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DESIGN OF DIGITAL MIMO CONTROL SYSTEMS WITH TWO-TIME-SCALE MOTIONS

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Abstract: The systematic approach of designing a robust sampled-data controller to solve output regulation problem for a MIMO nonlinear systems with unknown external disturbances and varying parameters is presented. The design methodology is based on the construction of two-time-scale motions in the closed-loop system. It has been shown that the proposed dynamical controller with a sufficiently small sampling period induces a two-time-scale separation of the fast and slow modes in the closed-loop system. Stability conditions imposed on the fast and slow modes and small sampling period can ensure that the full-order closed-loop system achieves the desired properties so that the output transient performances are desired and insensitive to parameter variations and external disturbances. Finally, an example with simulation results is presented. *Copyright ©2002 IFAC*

Keywords: sampled-data systems, robust control, output regulation, disturbance rejection, singular perturbation method

1. INTRODUCTION

In order to design a feedback controller to stabilize of nonlinear systems a two-step approach is widely used where a state feedback controller is designed and then a hign-gain observer is constructed to estimate the state of the nonlinear system (Esfandiary and Khalil, 1992; Teel and Praly, 1995; Isidori, 1999). The introducing of the fast hign-gain observer leads to the using of methods of asymptotic analysis of a singularly perturbed closed-loop systems (Kokotović and Khalil, 1986; Kokotović *et al.*, 1986; Litkouhi and Khalil, 1985; Saksena *et al.*, 1984). In (Teel *et al.*, 1998), the problem of digital implementation of nonlinear systems with input-to-state stabilizing controller is discussed and, in (Dabroom and Khalil, 2001), the performance of systems with state feedback controller and hign-gain observer under a sampled data is studied.

In contrast to the systems with state feedback controller (Isidori and Byrnes, 1990; Nijmeijer and van der Schaft, 1990; Esfandiary and Khalil, 1992), the discussed in this paper approach relates to the control systems based on the using of higher output derivative (or derivative of the state of nonlinear system, output derivative of the order which is the same as relative degree of nonlinear system) in feedback loop (Vostrikov, 1977; Vostrikov and Yurkevich, 1991) where fast differentiating filter (extended analog of the hign-gain observer) is used. The digital implementation of the (Vostrikov, 1977) controller was discussed in (Mutschkin, 1988; Fehrmann *et al.*, 1989).

The results of (Vostrikov, 1977) were extended in (Yurkevich, 1995a) by introducing, instead of separate differentiating filter, a fast dynamical controller with higher output derivative in feedback loop. In nonlinear control system with such fast dynamical controller, the output regulation and disturbance rejection are achieved under uncertainty by construction of desired fast and slow modes in the closed-loop system.

Design of digital MIMO control systems based on discretization of the (Yurkevich, 1995b) continuous-time fast dynamical controller was discussed in (Błachuta *et al.*, 1996; Yurkevich *et al.*, 1998) where a pseudo-continuous-time model of the control loop with a pure time delay is used for which a linear continuous-time controller is designed and then a digital controller follows from the continuous-time controller discretization. In this paper, as opposed to the above works, the discrete-time counterpart of the fast dynamical controller given by (Yurkevich, 1995a; Yurkevich, 1999) is used and extended in order to sampled-data controller design for MIMO uncertain nonlinear time-varying systems.

The paper is organized as follows. First, an approximate discrete-time model of output behavior for nonlinear system with zero-order hold is introduced. Second, the output tracking problem with prescribed output dynamics is transformed into insensitivity condition and then the control law structure is given. Third, the way of desired two-time-scale motion construction in the closed-loop system as well as expressions to choose of controller parameters and a sampling period are presented. Finally, an example with simulation results is given.

2. CONTROL PROBLEM

Let us consider a MIMO non-linear time-varying system given by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w}) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{w})\boldsymbol{u}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{w}) \tag{2}$$

where $\boldsymbol{y}(t)$ is the output available for measurement, $\boldsymbol{y} \in \boldsymbol{R}^p$, $\boldsymbol{x}(t)$ is the state, $\boldsymbol{x} \in \boldsymbol{R}^n$, $\boldsymbol{x}(0) = \boldsymbol{x}_0$ is the initial state, $\boldsymbol{x}_0 \in \Omega_{\boldsymbol{x}}$, $\Omega_{\boldsymbol{x}}$ is a bounded set, $\Omega_{\boldsymbol{x}} \subset \boldsymbol{R}^n$, $\boldsymbol{u}(t)$ is the control, $\boldsymbol{u} \in \Omega_{\boldsymbol{u}} \subset \boldsymbol{R}^p$, $p \le n$, $\boldsymbol{w}(t)$ is the vector of an unavailable for measurement external disturbances and varying parameters, $\boldsymbol{w} \in \Omega_{\boldsymbol{w}}$, $\Omega_{\boldsymbol{w}}$ is a bounded set and $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w})$, $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{w})$, $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{w})$ are smooth $\forall\ (\boldsymbol{x}, \boldsymbol{w}) \in \Omega_{\boldsymbol{x},\boldsymbol{w}} = \Omega_{\boldsymbol{x}} \times \Omega_{\boldsymbol{w}}$, t denotes time, $t > 0$. The influence of all external disturbances and varying parameters of the system is represented by dependence of $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{w})$, $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{w})$, $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{w})$ from $\boldsymbol{w}$.

Assumption 2.1 *Let as assume that a series connection of a zero-order hold (ZOH) and the continuous-time system (1), (2) takes place, where* $\boldsymbol{u}(t) = \boldsymbol{u}_k$, *for* $kT_0 \le t < (k+1)T_0$ *and* T_0 *is the sampling period.*

The control system is being designed to provide the following condition

$$\lim_{k\to\infty} \boldsymbol{e}_k = 0 \tag{3}$$

where $\boldsymbol{e}_k = \boldsymbol{e}(t)|_{\ t\,=\,kT_0}$ is the tracking error, $\boldsymbol{e}_k = \boldsymbol{r}_k - \boldsymbol{y}_k$, $\boldsymbol{y}_k = \boldsymbol{y}(t)|_{\ t\,=\,kT_0}$ is the sample point of the output $\boldsymbol{y}(t)$. $\boldsymbol{r}_k = \boldsymbol{r}(t)|_{\ t\,=\,kT_0}$ is the sample point of the reference input $\boldsymbol{r}(t)$.

Moreover, the controlled transients of the each i-th component $y_i(t)$ of the output vector $\boldsymbol{y}(t)$ should have a desired performance indices that are separately assigned such as overshoot σ_i^d, settling time t_i^d and system type. These transients should not depend on an external disturbances and varying parameters of the system (1), (2).

3. SYSTEM WITH ZERO-ORDER HOLD

By differentiating each component of (2) yields

$$\boldsymbol{y}_* = \boldsymbol{h}^*(\boldsymbol{x}, \bar{\boldsymbol{w}}) + \boldsymbol{g}^*(\boldsymbol{x}, \bar{\boldsymbol{w}})\boldsymbol{u} \tag{4}$$

where

$$\boldsymbol{y}_* = \left\{ \frac{d^{\alpha_1} y_1}{dt^{\alpha_1}}, \ldots, \frac{d^{\alpha_p} y_p}{dt^{\alpha_p}} \right\}', \ \boldsymbol{h}^* = \{h_1^*, \ldots, h_p^*\}'$$

$$\bar{\boldsymbol{w}} = \{\boldsymbol{w}', \ldots, [d^{\alpha}\boldsymbol{w}/dt^{\alpha}]'\}', \ \alpha = \max_i\{\alpha_i\}$$

and the set $\{\alpha_1, \ldots, \alpha_p\}$ is the relative degrees.

Assume that $\bar{\boldsymbol{w}} \in \Omega_{\bar{\boldsymbol{w}}}$, $\Omega_{\bar{\boldsymbol{w}}}$ is a bounded set.

Assumption 3.1 *Let assume that the sufficient invertibility condition of (1),(2)*

$$\det \boldsymbol{g}^*(\boldsymbol{x}, \bar{\boldsymbol{w}}) \neq 0 \quad \forall\ (\boldsymbol{x}, \bar{\boldsymbol{w}}) \in \Omega_{\boldsymbol{x},\bar{\boldsymbol{w}}}$$

is satisfied.

In order to receive an approximate discrete-time model of the system (1), (2) let us introduce the new time scale $t_0 = t/T_0$ depending on the sampling period T_0. Then

$$d\boldsymbol{x}/dt_0 = T_0\{\boldsymbol{f}(\cdot) + \boldsymbol{g}(\cdot)\boldsymbol{u}\}, \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{5}$$

follows from (1). After introducing of the matching matrix $\boldsymbol{K_0}$ such that

$$\boldsymbol{u} = \boldsymbol{K_0}\boldsymbol{v} \quad \text{and} \quad \boldsymbol{K_0} = \{\boldsymbol{g}^*\}^{-1} \tag{6}$$

from (4) the expression

$$\left\{ \frac{d^{\alpha_1} y_1}{dt_0^{\alpha_1}}, \frac{d^{\alpha_2} y_2}{dt_0^{\alpha_2}}, \ldots, \frac{d^{\alpha_p} y_p}{dt_0^{\alpha_p}} \right\}' = \boldsymbol{\mathcal{T}}\{\boldsymbol{h}^* + \boldsymbol{v}\} \tag{7}$$

follows where $\boldsymbol{\mathcal{T}} = diag\{T_0^{\alpha_1}, T_0^{\alpha_2}, \ldots, T_0^{\alpha_p}\}$.

If $T_0 \to 0$ then $d\boldsymbol{x}/dt_0 \to 0$ and $\boldsymbol{x} \approx const$, $\boldsymbol{h}^* \approx const$. So, if the sampling period T_0 is suffieciently small then it may be assumed that at least during the sampling period T_0 the condition $\boldsymbol{h}^*(\boldsymbol{x}, \bar{\boldsymbol{w}}) = const$ for $kT_0 \le t < (k+1)T_0$ is satisfied. Accordingly, as a result of the $\mathcal{Z}$-transform of the each i-th component of (7) it follows that

$$y_{i,k} = \frac{\mathcal{E}_{\alpha_i}(z)}{\alpha_i!\ (z-1)^{\alpha_i}} T_0^{\alpha_i} \left\{ h_{i,k}^* + v_{i,k} \right\} \tag{8}$$

where $y_{i,k} = y_i(t)|_{t\,=\,kT_0}$, $v_{i,k} = v_i(t)|_{t\,=\,kT_0}$, $h^*_{i,k} = h^*_i(\boldsymbol{x}, \bar{\boldsymbol{w}})|_{\,t\,=\,kT_0}$ and $\mathcal{E}_i(z)$ are Euler polynomials (Sobolev, 1977; Åström *et al.*, 1984; Błachuta, 1999).

$$\mathcal{E}_l(z) = \epsilon_{l,1} z^{l-1} + \epsilon_{l,2} z^{l-2} + \cdots + \epsilon_{l,l}, \qquad (9)$$

$$\epsilon_{l,j} = \sum_{\rho=1}^{j} (-1)^{j-\rho} \rho^l \binom{l+1}{j-\rho}, \qquad (10)$$

$$\mathcal{E}_l(1) = l\,!, \quad j = 1, 2, \ldots, l, \quad l = 1, 2, \ldots \quad (11)$$

Then for T_0 small enough the behavior of $y_{i,k}$ can be approximately discribed by the difference equation

$$y_{i,k} = \sum_{j=1}^{\alpha_i} (-1)^{j+1} \binom{\alpha_i}{\alpha_i - j} y_{i,k-j} + T_0^{\alpha_i} \sum_{j=1}^{\alpha_i} \frac{\epsilon_{\alpha_i, j}}{\alpha_i\,!} \left\{ h^*_{i,k-j} + v_{i,k-j} \right\} \quad (12)$$

Remark 3.1 *If $T_0 = 0$ then from (12) the difference equation*

$$y_{i,k} = \sum_{j=1}^{\alpha_i} (-1)^{j+1} \binom{\alpha_i}{\alpha_i - j} y_{i,k-j} \qquad (13)$$

follows where its characteristic polynomial is equal to $(z-1)^{\alpha_i}$.

4. DESIRED DIFFERENCE EQUATIONS

Let us construct such a continuous-time counterpart of reference model for desired behavior of output $y_i(t)$ that $y_i = G^d_i(s) r_i$ where parameters of the α_i-th order stable continuous-time transfer function

$$G^d_i(s) = \frac{b^d_{i,\rho_i} s^{\rho_i} + b^d_{i,\rho_i - 1} s^{\rho_i - 1} + \cdots + b^d_{i,0}}{s^{\alpha_i} + a^d_{i,\alpha_i - 1} s^{\alpha_i - 1} + \cdots + a^d_{i,0}} \quad (14)$$

are selected based on the required output transient performance indices and $b^d_{i,0} = a^d_{i,0}$. From (14) the desired stable differential equation

$$y_i^{(\alpha_i)} = F_i(\boldsymbol{y}_i, \boldsymbol{r}_i) \qquad (15)$$

follows where $\boldsymbol{y}_i = [y_i, \ldots, y_i^{(\alpha_i - 1)}]'$, $\boldsymbol{r}_i = [r_i, \ldots, r_i^{(\rho_i)}]'$, $\rho_i < \alpha_i$, $r_i = y_i$ at the equilibrium of (15). From (15) it follows that the reference model for the desired behavior of the output $\boldsymbol{y}(t)$ given by

$$\boldsymbol{y}_* = \boldsymbol{F}(\bar{\boldsymbol{y}}, \bar{\boldsymbol{r}}) \qquad (16)$$

where $\bar{\boldsymbol{y}} = \{y_1, \ldots, y_1^{(\alpha_1 - 1)}, y_2, \ldots, y_p^{(\alpha_p - 1)}\}'$, $\bar{\boldsymbol{r}} = \{r_1, \ldots, r_1^{(\rho_1)}, r_2, \ldots, r_p^{(\rho_p)}\}'$. In accordance with (14)

$$H^d_i(z) = \frac{z-1}{z} \mathcal{Z} \left\{ \frac{G^d_i(s)}{s} \right\} = \frac{B^d_i(z)}{A^d_i(z)} \quad (17)$$

is the desired pulse transfer function where

$$H^d_i(z)\big|_{z=1} = 1. \qquad (18)$$

From (17) the desired stable difference equation $y_{i,k} = F_i(Y_{i,k}, R_{i,k})$ follows which may be rewritten similar to (12) as the difference equation with small parameter

$$y_{i,k} = \sum_{j=1}^{\alpha_i} (-1)^{j+1} \binom{\alpha_i}{\alpha_i - j} y_{i,k-j} + T_0^{\alpha_i} \tilde{F}_i(Y_{i,k}, R_{i,k}, T_0) \qquad (19)$$

and $r_{i,k} = y_{i,k}$ at the equilibrium for all $i = 1, 2, \ldots, p$.

Theorem 4.1 *From (13) and (19) it follows that*

$$\lim_{T \to 0} \tilde{F}_i(Y_{i,k}, R_{i,k}, T_0) = F_i(y_i^{(\alpha_i - 1)}, \ldots, y_i, r_i^{(\rho_i)}, \ldots, r_i^{(1)}, r_i)|_{\,t\,=\,kT_0} \quad (20)$$

Proof. Obviously

$$\lim_{T_0 \to 0} \{y_{i,k} - y_{i,k-1}\}/T_0 = y_i^{(1)}(t)|_{\,t\,=\,kT_0} \quad (21)$$

Similar to (21), we have that

$$\lim_{T_0 \to 0} \{y_{i,k} - \sum_{j=1}^{\alpha_i} (-1)^{j+1} \binom{\alpha_i}{\alpha_i - j} y_{i,k-j}\}/T_0^{\alpha_i} = y_i^{(\alpha_i)}(t)|_{\,t\,=\,kT_0} \quad (22)$$

The exp.(20) follows from (15), (19) and (22).

5. INSENSITIVITY CONDITION

Denote $\boldsymbol{e}^F = \boldsymbol{F}(\bar{\boldsymbol{y}}, \bar{\boldsymbol{r}}) - \boldsymbol{y}_*$ is the realization error of the desired output dynamics assigned by (16). Accordingly, if the condition

$$\boldsymbol{e}^F = 0 \qquad (23)$$

is held then the behavior of $\boldsymbol{y}(t)$ is described by (16) and insensitive with respect to the external disturbances and varying parameters of the system (1), (2).

Assumption 5.1 *Let as assume that stability or at the least boundedness of the internal behavior of (1),(2) under condition (23) takes place.*

As the discrete-time counterpart of (23), denote

$$e^F_{i,k} = F_{i,k} - y_{i,k} \qquad (24)$$

is the realization error of the desired dynamics which is assigned by $F_{i,k} = F_i(Y_{i,k}, R_{i,k})$ where $i = 1, 2, \ldots, p$. Then if the requierement

$$e^F_{i,k} = 0 \quad \forall\ k = 0, 1, \ldots \qquad (25)$$

is held then the behavior of $y_{i,k}$ is desired and insensitive to external disturbances and parameter variations in (1),(2). As a result, the discussed control problem (3) has been reformulated as the insensitivity condition given by (25).

6. CONTROL LAW STRUCTURE

To fulfil the requirement of (25) the control law is constructed as the following difference equation

$$v_{i,k} = \sum_{j=1}^{q_i \geq \alpha_i} d_{i,j} v_{i,k-j} + \lambda_i(T_0)\, e^F_{i,k} \qquad (26)$$

where $i = 1, 2, \ldots, p$ and

$$\lambda_i(T_0) = T_0^{-\alpha_i}\tilde{\lambda}_i, \qquad \tilde{\lambda}_i \neq 0 \qquad (27)$$

$$d_{i,1} + d_{i,2} + \cdots + d_{i,q_i} = 1. \qquad (28)$$

Note that in accordance with (28) the requirement (25) is satisfied at the equilibrium of (26).

7. MAIN RESULTS

7.1 *Fast-motion subsystem*

Theorem 7.1 *Associated with the closed loop system (12), (26) as $T_0, \to 0$, the fast-motion subsystem (FMS) of the i-th channel governed by*

$$v_{i,k} = \sum_{j=1}^{q_i \geq \alpha_i} \beta_{i,j} v_{i,k-j} + \tilde{\lambda}_i\{\tilde{F}_i - \sum_{j=1}^{\alpha_i} \frac{\epsilon_{\alpha_i,j}}{\alpha_i\,!} h^*_{i,k-j}\} \qquad (29)$$

where it is assumed that $h^*_{i,k} - h^*_{i,k-j} \approx 0$, $y_{i,k} - y_{i,k-j} \approx 0 \;\; \forall\; j \;= 1, 2, \ldots, q_i$ and

$$\beta_{i,j} = d_{i,j} - \tilde{\lambda}_i \epsilon_{\alpha_i,j}\{\alpha_i\,!\}^{-1} \;\; \forall\; j = 1, \ldots, \alpha_i \qquad (30)$$

$$\beta_{i,j} = d_{i,j} \quad \forall\; j = \alpha_i + 1, \ldots, q_i. \qquad (31)$$

Proof. From (24) and (19) it follows that the closed-loop system eqns.(12), (26) may be rewritten in the form

$$y_{i,k} = \sum_{j=1}^{\alpha_i} (-1)^{j+1} \binom{\alpha_i}{\alpha_i - j} y_{i,k-j} + T_0^{\alpha_i} \sum_{j=1}^{\alpha_i} \frac{\epsilon_{\alpha_i,j}}{\alpha_i\,!}\{h^*_{i,k-j} + v_{i,k-j}\} \qquad (32)$$

$$v_{i,k} = \sum_{j=1}^{q_i \geq \alpha_i} \{d_{i,j} - \tilde{\lambda}_i \frac{\epsilon_{\alpha_i,j}}{\alpha_i\,!}\} v_{i,k-j} + \tilde{\lambda}_i\{\tilde{F}_i(Y_{i,k}, R_{i,k}, T_0) - \sum_{j=1}^{\alpha_i} \frac{\epsilon_{\alpha_i,j}}{\alpha_i\,!} h^*_{i,k-j}\} \qquad (33)$$

where $\epsilon_{\alpha_i,j} = 0$ if $j > \alpha_i$.

In accordance with (5) and (7) it is easy to see that if $T_0 \to 0$ then in the new time scale t_0 a rate of output transients of (32) is decreased. So, the increasing sampling rate induces the fast and slow modes in the closed loop system (32), (33). If T_0 is small enough then

$$h^*_{i,k} - h^*_{i,k-j} \approx 0, \;\; y_{i,k} - y_{i,k-j} \approx 0 \qquad (34)$$

for all $j = 1, 2, \ldots, q_i$. Finally, from (32), (33), (34) the equation (29) of FMS follows.

7.2 *Control law parameters*

The asymptotic stability and desired transient performance indices of $v_{i,k}$ as well as desired settling time of FMS can be achieved by a proper choice of the control law parameters $d_{i,j}$ and λ_i, for example, by assigning of desired pole distribution of the characteristic polynomial of the fast-motion subsystem (29) for the each i-th channel.

Let $q_i = \alpha_i$ then from (29) it follows the characteristic polynomial $A_i^{FMS}(z)$ of FMS in the form

$$A_i^{FMS}(z) = z^{\alpha_i} - \beta_{i,1} z^{\alpha_i - 1} - \cdots - \beta_{i,\alpha_i} \qquad (35)$$

For example, the settling time t_s^{FMS} of FMS of the i-th channel is equal to $\alpha_i T_0$ if the requirement

$$A_i^{FMS}(z) = z^{\alpha_i} \qquad (36)$$

is satisfied. From (36) and exps.(28),(30) the parameters of the digital controller follows where

$$d_{i,j} = \epsilon_{\alpha_i,j}\{\alpha_i\,!\}^{-1} \quad \forall\; j = 1, 2, \ldots, \alpha_i \qquad (37)$$

$$\tilde{\lambda}_i = 1, \quad i = 1, 2, \ldots, p. \qquad (38)$$

So, if (6) is held then the parameters $d_{i,j}$ of the digital controller (26) depend only on the relative degrees $\{\alpha_i\}_{i=1}^p$ of the continuous-time system (1),(2) and Euler polynomials (9).

Usually, the sampling period T_0 may be chosen in accordance with the following requirement

$$T_0 \leq \min_{i=1,p} \frac{t_i^d}{\alpha_i \theta}$$

where θ is a desired degree of time-scale separation between the fast and slow modes and $\theta \geq 10$.

7.3 *Slow-motion subsystem*

Theorem 7.2 *If a steady state (quasi-steady state) in the FMS (29) takes place, i.e.*

$$v_{i,k} - v_{i,k-j} = 0 \;\; \forall\; j \;= 1, 2, \ldots, q_i, \qquad (39)$$

then $v_{i,k} = v^s_{i,k}$ *where*

$$v^s_{i,k} = \tilde{F}_i(Y_{i,k}, R_{i,k}, T_0) - \sum_{j=1}^{\alpha_i} \frac{\epsilon_{\alpha_i,j}}{\alpha_i\,!} h^*_{i,k-j} \quad (40)$$

Proof. The proof follows from (28),(29)-(31),(39).

Theorem 7.3 *If* $T_0 \to 0$ *and the FMS of (29) is asymptotically stable then the SMS equation of* $y_{i,k}$ *in the closed loop system (32), (33) is the same as (19).*

Proof. From (36) it follows that FMS is stable. If $T_0 \to 0$ then after fast ending of FMS transients in (32), (33) we have that (39) and (40) are fulfilled. Substituting (39), (40) into (33) yields the SMS equation which is the same as (19).

Theorem 7.4 *If the sampling period* $T_0 \to 0$ *then a such limit* $v^s_{i,k} - v^{NID}_i(t)|_{\;t\,=\,kT_0} \to 0$ *takes place where*

$$v^{NID}_i(t) = F_i(\boldsymbol{y}_i(t), \boldsymbol{r}_i(t)) - h^*_i(\boldsymbol{x}(t), \bar{\boldsymbol{w}}(t)) \quad (41)$$

is the Nonlinear Inverse Dynamics problem solution which follows from (4), (6), (16), (23).

Proof. From (5) it follows that $\boldsymbol{x}_k - \boldsymbol{x}_{k-i} \to 0 \;\; \forall\; i \;= 1, 2, \ldots, q_j$ as $T_0 \to 0$. Then from (9), (20) and (40) the expr.(41) follows.

Corollary 7.1 *If* $T_0 \to 0$ *then from (41) it follows that the behavior of* $y_i(t)$ *tends to the solution of (15). Accordingly, after fast ending of FMS transients, the controlled output transients in the closed loop system have a desired output performance indices assigned by (16).*

8. EXAMPLE

Let us consider the non-linear time-varying system

$$\begin{aligned}
\dot{x}_1 &= x_2 + x_3(x_1 - x_3)(x_3 + x_4 - x_1) \\
&\quad +(2 + \sin(x_4))u_1 + u_2 \\
\dot{x}_2 &= -(x_1 - x_3)(x_3 + x_4 - x_1) + w(t) \\
&\quad +(-1)u_1 + (1 + 0.5\sin(x_3))u_2, \\
\dot{x}_3 &= x_3(x_1 - x_3)(x_3 + x_4 - x_1) \\
&\quad +(2 + \sin(x_4))u_1 + u_2, \\
\dot{x}_4 &= x_2 - 12(x_3 + x_4 - x_1) + u_1 + u_2, \\
y_1 &= x_1 - x_3, \quad y_2 = x_3.
\end{aligned} \quad (42)$$

From (42) it follows that $\alpha_1 = 2, \alpha_2 = 1$ and

$$\boldsymbol{g}^* = \begin{bmatrix} -1 & 1 + 0.5\sin(x_5) \\ 2 + \sin(x_4) & 1 \end{bmatrix} \quad (43)$$

The assumption 3.1 is held. It is easy to verify the boundedness of the internal behavior of (1),(2) under condition (23) on some bounded sets $\Omega_{\boldsymbol{x}}$, $\Omega_{\boldsymbol{r}}$.

Let us assume that $K_0 = \{k_{ij}\} \approx \{\boldsymbol{g}^*\}^{-1}$ where $k_{11} = -1/3$, $k_{12} = 1/3$, $k_{21} = 2/3$, $k_{22} = 1/3$. Require that the controlled outputs $y_1(t)$, $y_2(t)$ behave as step response of transfer functions

$$G^d_1(s) = 1/(\tau_1 s + 1)^2, \quad G^d_2(s) = 1/(\tau_2 s + 1) \quad (44)$$

Then pulse transfer functions $H^d_1(z)$, $H^d_2(z)$ of a series connection of a zero-order hold and continuous-time systems of (44) are the functions

$$H^d_1(z) = \frac{(1 - d_1 - \tau_1^{-1}T_0 d_1)z + d_1(d_1 - 1 + \tau_1^{-1}T_0)}{z^2 - 2d_1 z + d_1^2}$$

$$H^d_2(z) = (1 - d_2)/(z - d_2)$$

where $d_1 = \exp(-T_0/\tau_1)$, $d_2 = \exp(-T_0/\tau_2)$. As a result the discrete-time controller has the form

$$\begin{aligned}
v_{1,k} &= 0.5v_{1,k-1} + 0.5v_{1,k-2} \\
&\quad + T_0^{-2}\{-y_{1,k} + 2d_1 y_{1,k-1} - d_1^2 y_{1,k-2} \\
&\quad + (1 - d_1 - \tau_1^{-1}T_0 d_1)r_{1,k-1} \\
&\quad + d_1(d_1 - 1 + \tau_1^{-1}T_0)r_{1,k-2}\} \\
v_{2,k} &= v_{2,k-1} + T_0^{-1}\{-y_{2,k} + d_2 y_{2,k-1} \\
&\quad + (1 - d_2)r_{2,k-1}\}
\end{aligned} \quad (45)$$

where $\{u_{1,k}, u_{2,k}\}' = \boldsymbol{K_0}\{v_{1,k}, v_{2,k}\}'$.

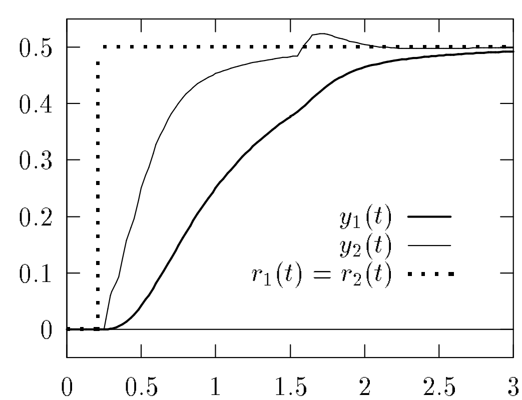

Fig. 1. Step response of outputs in the closed loop system of the example.

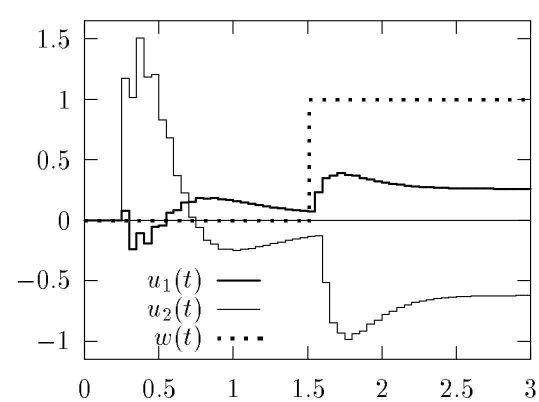

Fig. 2. Signals of inputs and disturbance in the closed loop system of the example.

The simulation results of (42) controlled by the algorithm (45) are displayed in Figs.1,2 for the time interval $t \in [0, 3]$ s, where $u(t) = u_k \; \forall \; kT_0 \leq t < (k+1)T_0$, $T_0 = 0.05$ s, $\tau_1 = 0.5$ s, $\tau_2 = 0.4$ s.

9. CONCLUSIONS

The proposed dynamical controller with the sufficiently small sampling period induces the two-time-scale separation of the fast and slow modes in the closed-loop system where after damping of the stabilized fast transients the behavior of the output $\boldsymbol{y}(t)$ is desired and insensitive to variation of parameters of the system and external disturbances. The main advantage of the presented method is that the knowledge about the relative degrees $\{\alpha_1, \ldots, \alpha_p\}$ and the matrix $\boldsymbol{g}^*$ is enough to controller design. Note that varying parameters and external disturbances don't need to be known as well as their way of entering in the system. It has been shown that the control signal $\boldsymbol{u}(t)$ converges to the continuous-time Nonlinear Inverse Dynamics solution as the sampling rate increases. Presented design methodology is the full discrete-time counterpart of the design methodology developed for continuous-time nonlinear control systems in (Yurkevich, 1995b).

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