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ROBUST BOUNDARY CONTROL OF AN AXIALLY MOVING STEEL STRIP

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Abstract: In this paper, a robust vibration control scheme of the axially moving steel strip in the zinc galvanizing line is considered. The boundary control force is applied to the strip through the two touch rolls connected to a hydraulic actuator. The mathematical model of the system, which consists of a partial differential equation describing the dynamics of the traveling steel strip and an ordinary differential equation describing the actuator dynamics, is derived by using the Hamilton's principle for the systems with changing mass. The total mechanical energy of the system is considered as a Lyapunov function candidate. For vibration suppression purpose, a robust boundary feedback control law is designed. The asymptotic stability of the closed loop system is verified through the Lyapunov analysis and the semigroup theory. *Copyright © 2002 IFAC.*

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1. INTRODUCTION

Fig. 1 shows a continuous hot-dip zinc galvanizing process. The steel strips, of order of 1m wide by 1mm thick, are preheated and passed at a constant speed through a pot of molten zinc at a temperature in the region of about $450\,°C$. A zinc film is entrained onto the strip as it emerges from the pot. In order to achieve the target deposited mass and maintain it over a range of process conditions, a pair of air knives, which direct a long thin wedge-shaped jet of high-velocity air at the strip, are generally used to control the deposited mass by stripping excess zinc back into the pot. The deposited film solidifies while the strip runs vertically upward, cooling as it goes, for a distance of the order of about 110 m, to a gauge that measures the mass of zinc deposited on the strip surfaces.

The control aims of the galvanizing line are to improve the uniformity of the zinc deposit on the strip surfaces and reduce the zinc consumption. The problem of regulating the hot-dip galvanizing process by adjusting the air knives has been studied by several researchers: McKerrow (1983) and Chen (1995).

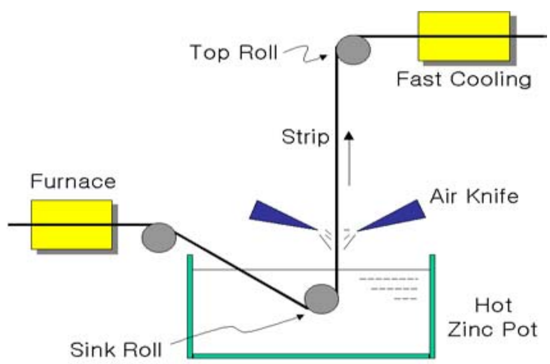

Fig. 1. The traveling steel strip in a zinc galvanizing line.

However, an immediate problem to adjust the air knives is that there is a lack of the strip positional information. Shifting strip position and vibration are the main causes of differences between the average deposited masses of the top and bottom strip surfaces and the non-uniformity of the deposited mass across the strip. Many galvanized steel producers such as POSCO (Korea) and U.S. Steel have attempted to measure the strip position directly by installing laser transducers near to the air knives. However, no success has been reported, because the high-temperature environment makes the transducers unreliable.

Thus, to improve the uniformity of the deposited mass of zinc on the steel strip surfaces and to reduce zinc consumption, the strip vibration should be directly suppressed by using a more practical, flexible, and reasonable control method. The external forces from the air knives and air cooler as well as the periodic excitation due to the support roller eccentricity can be treated as disturbances to this system. Robust control strategy is then needed to suppress vibrations. Since a hydraulic touch-roll actuator is used for exerting control force and the mathematical model describing the dynamics of the moving steel strip is represented as a partial differential equation, the stability of coupled ODE and PDE is analyzed using the semigroup theory.

The model proposed in this paper can further represent various physical systems such as high-speed magnetic tapes, band saws, belt drives, and paper sheets during processing. The systems are often subject to a stationary, one-sided constraint, such as a read/write head in magnetic tape drives or a guide bearing in band saws. The transient response of an axially moving strip subject to arbitrary external forces and boundary disturbances was investigated by Zhu and Mote (1994). The active vibration control of an axially moving string was studied by Lee and Mote (1996), Renshaw *et al.* (1998), Fung *et al.* (1999a,b), and Li *et al.* (2002).

The contributions of this paper are: The zinc galvanizing line is analyzed and a control-oriented model for the traveling steel strip is derived. The tension applied to the strip is considered as a spatiotemporally varying function. To the author's best knowledge, the paper is the first attempt on boundary control of the axially moving string subject to distributed external disturbance force. The robust boundary control law derived is implementable. The asymptotic stability of the closed loop system is assured through the Lyapunov analysis and semigroup theory.

2. PROBLEM FORMULATION: EQUATIONS OF MOTION

Fig. 2 shows a schematic of the axially moving steel strip for control system design purpose. The left boundary at the sink roll is assumed fixed. The two touch rolls linked to a hydraulic actuator in the middle section of the strip will play the right boundary, where the control input (force) is applied at this right boundary. In the zinc galvanizing line, the distance between the two supports is quite large compared to the strip thickness. Therefore, the moving steel strip can be modeled as a moving string.

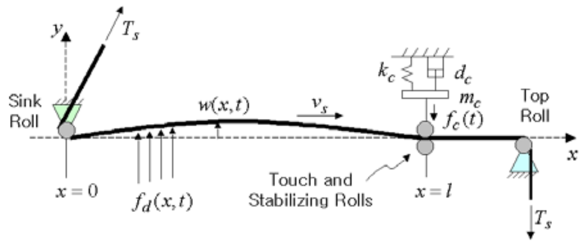

Fig. 2. An axially moving strip with two touch rolls connected to a hydraulic actuator.

Let t be the time, x be the spatial coordinate along the longitude of motion, v_s be the axial speed of the strip, $w(x,t)$ be the transversal displacements of the strip at time t and spatial coordinate x, and l be the length of the strip. Also, let ρ be the mass per unit area of the strip, $T_s(x,t)$ be the tension applied to the strip, and $f_d(x,t)$ be the distributed external force resulting from the aerodynamic excitation due to the air knives and air cooler.

As shown in Fig. 2, to suppress the vibrations of the strip, the two touch rolls are attached at $x=l$ and connected to a hydraulic actuator including a lumped mass m_c, a viscous damper with constant coefficient d_c, and a spring with constant stiffness k_c. The control force $f_c(t)$ is applied to this actuator. The partial derivatives denote $(\cdot)_t = \partial(\cdot)/\partial t$ and $(\cdot)_x = \partial(\cdot)/\partial x$. The total derivative operator with respect to time is defined as $\dot{(\cdot)} = d(\cdot)/dt = (\cdot)_t + v_s(\cdot)_x$.

The kinetic energy of the system is

$$T = \frac{1}{2}\int_0^l \rho\left\{v_s^2 + (v_s w_x + w_t)^2\right\}dx + \frac{1}{2}m_c w_t^2(l,t) \,. \quad (1)$$

The potential energy of the system is

$$V = \frac{1}{2}\int_0^l T_s(x,t) w_x^2\, dx + \frac{1}{2}k_c w^2(l,t) \,. \quad (2)$$

The virtual work by the external forces is

$$\begin{aligned}\delta W &= f_c \delta w(l,t) - \int_0^l f_d \delta w\, dx \\ &\quad - \int_0^l c_v(w_t + v_s w_x)\delta w\, dx - d_c w_t(l,t)\delta w(l,t) \,, \end{aligned} \quad (3)$$

where c_v is the viscous damping coefficient of the steel strip. By using the Hamilton's principle such that $\int_{t_0}^{t_1}(\delta T - \delta V + \delta W)\, dt = 0$, the governing equation and boundary conditions are derived as

$$\begin{aligned}&\rho w_{tt}(x,t) + 2\rho v_s w_{xt}(x,t) + \rho v_s^2 w_{xx}(x,t) \\ &\quad -(T_s(x,t) w_x(x,t))_x + c_v(w_t(x,t) \\ &\quad + v_s w_x(x,t)) + f_d(x,t) = 0 \,, \end{aligned} \quad (4)$$

$$w(x,0) = w_0(x),\ w_t(x,0) = w_{t0}(x), \quad w(0,t) = 0,$$

and

$$\begin{aligned} f_c &= m_c w_{tt}(l,t) + (d_c - \rho v_s) w_t(l,t) + k_c w(l,t) \\ &\quad + (T_s(l,t) - \rho v_s^2) w_x(l,t) \,, \end{aligned} \quad (5)$$

where $0 < x < l$. Note that the right boundary condition (5) is an ordinary differential equation that describes the equation of motion of the hydraulic actuator in compliance with the transversal force at $x = l$. Let $w(l) = w(l,t)$ and $T_s(l) = T_s(l,t)$.

Note that the tension $T_s(x,t)$ is described as a spatiotemporally varying function. Since the steel strip is moving vertically in the zinc galvanizing line as shown in Fig. 1, the gravitational force ρgx, which acts as an additional tension to the strip, cannot be neglected. Also the tension itself may be time-varying due to the eccentricity of the support roller, which causes a periodic excitation. Thus, the tension variation $T_s(x,t)$ in the strip should be considered as spatiotemporally varying function. Assume that $T_s(x,t)$ is sufficiently smooth and uniformly bounded as follows:

$$T_{s,\min} \le T_s(x,t) \le T_{s,\max}, \tag{6}$$

$$\left|(T_s(x,t))_t\right| \le (T_s)_{t,\max}, \quad \left|(T_s(x,t))_x\right| \le (T_s)_{x,\max}, \tag{7}$$

for all $x \in [0,l]$, $t \ge 0$, and some a priori known constants $T_{s,\min}$, $T_{s,\max}$, $(T_s)_{t,\max}$, and $(T_s)_{x,\max}$.

The boundary control problem of the traveling strip is now formulated. From (1) and (2), the total mechanical energy $V_0(t)$ of the strip is given by

$$V_0(t) = \frac{1}{2}\int_0^l \rho(v_s w_x(x,t) + w_t(x,t))^2\, dx + \frac{1}{2}\int_0^l T_s w_x^2\, dx + \frac{1}{2}m_c w_t^2(l,t) + \frac{1}{2}k_c w^2(l,t). \tag{8}$$

The third and fourth terms in (8) denote the mechanical energy of the hydraulic touch-roll actuator. The objective of the control system design is to stabilize asymptotically the transverse vibration of the axially moving strip in spite of the existence of distributed external disturbances.

Assume first that the strip is traveling at a constant transport velocity v_s between two fixed rolls, i.e., there is no actuator at $x = l$. Then, the time derivative of $V_0(t)$ in (8) yields:

$$\dot V_0(t) = \int_0^l (w_t + v_s w_x)((T_s w_x)_x - c_v(w_x + v_s w_x) - f_d)dx + \int_0^l T_s w_x (w_{xt} + v_s w_{xx})dx + \frac{1}{2}\int_0^l \{(T_s)_t + v_s (T_s)_x\} w_x^2 dx. \tag{9}$$

The terms in (9) are simplified via integration by parts and the boundary conditions as follows:

$$\int_0^l \{w_t (T_s w_x)_x + T_s w_x w_{xt}\}\, dx = \left[w_t (T_s w_x)\right]_0^l, \tag{10}$$

$$\int_0^l w_x (T_s w_x)_x dx = \left[T_s w_x^2\right]_0^l - \int_0^l T_s w_x w_{xx} dx, \tag{11}$$

The substitution of (10) and (11) into (9) yields:

$$\dot V_0(t) = -c_v\int_0^l (w_t + v_s w_x)^2 dx - v_s T_s(0) w_x^2(0) + v_s T_s(l) w_x^2(l) + \frac{1}{2}\int_0^l \{(T_s)_t + v_s (T_s)_x\} w_x^2 dx - \int_0^l (w_t + v_s w_x) f_d dx, \tag{12}$$

where $T_s(0) = T_s(0,t)$.

From (12), the followings are concluded: Even though the transverse velocity w_t at the boundary is zero, the instantaneous transverse velocity of a material particle at the boundary is $v_s w_x$. Consequently, at each boundary, the transverse component of the strip tension does work on the string. For the traveling strip with fixed boundary conditions, any traveling wave impinging on the boundaries causes the decay of the total energy at $x = 0$ and the increase of the energy at $x = l$. Also, the derivative of $T_s(x,t)$ with respect to time generates an energy flux to increase the total energy by $(T_s)_t$ and $(T_s)_x$. This shows that the time rate of the change of T_s cannot be neglected for the stabilization of the axially moving strip system.

Thus, it can be concluded for the traveling strip with fixed boundary conditions that the traveling wave impinging on the right boundary $x = l$, the time rate of the tension T_s, and the distributed external force f_d cause the increase of the total mechanical energy $V_0(t)$ in (8). To determine a boundary controller for the stabilization of the vibration energy, the positive definite, total energy $V_0(t)$ can be considered as a Lyapunov function candidate.

3. ROBUST BOUNDARY CONTROL

The distributed external force $f_d(x,t)$ resulting from the aerodynamic excitation of the air knives and air cooler can be treated as disturbances. Assume that $\int_0^l f_d(x,t)^2 dx$ is uniformly bounded. Therefore, a robust control algorithm that assures the boundedness of all the signals and the asymptotic stability of the system is needed. The main idea is to consider the worst case of the uncertainties in the form of possible bounds. Based upon the worst case, the robust boundary control algorithm is designed.

Consider a modified functional $V(t)$ such that

$$V(t) \overset{\Delta}{=} \alpha V_0(t) + 2\beta\int_0^l \rho\, x w_x (v_s w_x + w_t)\, dx, \tag{13}$$

where $\alpha > 0$ and $\beta > 0$. Then by using Cauchy-Schwarz inequality, it can be shown easily that there exists a positive constant C such that

$$2\beta\int_0^l \rho\, x w_x (v_s w_x + w_t)\, dx \le C V_0(t). \tag{14}$$

From (14) and (15), the following holds:

$$(\alpha - C)V_0(t) \le V(t) \le (\alpha + C)V_0(t), \tag{15}$$

where $\alpha > C$. From (15), it can be concluded that $V(t)$ is equivalent to the Lyapunov function candidate $V_0(t)$ in (8) if $\alpha > C$.

The time derivative of $V(t)$ along (4)-(5) yields:

$$\dot{V}(t) = \alpha\dot{V}_0(t) + \beta\rho\Big[x(w_t + v_s w_x)^2\Big]_0^l - \beta\rho\int_0^l (w_t + v_s w_x)^2 dx + 2\beta\int_0^l x w_x((T_s w_x)_x - c_v(w_x + v_s w_x) - f_d)dx \,. \quad (16)$$

The integration by parts yields:

$$2\int_0^l x w_x (T_s w_x)_x dx = \Big[x(T_s w_x^2)\Big]_0^l - \int_0^l T_s w_x^2 dx + \int_0^l x(T_s)_x w_x^2 dx \,. \quad (17)$$

The following inequality is also utilized.

$$u\,v \le \gamma\, u^2 + \frac{1}{\gamma} v^2 \qquad \text{for any } \gamma > 0 \,. \quad (18)$$

Thus, by substituting (6)-(7), (17), and (18) into (16), the time derivative of the energy $V(t)$ in the strip, with a right boundary actuator at $x = l$ and the fixed condition at left boundary, becomes:

$$\begin{aligned}\dot{V}(t) \le & -\left(\alpha c_v + \beta\rho - \alpha\gamma_1 - \frac{2\beta c_v\, l}{\gamma_2}\right)\int_0^l (w_t + v_s w_x)^2 dx \\ & - \Big\{ \beta\, T_{s,\min} - \left(\beta\, l + \frac{\alpha v_s}{2}\right)(T_s)_{x,\max} - \frac{\alpha}{2}(T_s)_{t,\max} \\ & - 2\beta\, l(c_v\gamma_2 + \gamma_3)\Big\}\int_0^l w_x^2 dx + \left(\frac{\alpha}{\gamma_1} + \frac{2\beta\, l}{\gamma_3}\right)\int_0^l f_d^2 dx \\ & - \{\alpha(\psi\, d_c - \rho v_s) - \beta\rho\, l\}\, w_t^2(l) + (\psi - 1)\,\alpha\, d_c w_t^2(l) \\ & + \left\{(\beta\, l + \alpha v_s)T_s(l) + \beta\rho\, l v_s^2\right\} w_x^2(l) \\ & + (2\beta\rho\, l v_s + \alpha\rho v_s^2)\, w_x(l)\, w_t(l) + \alpha\, f_c w_t(l)\,, \end{aligned} \quad (19)$$

where $\psi > 0$ such that $\{\alpha(\psi\, d_c - \rho v_s) - \beta\rho\, l\} > 0$.

If $T_{s,\min}$ is sufficiently large, the positive values α, β, and γ_i, $i = 1,2,3$, can be chosen to satisfy

$$\left(\alpha c_v + \beta\rho - \alpha\gamma_1 - \frac{2\beta c_v\, l}{\gamma_2}\right) > 0 \,, \quad (20)$$

$$\Big\{ \beta\, T_{s,\min} - \left(\beta\, l + \frac{\alpha v_s}{2}\right)(T_s)_{x,\max} - \frac{\alpha}{2}(T_s)_{t,\max} - 2\beta\, l(c_v\gamma_2 + \gamma_3)\Big\} > 0 \,. \quad (21)$$

By the assumption of the uniform boundedness of $f_d(x,t)$, the upper bound of the third term in (19) can be given by

$$\left(\frac{\alpha}{\gamma_1} + \frac{2\beta\, l}{\gamma_3}\right)\int_0^l f_d^2 dx \le \sigma\,, \quad \sigma > 0 \,. \quad (22)$$

The robust boundary control laws, which make the time derivative of the total energy negative semi-definite, $\dot{V}(t) \le 0$, are then proposed as follows:

Case 1: $|w_x(l)| \ge \varepsilon$,

$$w_t(l) = -\eta_1 w_x(l)\,, \quad \eta_1 > 0\,, \quad (23)$$

$$f_c = k_0 w_x(l) + \sigma_m(\alpha\,\eta_1 w_x(l))^{-1}\,, \quad k_0 > 0\,, \quad (24)$$

Case 2: $|w_x(l)| < \varepsilon$,

$$w_t(l) = \sqrt{\sigma_m \Theta^{-1}}\,, \quad (25)$$

$$f_c = -k_1 w_x(l) - k_2 \Omega\alpha^{-1}\sqrt{\sigma_m \Theta^{-1}}\,, \quad k_1, k_2 > 0\,, \quad (26)$$

where $0 < \varepsilon << 1$, $\sigma_m \ge \sigma$, $\Omega = 2\beta\rho\, l v_s + \alpha\rho\, v_s^2$, and $\Theta = \alpha(\psi\, d_c - \rho v_s) - \beta\rho\, l$.

Note that the slope measurement $w_x(l,t)$ is used as input to the velocity control law (23) and the force control laws (24), (26) to dissipate energy. Lee and Mote (1996) and Li *et al.* (2002) presented experimental results for controlling the vibrations with the slope $w_x(l,t)$ as input signal for an axially moving string system.

For the case 1, substituting (23)-(24) into (19) yields:

$$\begin{aligned}\dot{V}(t) \le & -c_0\left(\int_0^l (w_t + v_s w_x)^2 dx + \int_0^l w_x^2 dx\right) \\ & + \sigma - \{\alpha(\psi\, d_c - \rho v_s) - \beta\rho\, l\}\,\eta_1^2 w_x^2(l) \\ & + (\psi - 1)\,\alpha\, d_c \eta_1^2 w_x^2(l) \\ & + \left\{(\beta\, l + \alpha v_s)T_s(l) + \beta\rho\, l v_s^2\right\} w_x^2(l) \\ & - (2\beta\rho\, l v_s + \alpha\rho v_s^2)\,\eta_1 w_x^2(l) - \alpha\eta_1 k_c w_x^2(l) \\ & - \alpha\eta_1 w_x(l)\frac{\sigma_m}{\alpha\eta_1 w_x(l)}\,, \end{aligned} \quad (27)$$

where $c_0 = \min\Big\{\alpha c_v + \beta\rho - \alpha\gamma_1 - \frac{2\beta c_v\, l}{\gamma_2},\ \beta\, T_{s,\min} - \left(\beta\, l + \frac{\alpha v_s}{2}\right)(T_s)_{x,\max} - \frac{\alpha}{2}(T_s)_{t,\max} - 2\beta\, l(c_v\gamma_2 + \gamma_3)\Big\}$.

The control gains η_1 and k_0 from (27) can be chosen to satisfy

$$(\psi - 1)\,\alpha\, d_c \eta_1^2 + \left\{(\beta\, l + \alpha v_s)T_s(l) + \beta\rho\, l v_s^2\right\} - \alpha\eta_1 k_0 \le 0 \,. \quad (28)$$

Also, for the case 2, substituting (25)-(26) into (19) yields:

$$\begin{aligned}\dot{V}(t) \le & -c_0\left(\int_0^l (w_t + v_s w_x)^2 dx + \int_0^l w_x^2 dx\right) \\ & + \sigma - \sigma_m + \left\{(\beta\, l + \alpha v_s)T_s(l) + \beta\rho\, l v_s^2\right\}\varepsilon^2 \\ & - \left\{\alpha\, k_1 - (2\beta\rho\, l v_s + \alpha\rho v_s^2)\right\}\sqrt{\sigma_m \Theta^{-1}}\; w_x(l) \\ & - \{k_2\Omega - (\psi - 1)\,\alpha\, d_c\}\sigma_m \Theta^{-1}\,. \end{aligned} \quad (29)$$

The control gains k_1 and k_2 from (29) can be chosen to satisfy

$$k_1 \ge \alpha^{-1}(2\beta\rho\, l v_s + \alpha\rho v_s^2)\,, \quad (30)$$

$$k_2 \ge (\psi - 1)\,\alpha\, d_c \Omega^{-1} + \left\{(\beta\, l + \alpha v_s)T_s(l) + \beta\rho\, l v_s^2\right\}\varepsilon^2 \sigma_m^{-1}\Theta \,. \quad (31)$$

From the above results, i.e., (27)-(31), the following is then obtained:

$$\dot{V}(t) \le -c_0\left(\int_0^l (w_t + v_s w_x)^2 dx + \int_0^l w_x^2 dx\right). \quad (32)$$

Thus, the functional $V(t)$ given by (13) is nonincreasing and is a Lyapunov function, since $\dot{V}(t)$ is negative semidefinite. Hence, it can be concluded that all the signals in the closed loop system are bounded.

4. STABILITY ANALYSIS

The boundary control laws (23)-(24) and (25)-(26) show that $\dot{V}(t)$ is negative semidefinite, ensuring the stability but not the asymptotic stability. In this section, the asymptotic stability of the axially moving strip under the boundary control laws is proved.

In other to analyze the asymptotic stability of the system (4)-(5), the state space $\Im$ of the system is defined as follows:

$$\Im \overset{\Delta}{=} \left\{ (w, \dot{w}, w(l), w_t(l))^T \mid w \in H_L^1, \dot{w} \in L^2, w(l), w_t(l) \in R \right\},$$

where the superscript T stands for transpose. The spaces L_2 and H_L^k are defined as follows:

$$L^2 \overset{\Delta}{=} \left\{ f : [0,1] \to R \mid \int_0^l f^2 dx < \infty \right\},$$

$$H_L^k \overset{\Delta}{=} \left\{ f \in L^2 \,\middle|\, f', f'', \cdots, f^{(k)} \in L^2, \text{and } f(0) = 0 \right\}.$$

In the space $\Im$, the inner-product is defined as follows:

$$\langle z, \bar{z} \rangle_\Im \overset{\Delta}{=} \frac{1}{2} \int_0^l (\rho \dot{w} \dot{\bar{w}} + T_s w_x \bar{w}_x) dx + \frac{1}{2} (m_c w_t(l) \bar{w}_t(l) + k_c w(l) \bar{w}(l)), \qquad (33)$$

where $z = (w, \dot{w}, w(l), w_t(l))^T$, $\bar{z} = (\bar{w}, \dot{\bar{w}}, \bar{w}(l), \bar{w}_t(l))^T \in \Im$. The norm induced by the inner-product (33) is equivalent to $V_0(t)$ in (8), i.e.,

$$V_0(t) \overset{\Delta}{=} \langle z, z \rangle_\Im = \| z(t) \|_\Im^2 = \frac{1}{2} \int_0^l \rho (v_s w_x + w_t)^2 \, dx + \frac{1}{2} \int_0^l T_s(x) w_x^2 \, dx + \frac{1}{2} m_c w_t^2(l,t) + \frac{1}{2} k_c w^2(l,t). \qquad (34)$$

By using $\frac{d}{dt}(w_t + v_s w_x) = w_{tt} + 2 v_s w_{xt} + v_s^2 w_{xx}$, the system (4)-(5) can be rewritten in the following abstract form.

$$\dot{z} = Az + F, \quad z(0) \in \Im, \qquad (35)$$

where $z = (w, \dot{w}, w(l), w_t(l))^T \in \Im$ and the operator $A : \Im \to \Im$ is an unbounded linear operator. From (24) and (26), A and F in (35) are defined, respectively, as follows:

Case 1: $|w_x(l)| \ge \varepsilon$,

$$A \overset{\Delta}{=} \begin{bmatrix} 0 & 1 & 0 & 0 \\ \rho^{-1} \frac{\partial}{\partial x}\left(T_s \frac{\partial}{\partial x} \right) & -\rho^{-1} c_v & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -m_c^{-1}(T_s - \rho v_s^2 - k_0) \frac{\partial}{\partial x}\Big|_{x=l} & 0 & -m_c^{-1} k_c & -m_c^{-1}(d_c - \rho v_s) \end{bmatrix},$$

and

$$F \overset{\Delta}{=} \begin{bmatrix} 0 \\ -\rho^{-1} f_d \\ 0 \\ m_c^{-1} \sigma_m (\alpha \eta_1 w_x(l))^{-1} \end{bmatrix}. \qquad (36)$$

Case 2: $|w_x(l)| < \varepsilon$,

$$A \overset{\Delta}{=} \begin{bmatrix} 0 & 1 & 0 & 0 \\ \rho^{-1} \frac{\partial}{\partial x}\left(T_s \frac{\partial}{\partial x} \right) & -\rho^{-1} c_v & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -m_c^{-1}(T_s - \rho v_s^2 + k_1) \frac{\partial}{\partial x}\Big|_{x=l} & 0 & -m_c^{-1} k_c & -m_c^{-1}(d_c - \rho v_s + k_2 \Omega \alpha^{-1}) \end{bmatrix},$$

and

$$F \overset{\Delta}{=} \begin{bmatrix} 0 \\ -\rho^{-1} f_d \\ 0 \\ 0 \end{bmatrix}. \qquad (37)$$

The domain $D(A)$ of the operator A is defined as

$$D(A) \overset{\Delta}{=} \left\{ (w, \dot{w}, w(l), w_t(l))^T \mid w \in H_0^2, \dot{w} \in H_0^1, w(l), w_t(l) \in R \right\}.$$

From (36) and (37), the followings are obtained:

Case 1: $|w_x(l)| \ge \varepsilon$,

$$\langle z, Az \rangle_\Im = -v_s T_s(0) w_x^2(0) - c_v \int_0^l (w_t + v_s w_x)^2 dx - \left\{ \eta_1 \rho v_s^2 + \eta_1 k_0 + (d_c - \rho v_s) \eta_1^2 - v_s T_s(l) \right\} w_x^2(l) \le 0, \qquad (38)$$

where the control gains η_1 and k_0 can be chosen to satisfy

$$\left\{ \eta_1 \rho v_s^2 + \eta_1 k_0 + (d_c - \rho v_s) \eta_1^2 - v_s T_s(l) \right\} \ge 0.$$

Case 2: $|w_x(l)| < \varepsilon$,

$$\langle z, Az \rangle_\Im = -v_s T_s(0) w_x^2(0) - c_v \int_0^l (w_t + v_s w_x)^2 dx - \left\{ k_1 - \rho v_s^2 - v_s T_s(l) \varepsilon \left(\sqrt{\sigma_m \Theta^{-1}} \right)^{-1} \right\} \varepsilon \sqrt{\sigma_m \Theta^{-1}} - \left\{ k_2 \Omega \alpha^{-1} + (d_c - \rho v_s) \right\} \sigma_m \Theta^{-1} \le 0, \qquad (39)$$

where the control gains k_1 and k_2 can be chosen to satisfy

$$k_1 > \rho v_s^2 + v_s T_s(l) \varepsilon \left(\sqrt{\sigma_m \Theta^{-1}} \right)^{-1} \quad \text{and}$$

$$k_2 > (d_c - \rho v_s) / (\Omega \alpha^{-1}).$$

From (38) and (39), it can be concluded that the unbounded linear operator A is dissipative. Hence, $A : D(A) \subset \Im \to \Im$ is an infinitesimal generator of the linear process $\{T(t)\}_{t \ge 0} = \{ (\Phi(t,0), B(t)) \}_{t \ge 0}$ on $\Im$, see Theorem 3.2, p. 92, of Walker (1980). Note that the first component $\Phi(t,0)$ is generated by

$$\dot{s}(t) = A_0 s(t) + F_0, \quad s(0) = s_0, \qquad (40)$$

where $s = (w, \dot{w})^T \in (H_L^1 \times L^2)$,

$$A_0 \overset{\Delta}{=} \begin{bmatrix} 0 & 1 \\ \rho^{-1} \frac{\partial}{\partial x}\left(T_s \frac{\partial}{\partial x} \right) & -\rho^{-1} c_v \end{bmatrix}, \text{ and } F_0 \overset{\Delta}{=} \begin{bmatrix} 0 \\ -\rho^{-1} f_d \end{bmatrix}.$$

Note also that $\Phi(t,0) s_0$ is the strong solution of the evolution equation $\dot{s}(t) = A_0 s(t)$ for every $s_0 \in D(A_0)$. Finally, the solution of (40) can be written in the following variation of constant formula (Pazy, 1983)

$$s(t) = \Phi(t,0) s_0 + \int_0^t \Phi(t,\tau) F_0(\tau) \, d\tau, \qquad (41)$$

where $\Phi(t,s)$ is the evolution operator associated with A_0 in the space $(H_L^1 \times L^2)$.

Theorem 1. Consider the system (4)-(5) with the boundary control laws (23)-(24) and (25)-(26). If there exist the positive values α, β, γ_i, $i = 1,2,3$, η_1, k_0, k_1, and k_2 such that

$$\alpha > C \,, \quad \left(\alpha c_v + \beta\rho - \alpha\gamma_1 - \frac{2\beta c_v l}{\gamma_2} \right) > 0 \,,$$

$$\left\{ \beta T_{s,\min} - \left(\beta l + \frac{\alpha v_s}{2} \right)(T_s)_{x,\max} - \frac{\alpha}{2}(T_s)_{t,\max} \right.$$
$$\left. - 2\beta l(c_v\gamma_2 + \gamma_3) \right\} > 0 \,,$$

$$(\psi - 1)\,\alpha\, d_c \eta_1^2 + \left\{ (\beta l + \alpha v_s) T_s(l) + \beta\rho l v_s^2 \right\} - \alpha\eta_1 k_0 \le 0 \,,$$

$$\left\{ \eta_1 \rho v_s^2 + \eta_1 k_0 + (d_c - \rho v_s)\eta_1^2 - v_s T_s(l) \right\} \ge 0 \,,$$

$$k_1 \ge \max\left\{ \alpha^{-1}(2\beta\rho l v_s + \alpha\rho v_s^2),\ \rho v_s^2 + v_s T_s(l)\varepsilon\left(\sqrt{\sigma_m \Theta^{-1}} \right)^{-1} \right\},$$

and

$$k_2 \ge \max\left[\frac{d_c - \rho v_s}{\Omega\,\alpha^{-1}},\ (\psi - 1)\,\alpha\, d_c \Omega^{-1} \,,\right.$$
$$\left. + \left\{ (\beta l + \alpha v_s) T_s(l) + \beta\rho l v_s^2 \right\} \varepsilon^2 \sigma_m^{-1} \Theta \right],$$

then

$\int_0^l w_x^2 dx \to 0$ and $\int_0^l (w_t + v_s w_x)^2 dx \to 0$ as $t \to \infty$.

Proof: Denoting (41) as $s(t) = s(t, s(0), 0)$, define a two parameter family of map $M(t,0)$ on $(H_L^1 \times L^2)$ as

$$M(t,0)s(0) = s(t, s(0), 0)\,, \quad 0 \le t < \infty\,, \qquad (42)$$

where the mapping $M(t,0)$ on $(H_L^1 \times L^2)$ denotes an evolution process (Walker, 1980, p. 12, p. 49). Finally, (32) implies that the following integral has to be finite, i.e.,

$$c_0 \int_0^\infty \|s(t)\|_{H_L^1 \times L^2}^2 dt = c_0 \int_0^\infty \|M(t,0)s_0\|_{H_L^1 \times L^2}^2 dt$$
$$\le V(0) - V(\infty) < \infty \,. \qquad (43)$$

Thus, by Theorem 1 in Hong (1997), it can be concluded from (43) that $\|s(t)\|_{H_L^1 \times L^2} \to 0$ as $t \to \infty$, i.e.,

$\int_0^l w_x^2 dx \to 0$ and $\int_0^l (w_t + v_s w_x)^2 dx \to 0$ as $t \to \infty$.

5. CONCLUSIONS

In this paper, a robust boundary control scheme has been investigated to suppress the transverse vibration of an axially moving steel strip in the hot-dip galvanizing line. Due to the gravitational force added to the tension of the strip and the eccentricity of the support roll, which causes a periodic excitation, the tension variation of the strip is considered as a spatiotemporally varying function. In the traveling strip with fixed boundary conditions, the elements that cause the increase of the total mechanical energy are the traveling wave impinging on the right boundary, the time rate of the tension, and the distributed external force. The distributed external force due to the aerodynamic excitation from the air knives and air cooler is treated as disturbances. By using the robust boundary feedback control laws proposed, the asymptotic stability of the axially moving strip has been obtained through the Lyapunov analysis and the semigroup theory.

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