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A CLASS OF RATIONAL FUNCTION MATRICES WITH SOME PROPERTIES AND STRUCTURAL CONTROLLABILITY[1]

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Abstract: In this paper, a definition of a class of rational function matrices called a type-1 matrix is given. Two illustrative examples show that the type-1 matrix can describe most of linear physical systems. It is proven that the type-1 matrix satisfies two properties that its characteristic polynomial in the ring $F(z)[\lambda]$ has no nonzero constant eigenvalues or nonzero multiple factors in $F(z)[\lambda]$. The author present some controllability criteria of the linear systems whose characteristic polynomials have no nonzero multiple factors in $F(z)[\lambda]$. The applications of the type-1 matrix and system to structural controllability are indicated. *Copyright 2002 IFAC*

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1 INTRODUCTION

Consider a linear system

$$\dot{X}=AX+BU,\ Y=CX+DU, \tag{1.1}$$

where $X\in R^n$, $U\in R^g$, and $Y\in R^e$. Let $z_1, z_2, .., z_q$ be q independently variable parameters (independent parameters or paraemters for short). Let $z=(z_1, .., z_q)$. The domain of z is R^q. R^q is also said to be the parameter space. Let $F(z)$ denote the field of the rational functions with real coefficients in the q parameters $z_1, .., z_q$. A matrix $M(z)$ is called a rational function matrix (RFM) or a matrix over $F(z)$ if each entry in $M(z)$ is a member of $F(z)$. The system (1.1) is called a rational function system (RFS) or a system over $F(z)$ if all the coefficient matrices A, B, C and D are RFMs. Let $F(z)[\lambda]$ denote the ring of all the $F(z)$ coefficient polynomials in λ.

The following two properties were introduced by Lu and Wei (1991).

Definition: Let A be an $n\times n$ matrix over $F(z)$. Then its characteristic polynomial $\det(\lambda I-A)\in F(z)[\lambda]$. The martix A is said to be of Property 1 if m* $\{z\in R^q \mid \det(\lambda I-A)=0$ has nonzero multiple roots$\}=0$; A is of property 2 if m* $\{z\in R^q|\det(rI-A)=0\}=0$, where r is a nonzero constant, m*{..}=0 denotes that the Lebesgue measure of the point set {..} is zero. Property 1 means that the nonzero eigenvalues of A have multiplicity 1 for almost all $z\in R^q$. Property 2 means that a nonzero constant r is not an eigenvalue of A for almost all $z\in R^q$.

Definition: An $n\times n$ matrix $A=(a_{ij})$ containing q independent parameters $z_1, .., z_i, .., z_n, z_{n+1}, .., z_q$ is called a type-1 matrix if when $i<j$, either a_{ij}= a constant or $a_{ij} = a'_{ij}\ z_i$ and when $i\geq j$, $a_{ij}=a'_{ij}\ z_i$, where $a'_{ij}=a'_{ij}(z_{n+1}, .., z_q)$ is a rational function in only $z_{n+1}, .., z_q$ in both cases $i<j$ and $i\geq j$.

It will be proven in Section 4 that the type-1 matrix is of the two properties.

It is well known that many important properties of linear systems such as stability and controllability depend on their characteristic polynomials. Since Properties 1 and 2 just are the ones about $\det(\lambda I-A)$ in $F(z)[\lambda]$, they have an important application to the

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problem of structural controllability (see Section 6). The type-1 matrix can describe most of linear physical systems (see Section 2) although it is a class of RFMs. Therefore, it is necessary to prove Properties 1 and 2 of the type-1 matrix so as to analyze structural controllability and observability of linear physical systems. A linear system applied to engineering is a physical system. Thus, the investigation may be of significance from the viewpoint of physics.

In Section 2, the author presents three generic examples to show that the type-1 matrix can describe most of linear physical systems. Some lemmas are derived in Section 3. It is proven in Section 4 that the type-1 matrix satisfies the two properties. Some questions are mentioned in Section 5. The structural controllability criteria for the rational function systems with Property 1 are stated in Section 6. In Section 7, as an example, the controllability of a type-1 system (its coefficient matrix *A* is a type-1 matrix) is analyzed.

It should be emphasized that there have been some parametrizations: Lin (1974) introduced a structured matrix (SM). A matrix is called an SM if each entry of the matrix is either fixed zero or free nonzero where its nonzero entries are considered to be mutually independent parameters. Corfmat and Morse (1976), Anderson and Hong (1982), and Willems (1986) proposed three kinds of matrices whose entries are one-degree polynomials in independent parameters. They are said to be one-degree polynomial matrices for short. A matrix is called a column-structured matrix (CSM) if the different entries in a column of the matrix contain the same parameter factor, but the factors in distinct columns are independent of each other (Yamada and Luenberger (1985)). A matrix of the form $M=T+G$ is a mixed matrix if the 6nonzero entries of T are algebraically independent over the field to which the entries of G belong (Murota (1987,1998)). Lu and Wei (1994) defined a class of RFMs with the form $A=(C+V)^{-1}U$, where $C=\mathrm{diag}(z_1,z_2,\ldots,z_n)$, V and U are two $n\times n$ matrices over $F(z_{n+1},\ldots,z_q)$, $F(z_{n+1},\ldots,z_q)$ denote the field of rational functions in $z_{n+1},\ldots,z_q$, and $z_1,\ldots, z_n, z_{n+1},\ldots,z_q$ are q independent parameters. The matrix is here called the one $(C+V)^{-1}U$ for simplicity.

According to the above definitions, SM, CSM and $(C+V)^{-1}U$ do not contain any nonzero constant entries, but the type-1 matrix may contain. Generally, a type-1 matrix is not a one-degree polynomial matrix in independent parameters, but it is an RFM. Clearly, it is not a mixed matrix. For instance, let us consider a type-1 matrix

$$\begin{pmatrix} (z_3+1)z_1 & 8 \\ \dfrac{z_5+z_6}{z_3+2}z_2 & \dfrac{z_3+3z_5}{z_4+9}z_2 \end{pmatrix} .$$

Obviously, SM, CSM, one-degree polynomial matrix, mixed matrix or $(C+V)^{-1}U$ can not express it.

2. TWO KINDS OF PHYSICAL SYSTEMS

The following examples show that the type-1 matrix can describe most of linear physical systems.

Example 1: Consider a linearized system consisting of mechanical, electric, thermal, pneumatic, liquid level, and hydraulic, etc., components in which there are n energy storage elements. The initial conditions of the n energy storage elements are considered to be independent. This assumption is very mild since it is satisfied in most linear physical systems.

One can choose the (angular) velocities of (inertia moments) masses, the capacitor voltages, the inductor currents, the temperatures of thermal capacitances, the air pressures of capacitances of vessels and the liquid heads of capacitances of tanks, etc., as the state variables of the system. One has the force (torque), current, voltage, heat flow, air flow rate and liquid rate, etc., balance equations (Ogate,1970)

$$\sum_{j=1}^{n} \frac{l_{ij}}{z_j}\dot{x}_j = \sum_{j=1}^{n} a''_{ij}\, x_j + \sum_{r=1}^{p} b_{ir}u_r , \qquad (2.1)$$

where $a''_{ij}\in R$, $l_{ij}\in R$, $b_{ir}\in R$, $i\in\{1,\ldots,n\}, j\in\{1,\ldots,p\}$; z_1^{-1} ,..., z_j^{-1}, ..., z_n^{-1} are the masses (inertia moments), capacitances, inductances, and thermal, air and liquid capacitances, etc., which are the parameters of n energy storage elements. Let

$$\dot{x}_j z_j^{-1} = y_j,\quad \bar{y} = (y_1,\ldots,y_n)^T,$$
$$a''_j = \sum_{j=1}^{n} a''_{ij}\, x_j + \sum_{r=1}^{p} b_{ir}u_r,\quad \bar{a} = (a''_1,\ldots,a''_n)^T . \qquad (2.2)$$

Then $(l_{ij})\bar{y} = \bar{a}$ by (2.1) and (2.2). Since the initial conditions are independent, $\det(l_{ij})\neq 0$ and

$$\bar{y} = (l_{ij})^{-1}\bar{a}. \qquad (2.3)$$

When $i=j$, $y_j = \dot{x}_j z_j^{-1}$ can be written as

$$y_i = \dot{x}_i z_i^{-1}. \qquad (2.4)$$

Substituting (2.2) and (2.4)into (2.3)yields

$$\dot{X} = AX + BU,\quad A = (a_{ij}), \qquad (2.5)$$

where $a_{ij} = a'_{ij} z_i , a'_{ij} = a'_{ij}(z_{n+1},\ldots,z_q)$ is the rational function in $z_{n+1},\ldots, z_q$ which denote the physical parameters of q-n non-energy storage elements. Obviously, the matrix a is a type-1 matrix.

Example 2: Consider a linear RLC network with n energy storage elements whose network without sources has not any capacitor-only loops or inductor-only cut-sets. According to the circuit theory (Kuh and Rohrer, 1965), the state equation is

$$\begin{pmatrix} \dot{i}_1 \\ \vdots \\ \dot{i}_m \\ \dot{v}_{m+1} \\ \vdots \\ \dot{v}_n \end{pmatrix} = \begin{pmatrix} d_{11}/L_1 & d_{12}/L_1 & \cdots & d_{1n}/L_1 \\ \vdots & \vdots & \cdots & \vdots \\ d_{m1}/L_m & d_{m2}/L_m & \cdots & d_{mn}/L_m \\ d_{m+1,1}/C_{m+1} & d_{m+1,2}/C_{m+1} & \cdots & d_{m+1,n}/C_{m+1} \\ \vdots & \vdots & \cdots & \vdots \\ d_{n1}/C_n & d_{n2}/C_n & \cdots & d_{nn}/C_n \end{pmatrix} \begin{pmatrix} i_1 \\ \vdots \\ i_m \\ v_{m+1} \\ \vdots \\ v_n \end{pmatrix} + BU \tag{2.6}$$

where $i_1,\cdots,i_m$ are, respectively, the currents through the inductors $L_1,\cdots,L_m$; $v_{m+1},\cdots,v_n$ the voltages across the capacitors $C_{m+1},\cdots,C_n$; $a'_{ij}\in R$, $i\in\{1,\cdots,n\}$, $j\in\{1,\cdots,n\}$. Let

$$(L_1^{-1},\cdots,L_m^{-1},C_{m+1}^{-1},\cdots,C_n^{-1})=(z_1,\cdots,z_n)\ .$$

Then, $A=(a_{ij}),a_{ij}=a'_{ij}\,z_i$. Obviously, there do not exist any relations between z_{i_1} and $a'_{ij}\,,i_1=1,\cdots,n$. A is a type-1 matrix.

3. SOME LEMMAS FOR $P(\lambda)$ IN $F(z)[\lambda]$

It is necessary to derive some lemmas before proving that a type-1 matrix is of the two properties.

Let R denote the field of all the real numbers and $R[z_1,\cdots,z_q]$ denote the ring of all the real coefficient polynomials in q parameters $z_1,...,z_q$. $R[z_1,...,z_q]$ can be simply written as R_Z or $R[z]$, where $z=(z_1,...,z_q)$. Let $R_Z[\lambda]$ denote the ring of the R_Z coefficient polynomials in λ. The following lemma is a conclusion in the algebraic theory.

Lemma 1: If a polynomial $P(\lambda)$ in $R_Z[\lambda]$ can be decomposed in $F(z)[\lambda]$, then $P(\lambda)$ can be decomposed in $R_Z[\lambda]$.

Let $P(\lambda)$ be an n-degree polynomial in $F(z)[\lambda]$. If $P(\lambda)=\lambda^m\varphi(\lambda)$,$0\leqslant m<n$, $(\lambda,\ \varphi(\lambda))=1$, $\varphi(\lambda)=\varphi_1(\lambda)\varphi_2(\lambda)\ldots\varphi_s(\lambda)$, $\varphi_j(\lambda)\in F(z)[\lambda]$, $\deg\varphi_j(\lambda)\geqslant 1$,$1\leqslant j\leqslant s$, then $\varphi(\lambda)$ is called the nonzero part of $P(\lambda)$ and $\varphi_j(\lambda)$ the nonzero factor. If $\varphi_1(\lambda)=\varphi_2(\lambda)$, $\varphi_1(\lambda)$ and $\varphi_2(\lambda)$ are called nonzero multiple factors.

Lemma 2: If $P(\lambda)$ is an irreducible polynomial in $F(z)[\lambda]$ (i.e., $P(\lambda)$ can not decomposed in $F(z)[\lambda]$), then m*$\{z\in R^q\,|\,P(\lambda)=0$ has multiple roots$\}$=0.

Proof: Since $P(\lambda)$ is irreducible, $P(\lambda)$ has no multiple factors in $F(z)[\lambda]$. Then, $(P(\lambda),\dot{P}(\lambda))=1$, where $\dot{P}(\lambda)\in F(z)[\lambda]$ denotes the derivative of $P(\lambda)$. Thus m*$\{z\in R^q\,|\,(P(\lambda),\dot{P}(\lambda))\neq 1\}=0$ by Lemma 2 in [1], which implies m*$\{z\in R^q\,|\,P(\lambda)=0$ has multiple roots$\}$ =0. □

Lemma 3: Let $P(\lambda)\in F(z)[\lambda]$. $P(\lambda)$ has no multiple factors in $F(z)[\lambda]$ iff m*$\{z\in R^q\,|\,P(\lambda)=0$ has multiple roots$\}$=0.

Proof: Sufficiency is obvious. It is only necessary to prove necessity. Since $P(\lambda)$ has no multiple factors, we let $P(\lambda)$ be irreducible. Then necessity holds by Lemma 2. Suppose $P(\lambda)=\varphi_1(\lambda)\ldots\varphi_h(\lambda)$, where $h\geq 2$, $\deg(\varphi_j(\lambda))\geq 1$, $\varphi_j(\lambda)\in F(z)[\lambda]$ is irreducible and $(\varphi_i(\lambda),\varphi_j(\lambda))=1$, $i\neq j$, $h\geq i,j\geq 1$. Thus m*$\{z\in R^q\,|\,(\varphi_i(\lambda),\varphi_j(\lambda))\neq 1\}$=0 by Lemma 2 in [1], which means that m*$\{z\in R^q\,|\,P(\lambda)=0$ has multiple roots$\}$=0. □

Lemma 4: (i) m*$\{z\in R^q\,|\,\det(\lambda I\text{-}A)=0$ has nonzero multiple roots$\}$ = 0 iff $\det(\lambda I\text{-}A)$ has no nonzero multiple factors in $F(z)[\lambda]$; (ii) m*$\{z\in R^q\,|\det(rI\text{-}A)=0\}$=0 iff $\det(rI\text{-}A)$ is a nonzero member in $F(z)$, where r is a nonzero constant.

Proof : (i) Clearly, this is the special case of Lemma 3. (ii) For necessity, if $\det(rI\text{-}A)$ is a zero member of $F(z)$, then m*$\{z\in R^q\,|\det(rI\text{-}A)=0\}\neq 0$. For sufficiency, if $\det(rI\text{-}A)$ is a nonzero member of $F(z)$, it is obvious that m*$\{z\in R^q\,|\det(rI\text{-}A)=0\}$=0. □

According to Lemma 4, in other words, the matrix A is said to be of Property 1 if $\det(\lambda I\text{-}A)$ has no nonzero multiple factors in $F(z)[\lambda]$; A is of property 2 if $\det(rI\text{-}A)$ is a nonzero member in $F(z)$, where r is a nonzero constant.

4. TWO PROPERTIES OF TYPE-1 MATRIX

Theorem 1: Consider the polynomial (in λ) $P(\lambda)=\lambda^n+a_1\lambda^{n-1}+\cdots+a_k\lambda^{n-k}+\cdots+a_{n-1}\lambda+a_n$, where the coefficient $a_k=a_k(z)$ is a polynomial in q parameters $z_1,\cdots,z_q$ and in $a_k(z)$ there are no nonzero constant terms, $1\leq k\leq n$. Then $P(r)$ is a nonzero member of $F(z)$, where r is a nonzero constant.

Proof: i) If $a_1 = a_2 = \cdots = a_n = 0$, $P(l) = l^n$ and any nonzero constant is not its root. ii) Let $P(l) = l^m f(l)$, where $f(l) = l^{n-m} + a_1 l^{n-m-1} + \cdots + a_{n-m}, a_{n-m} \neq 0,\ 0 \le m < n$. Then $P(r) = r^m f(r) = r^m (r^{n-m} + a_1 r^{n-m-1} + \cdots + a_{n-m})$. Since a_k, $1 \le k \le n$, does not contain any nonzero constant terms, $a_1 r^{n-m-1} + ... + a_{n-m}$ is not a nonzero constant. Since r^{n-m} is a nonzero constant but $a_1 r^{n-m-1} + a_2 r^{n-m-2} + \cdots + a_{n-m+1} r + a_{n-m} \neq$ a nonzero constant, $P(r) = r^m f(r) \neq 0$. □

Theorem 2: Consider a polynomial (in λ) $P(l) = l^n + a_1 l^{n-1} + \cdots + a_k l^{n-k} + \cdots + a_{n-1} l + a_n$, where the coefficient $a_k = a_k(z)$, $1 \le k \le n$, is a polynomial in q parameters $z_1, \cdots, z_q$. If any term

$$b_r z_1^{r_1} z_2^{r_2} \cdots z_q^{r_q}$$

of a_k satisfies that

$$r_i \in \{0,1\},\ 1 \le i \le q, \sum_{i=1}^{q} r_i \ge 1$$

and b_r is a constant including zero, then $P(l)$ has no nonzero multiple factors in $F(z)[l]$.

Proof: i) If $a_1 = a_2 = \cdots = a_n = 0$, $P(l) = l^n$. The theorem is true. ii) Let $P(l) = l^m f(l)$, where $f(l) = l^{n-m} + a_1 l^{n-m-1} + \cdots + a_{n-m}, a_{n-m} \neq 0,\ 0 \le m < n, f(l)$ is the nonzero part of $P(l)$. If $f(l)$ is irreducible, this theorem is true. If $f(l)$ is a reducible polynomial in $F(z)[l]$, $f(l)$ is also reducible in $R_z(l)$ by Lemma 1. Let $f(l) = f_1(l) \cdots f_h(l)$, where $2 \le h \le n-m$, $f_j(l) \in R_z[l]$ is irreducible, $1 \le j \le h$. Since the leading coefficient of $f(l)$ is one, the leading coefficient of $f_j(l)$ is a nonzero constant. It is clear from the assumption $1 \le r_1 + r_2 + ... + r_q$ that $a_k(z)$ has no nonzero constant term. So it is impossible that each of the coefficients of $f_j(l)$ is a constant. Conversely, suppose that $f(l)$ has nonzero multiple factors. Then there exist at least two integers $i, j, 1 \le i, j \le h, i \neq j$ such that $f_i(l) = f_j(l)$, which implies that $P(l)$ has at least one nonzero coefficient $a_k(z), 1 \le k \le n$, containing at least one term which does not satisfy $r_i \in \{0,1\}$. This contradicts the assumption. □

Lemma 5: Let the $n \times n$ matrix $A = (a_{ij})$ satisfy that when $i < j$, either a_{ij} =constant or $a_{ij} = a'_{ij} z_i$ and when $i \ge j, a_{ij} = a'_{ij} z_i$, where a'_{ij} is a constant including zero and $z_1, \cdots, z_n$ are n parameters (q=n). Then in its characteristic polynomial det(lI-A)= $l^n + a_1 l^{n-1} + \cdots + a_{n-1} l + a_n$, the coefficient

$$a_k = \sum b_{i_1 i_2 \cdots i_k} z_{i_1}^{r_{i_1}} z_{i_2}^{r_{i_2}} \cdots z_{i_k}^{r_{i_k}},$$

where $1 \le k \le n$,

$$i_j \in \{1, \cdots, n\}, i_j \neq i_h\ (j \neq h),\ 1 \le j, h \le k, b_{i_1 i_2 \cdots i_k}$$

is a constant, $r_{i_j} \in \{0,1\}, 1 \le j \le k, \sum_{j=1}^{k} r_{i_j} \ge 1$.

Proof: It is well-known that det(lI-A)= $l^n + \sum_{k=1}^{n} (-1)^k D_k l^{n-k}$, where D_k is the sum of all the principal minors of order k in $A, 1 \le k \le n$; each of principal minors of order k in D_k can be denoted by

$$\det \begin{pmatrix} a_{i_1 i_2} & \cdots & a_{i_1 i_k} \\ \vdots & & \vdots \\ a_{i_k i_1} & \cdots & a_{i_k i_k} \end{pmatrix},$$

where $1 \le i_1 < i_2 < \cdots < i_k \le n$. Since $a_{i_j i_h} = a'_{i_j i_h} z_{i_j} (i_j \ge i_h)$ and when $i_j < i_h, a_{i_j i_h}$ = constant or $a_{i_j i_h} = a'_{i_j i_h} z_{i_j}$, the conclusion is obvious by the definition of determinant. □

Remark 1: Obviously, the coefficient a_k in Lemma 5 does not contain any nonzero constant terms.

From Theorems 1, 2 and Lemma 5, Theorem 3 is immediate.

Theorem 3: The $n \times n$ matrix A with n parameters $z_1, \cdots, z_n\ (q = n)$ in Lemma 5 is of Properties 1 and 2.

Theorem 4: The $n \times n$ type-1 matrix A with q parameters $z_1, \cdots, z_q (q > n)$ is of Properties 1 and 2.

Proof: Arbitrarily, we fix $(z_{n+1}, ..., z_q)$= $(z_{n+1}^*, \cdots, z_q^*)$ = a constant vector and Theorem 3 holds. This means that Theorem 4 holds. □

5. QUESTIONS

It has been proved in Section 4 that a type-1 matrix satisfies Properties 1 and 2. In this section, the author would like to mention some relevant questions.

We know that many matrices are of Property 1 and/or Property 2. It was derived (Murota [7]) that if a mixed matrix has an eigenvalue which is transcendental over the subfield K, then it is a simple root, which is similar to Property 1. It is well known that SM and CSM are of Property 1 and it is not difficult to prove that they satisfy Property 2. The matrix $(C+V)^{-1}U$ defined in [9] is also of the two properties. The type-1 matrix satisfies the two properties and can describe most of linear physical systems. Then, we have some questions. Does any linear physical system satisfy the two properties (if all of its physical parameters are regarded as independent parameters)? Are the two properties two fundamental properties depending on the structures of linear physical systems? The author hopes that the questions will invite further discussion.

6. SOME CONTROLLABILITY CRITERIA OF RFSS WITH PROPERTY 1

Let the system described by (1.1) be an RFS, $T=(B, AB, \cdots, A^{n-1}B)$ and $T_0=(C^T, A^TC^T, \cdots, (A^{n-1})^TC^T)^T$ be its controllability and observability matrices respectively. Since they are dependent on z, T and T_0 are denoted by $T(z)$ and $T_0(z)$. Let

$$N_1=\{z\in R^q \mid \det(T(z)T^T(z))=0\},$$

$$N_2=\{z\in R^q \mid \det(T_0^T(z)T_0(z))=0\}.$$

Let S be a point set and m*S denote the Lebesgue measure of the set S.

Definition: RFS (1.1) is structurally controllable if m* $N_1=0$; otherwise, it is not structurally controllable. RFS (1.1) is structurally observable if m* $N_2=0$; otherwise, it is not structurally observable. (This definition was introduced in [1]).

Definition: Since $T(z)$ and $T_0(z)$ are two matrices over $F(z)$, $\det(T(z)T^T(z))\in F(z)$ and $\det(T_0^T(z)T_0(z))\in F(z)$. RFS (1.1) is controllable over $F(z)$ if $\det(T(z)T^T(z))$ is a nonzero member of $F(z)$(i.e.,$T(z)$ has n column vectors which are linearly independent over $F(z)$); otherwise, it is uncontrollable over $F(z)$. RFS (1.1) is observable over $F(z)$ if $\det(T_0^T(z)T_0(z))$ is a nonzero member of $F(z)$; otherwise, it is unobservable over $F(z)$.

Lemma 6: Let $f(z)\in F(z)$. If $f(z)$ is a zero member of $F(z)$ (simply $f(z)=0$), then $f(z)=0$ for all $z\in R^q$; if $f(z)$ is a nonzero member of $F(z)$ (simply $f(z)\neq 0$), m*$\{z\in R^q \mid f(z)=0\}=0$.

This lemma is obvious form algebraic theory.

Remark 2: By the above definitions and Lemma 6, the structural controllability (structural observability) is equivalent to the controllability (observability) over $F(z)$ for RFS (1.1).

Let M be an $n\times n$ matrix over $F(z)$. M is said to be reducible over $F(z)$ or a reducible matrix over $F(z)$ if there exists some nonsingular matrix P over $F(z)$ such that

$$PMP^{-1}=\begin{pmatrix} M_1 & 0 \\ M_{21} & M_2 \end{pmatrix},$$

where M_i is an $n_i\times n_i$ matrix, $i=1,2, 1\le n_1<n$; otherwise M is irreducible over $F(z)$ or an irreducible matrix over $F(z)$. (Note: The words "over $F(z)$" are often omitted for simplicity).

The proofs of the following theorems are omitted because of the length limitation.

Theorem 5: Let A be an $n\times n$ matrix over $F(z)$. (A,B) is controllable over $F(z)$ for any $n\times m$ matrix B over $F(z)$, $B\neq 0$, iff A is irreducible.

Theorem 6: Let $A=\text{diag}(A_1, A_2)$, $B=(B_1^T, B_2^T)^T$, where A_i and B_i are, respectively, $n_i\times n_i$ and $n_i\times m$ matrices over $F(z)$, i=1,2, and the polynomials det $(lI-A_1)$ and det $(lI-A_2)$ are relatively prime. (A,B) is controllable over $F(z)$ iff (A_i, B_i) is controllable over $F(z)$, i=1,2.

Corollary 1: Consider $A=\text{diag}(A_1,\ldots,A_k)$, $B=(B_1^T,\cdots,B_k^T)^T$, where A_i and B_i are respectively an $n_i\times n_i$ and an $n_i\times m$ matrices over $F(z)$, $i=1,\cdots,k$, det $(lI-A_i)$ and det $(lI-A_j)$, $i\neq j$, are relatively prime. Then (A,B) is controllable over $F(z)$ iff (A_i,B_i) is controllable over $F(z)$, $i=1,\cdots,k$.

Theorem 7: Let the $n\times n$ matrix A be of Property 1 and B an $n\times m$ matrix over $F(z)$. Then there exists some invertible matrix P over $F(z)$ such that

$$PAP^{-1}=\text{diag}[A_0, A_1, \cdots, A_k],$$

$$PB=(B_0^T, B_1^T, \cdots, B_k^T]^T,$$

where A_0 is an $n_0\times n_0$ nilpotent matrix and A_i $(1\le i\le k)$ is an $n_i\times n_i$ irreducible matrix over $F(z)$, B_i $(0\le i\le k)$ is an

$n_i \times m$ matrix over $F(z)$, $n=n_0+n_1+...+n_k$.. Then, (A,B) is controllable over $F(z)$ iff (A_0,B_0) is controllable and $B_i \neq 0$, $1 \leq i \leq k$.

7. APPLICATIONS TO STRUCTURAL CONTROLLABILITY

Since the structural controllability is equivalent to the controllability over $F(z)$, it is only necessary to discuss the controllability over $F(z)$ for an RFS.

Example 3: Consider an RLC network shown in Fig.2 after the references, whose state equation is

$$\dot{X} = AX + Bv_e,$$

where $X=(X_1^T,X_2^T)^T$, $B=(B_1^T,B_2^T)^T$, v_e is the voltage of a voltage source,

$$A=\mathrm{diag}(A_1,A_2), \quad X_1=(v_1, v_2, i_1)^T, \quad X_2=(v_3, i_2, i_3)^T,$$

$$A_1 = \begin{pmatrix} 0 & 0 & z_1 \\ 0 & -z_7^{-1}z_2 & z_2 \\ -z_3 & -z_3 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & z_4 & 0 \\ -z_5 & -z_8 z_5 & z_8 z_5 \\ 0 & z_8 z_6 & -z_8 z_6 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0 \\ 0 \\ z_3 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ z_5 \\ 0 \end{pmatrix},$$

$z=(z_1,....,z_8)=(C_1^{-1},C_2^{-1},L_1^{-1},C_3^{-1}, L_2^{-1},L_3^{-1},R_1,R_2)$.

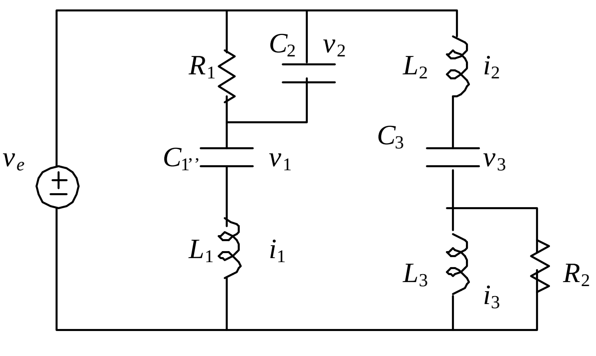

Fig.2

Clearly, $A=\mathrm{diag}(A_1,A_2)$ and $B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$ are two matrices over $F(z)$ and A is a type-1 matrix by the definitions. So, A is of Properties 1 and 2.

Obviously, det $(lI - A_i)$ is an irreducible polynomial in $F(z)[l](i=1,2)$. Thus A_1 and A_2 are two irreducible matrices over $F(z)$.

Since A is a type-1 matrix with Property 1, A_i is irreducible over $F(z)$ and $B_i \neq 0 (i=1,2)$, (A,B) is controllable over $F(z)$ (that is, structurally controllable) by Theorem 7.

In addition, some applications of Property 2 to structural controllability and observability were presented in the paper (Lu and Wei,1991).

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