using (17) produced produced the model and data $|FFT|$ comparison shown in Figure 5. The parameters, $\hat{\theta}_{OL}$, that minimized this measure are shown in Table 3.

The parameter sensitivity matrix was created by perturbing the individual parameters of $\hat{\theta}$ by 0.1% and evaluating Equation (16). The eigenvalues and eigenvectors for the sensitivity matrix are shown in Table 2. The strongest confidence is obtained for λ_6 and λ_7, as these two eigenvalues are well bounded away from the others. The parameters in the associated eigenvalues, shown in boldface, correspond to ξ and τ. Thus we speculate that our estimated values for ξ and τ are accurate. As for the other parameters, their eigenvalues are much smaller and hence inspire a lower confidence in their accuracy.

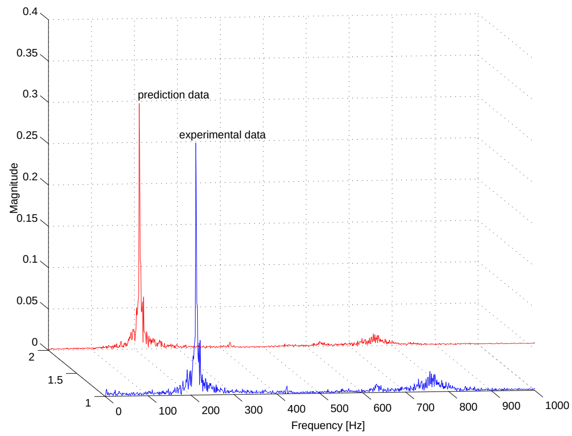

Fig. 5. Comparison of $|FFT|$ of experimental data, p_t, and open loop predicted data, $\hat{p}_t$, using an open loop prediction error measure.

Table 2. Eigenvalues (λ_i) and eigenvectors (v_i) of the O.L. sensitivity matrix.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
0.01	0.02	0.04	0.23	0.52	216.21	328.24
v_1	v_2	v_3	v_4	v_5	v_6	v_7
-0.02	0.06	-0.28	0.76	-0.55	0.10	0.16
-0.04	0.02	-0.18	0.53	0.83	0.00	0.01
0.98	-0.12	-0.14	-0.00	0.02	0.00	-0.00
0.01	-0.00	0.07	0.20	-0.10	$-\mathbf{0.43}$	$-\mathbf{0.87}$
0.01	0.00	0.01	0.02	-0.01	$-\mathbf{0.90}$	$\mathbf{0.45}$
0.17	0.39	0.86	0.27	0.02	0.07	0.09
-0.06	-0.91	0.36	0.18	-0.01	0.03	0.05

3.3 Closed Loop Simulation Measure & Confidence

The simulation, performed in closed loop as shown in Figure 3, produces $\hat{p}_t$. The spectrum of the simulation $\Phi_{\hat{p}}(\omega)$ is compared to the spectrum of the experimental data $\Phi_p(\omega)$, using

$$V_N^{CL}(\theta) = \sum_{\omega\in\Omega}\frac{\left[\Phi_p(\omega) - \Phi_{\hat{p}}(\omega)\right]^2}{\left[\Phi_p(\omega) + \epsilon I\right]^2} \times W(\omega) \tag{18}$$

where Ω is the frequency grid of interest and $W(\omega)$ is a weighting function which captures important features.

In this example, Ω is defined over the range 0 - 1000Hz and the weighting function captures the dominant peaks. The minimum measure using (18) produced the model and data $|FFT|$ comparison shown in Figure 6. The parameters, $\hat{\theta}_{CL}$, that minimized this measure are shown in Table 3.

The parameter sensitivity matrix was created by perturbing the individual parameters of $\hat{\theta}$ by 1% and evaluating Equation (16). The eigenvalues and associated eigenvectors of the sensitivity matrix are shown in Table 3. Notice that λ_1 is almost $30\times$ larger than the second highest eigenvalue, λ_2. This leads us to again speculate that the parameters associated with v_1, namely ξ, are accurate. Indeed this is one of the same parameters we had confidence in when using the open loop measure. As for the other parameters,

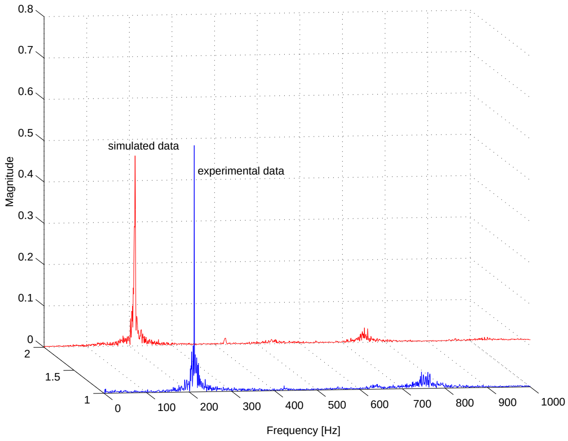

Fig. 6. Comparison of $|FFT|$ of experimental data, p_t, and closed loop simulated data, $\hat{p}_t$, using a closed loop simulation measure.

Table 3. Eigenvalues (λ_i) and eigenvectors (v_i) of the C.L. sensitivity matrix.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
2×10^5	7×10^3	7×10^3	4×10^3	1×10^3	1×10^2	1×10^{-1}	6×10^2
v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
0.07	0.58	-0.71	-0.03	0.11	-0.38	0.00	-0.03
0.01	-0.77	-0.63	-0.02	-0.01	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.01
$-\mathbf{1.00}$	0.05	-0.06	0.00	-0.02	0.00	0.00	0.00
0.00	-0.02	0.03	0.01	0.12	0.03	-0.01	-0.99
0.03	0.09	-0.09	0.00	-0.98	0.03	0.00	-0.12
0.03	0.23	-0.29	-0.01	0.07	0.93	0.00	0.03
0.00	0.00	-0.04	1.00	0.01	0.00	0.00	0.01

their eigenvalues are much smaller and hence inspire a lower confidence in their accuracy.

4. CONCLUSION

In this paper we have calculated empirically the second order derivative of the loss function with respect to the parameters, around the minimizing parameter values. We interpret this as a measure of confidence in the parameter values.

In future work we explicitly calculate the parameter distributions using sub-sampling and re-sampling techniques as a measure of confidence. This later definition of confidence captures the sensitivity of the parameters to variations in the data as seen in samples from a single data record. The reader is referred to Dunstan and Bitmead (2002).

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