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A ROBUST ITERATIVE LEARNING OBSERVER–BASED FAULT DIAGNOSIS OF TIME DELAY NONLINEAR SYSTEMS

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Abstract: An Iterative Learning Observer (ILO) updated successively and iteratively by immediate past system output error and ILO input is proposed for a class of time-delay nonlinear systems for the purpose of robust fault diagnosis. The proposed observer can estimate the system state as well as disturbances and actuator faults so that ILO can still track the post-fault system. In addition, the observer can attenuate slow varying output measurement disturbances. The ILO fault detection approach is then applied to automotive engine fault detection and estimation. Simulations show that the proposed ILO fault detection and estimation strategy is successful.

1. INTRODUCTION

Over the years, analytical redundancy approaches for fault detection and isolation (FDI) has been a subject of great deal of research studies (Beard 1971, Jones 1973, Chen and Saif 2000, Chen and Patton 1996, Polycarpou and Helmicki 1995, Saif and Guan 1993, Xiong and Saif 2001). Many practical biological, mechanical, or chemical processes involve delays that may cause instability or affect the performance of the control systems (Aggoune and Darouach 1999). Research in fault diagnosis for this class of systems has been scarce (Kratz and Ploix 1998, Yang and Saif 1998). In fact (Yang and Saif 1998) is believed to be the first work that addressed the FDI problem in time delay systems. (Yang and Saif 1998) proposed a robust observer for state estimation in a class of state delayed dynamic systems. The existence condition of the proposed observer and the convergence proof are derived based on the Razumikhin type theorem and this observer is then used to detect and isolate actuator and sensor faults in a class of time-delay systems. An alternative parity space approach is utilized to synthesize a residual generator for time-delay systems in (Kratz and Ploix 1998).

Iterative learning observer (ILO) was first proposed for FDI in (Chen and Saif 2001). In this paper, an ILO-based robust fault diagnosis strategy using the immediate past output estimation error and ILO input is presented for fault detection and estimation in a class of time-delay nonlinear systems. This ILO approach is then applied to fault detection and estimation of automotive engine that is employed as an application example. The main property of this ILO scheme is that it can compensate for both system disturbances and actuator faults. This allows it to follow the post-fault model after occurrence of an actuator fault. Additionally, any output measurement disturbance that is usually amplified by a classical Luenberger observer can be attenuated by ILO (Busawon and Kabore 2001).

2. PROBLEM STATEMENT

Consider a time-delay nonlinear system described by

$$\begin{aligned} \dot{x} &= Ax + \Phi(x,u) + Bx(t-t_h) + d(t) \\ y &= Cx \end{aligned} \qquad (1)$$

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where $x \in R^n$ is the state; $y(t)$ is the measurable output; $d(t)$ is an unmeasurable disturbance; t_h is a fixed known delay; and $\Phi(x,u)$ is a Lipschitz nonlinearity.
For the above system, we propose an ILO whose states are updated by the previous system output estimation errors and the previous ILO input $v(t-\tau)$. The proposed ILO differs from a classical Luenberger observer which is driven by the system input and output error of the current sampling time as described below:

$$\dot{\hat{x}}(t) = A\hat{x} + \Phi(\hat{x},u) + B\hat{x}(t-t_h) + Le_y(t) \quad (2)$$

where $e_y(t) = y(t) - \hat{y}(t)$. By subtracting the equation above from the system equation (1), the estimation error dynamics can be obtained

$$\dot{\tilde{x}} = (A-LC)\tilde{x} + [\Phi(x,u) - \Phi(\hat{x},u)] + B\tilde{x}(t-t_h) + d(t) \quad (3)$$

where $\tilde{x} = x - \hat{x}$.
Obviously, disturbance $d(t)$ has an impact on error dynamics, thereby, the main drawback of classical Luenberger observers is the lack of robustness. As a result, the ILO is designed to be robust so that the effect of disturbance $d(t)$ on error dynamics is compensated by ILO input $v(t)$. In addition, Luenberger observer usually amplifies the effect of output disturbances on the error dynamics. This can be illustrated by considering the time-delay nonlinear system (1) with output disturbances as follows

$$\begin{aligned} \dot{x} &= Ax + \Phi(x,u) + Bx(t-t_h) \\ y &= Cx + d(t). \end{aligned} \quad (4)$$

The error dynamics can be obtained using (2) and (4)

$$\dot{\tilde{x}} = (A-KC)\tilde{x} + [\Phi(x,u) - \Phi(\hat{x},u)] + B\tilde{x}(t-t_h) - Kd(t) \quad (5)$$

It is seen from the above that the disturbance $d(t)$ is amplified by gain K. On the other hand, if the gain K is chosen small to attenuate the effect of disturbance, then the stability of the observer may be affected. Busawon and Kabore (Busawon and Kabore 2001) proposed a PI observer to deal with this problem. Though the disturbance $d(t)$ is not amplified by PI observer gain, the disturbance itself still has an influence on the error dynamics. In this paper, a simple ILO is presented to attenuate output disturbance.

3. MAIN RESULTS

First, we construct a robust ILO with the property of disturbance compensation and estimation, furthermore, output disturbance attenuation by this ILO is discussed. The application of this ILO to robust fault diagnosis issue will be further introduced in section (4).

3.1 *ILO and Disturbance Estimate*

In this investigation, following assumptions are required.

A1: Disturbance $d(t)$ and its derivative $\dot{d}(t)$ are bounded with known bounds

$$\|d(t)\| \le b_d, \quad \|\dot{d}(t)\| \le b_{du}. \quad (6)$$

A2: System is bounded input bounded state stable, and the derivative of system input u is bounded.

A3: $\Phi(t)$, $\frac{\partial\Phi}{\partial x}$ and $\frac{\partial\Phi}{\partial u}$ are bounded and satisfy Lipschitz conditions as follows:

$$\|\Phi(x,u) - \Phi(\hat{x},u)\| \le \eta_1\|x-\hat{x}\|, \quad (7)$$

$$\left\|\frac{\partial\Phi}{\partial x}(x,u) - \frac{\partial\Phi}{\partial x}(\hat{x},u)\right\| \le \eta_2\|x-\hat{x}\|, \quad (8)$$

and

$$\left\|\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u}\right\| \le \eta_3\|x-\hat{x}\| \quad (9)$$

Based on system equation (1), an ILO is proposed as follows

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \Phi(\hat{x},u) + B\hat{x}(t-t_h) + L(y-\hat{y}) + v(t) \\ v(t) &= K_1 v(t-\tau) + K_2[y(t-\tau) - \hat{y}(t-\tau)] \end{aligned} \quad (10)$$

where $\hat{x}$ is estimated system state; τ is sampling time interval; $y(t-\tau)$ is the immediate past measurable output, i.e. the output at time $t-\tau$; $v(t)$ is called ILO input; L and $K_i's$ are some gain matrices to be determined. Note that the main characteristic of this ILO is that its states are updated by the previous system output estimation errors and the previous ILO input $v(t-\tau)$ as can be seen in equation (10).
Subtracting observer equation (10) from system equation (1), we have:

$$\dot{\tilde{x}} = (A-LC)\tilde{x} + [\Phi(x,u) - \Phi(\hat{x},u)] + B\tilde{x}(t-t_h) + d(t) - v(t) \quad (11)$$

where $\tilde{x} = x - \hat{x}$ is state estimation error, matrix $(A - LC)$ can be made Hurwitz by selecting an appropriate gain L.

Remark 1. To gain an understanding of this ILO, one can regard the nonlinear systems (1) as a reference model, where the observer tracks the reference model driven by the iterative input $v(t)$.

Following lemma will be helpful for the proof of theorem 1.

Lemma 1. If ILO input $v(t)$ is defined in equation (10), then following inequality holds

$$\begin{aligned} v^T(t)v(t) \le\ & 2v^T(t-\tau)K_1^T K_1 v(t-\tau) \\ & + 2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \end{aligned}. \quad (12)$$

The proof is omitted for brevity.

Theorem 1. Consider time delay nonlinear systems (1) satisfying assumptions A1-A3, and its ILO given in equation (10), if equations (19) and (20) and inequality (22) hold, then the error dynamics (11) is stable.

Proof. Consider the following Lyapunov function candidate:

$$V = \tilde{x}^T P\tilde{x} + \int_{t-\tau}^{t} \tilde{x}^T(\theta)R\tilde{x}(\theta)d\theta + \int_{t-t_h}^{t} \tilde{x}^T(\beta)\Gamma\tilde{x}(\beta)d\beta + \int_{t-\tau}^{t} v^T(\alpha)v(\alpha)d\alpha \tag{13}$$

where P, R and Γ are symmetric positive definite matrices. Substituting estimate error equation (11) into the derivative of Lyapunov function candidate V, we have

$$\begin{aligned}\dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T(t)R\tilde{x}(t) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau)\\ &\quad -\tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) + \tilde{x}^T(t)\Gamma\tilde{x}(t)\\ &\quad +v^T(t)v(t) - v^T(t-\tau)v(t-\tau)\\ &= \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma)\tilde{x} + 2\tilde{x}^T Pd(t)\\ &\quad +2\tilde{x}^T PB\tilde{x}(t-t_h) + 2\tilde{x}^T P(\Phi(x,u) - \Phi(\hat{x},u))\\ &\quad -2\tilde{x}^T Pv(t) + v^T(t)v(t) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau)\\ &\quad -\tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) - v^T(t-\tau)v(t-\tau).\end{aligned} \tag{14}$$

Combining inequalities

$$2\|\tilde{x}^T P\|\|v(t)\| \le \tilde{x}^T PP\tilde{x} + v^T(t)v(t) \tag{15}$$

$$2\tilde{x}^T PB\tilde{x}(t-t_h) \le \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-t_h)B^T B\tilde{x}(t-t_h) \tag{16}$$

into equation (14) yields

$$\begin{aligned}\dot{V} &\le \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + 2PP)\tilde{x}\\ &\quad +2\tilde{x}^T P(\Phi(x,u) - \Phi(\hat{x},u)) + \tilde{x}^T(t-t_h)B^T B\tilde{x}(t-t_h)\\ &\quad +2v^T(t)v(t) + 2b_d\|P\|\|\tilde{x}\| - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau)\\ &\quad -\tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) - v^T(t-\tau)v(t-\tau).\end{aligned} \tag{17}$$

Considering equation (7) of assumption A3) and lemma 1, equation (17) can be further extended as:

$$\begin{aligned}\dot{V} &\le \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + 2PP)\tilde{x}\\ &\quad +\tilde{x}^T(t-t_h)B^T B\tilde{x}(t-t_h) + 4v^T(t-\tau)K_1^T K_1 v(t-\tau)\\ &\quad -\tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) - \tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h)\\ &\quad +4\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) + 2b_d\|P\|\|\tilde{x}\|\\ &\quad +2\eta_1\|P\|\|\tilde{x}\|^2 - v^T(t-\tau)v(t-\tau)\\ &\le \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + 2PP)\tilde{x}\\ &\quad +2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + \tilde{x}^T(t-t_h)(B^T B - \Gamma)\tilde{x}(t-t_h)\\ &\quad +v^T(t-\tau)(4K_1^T K_1 - I)v(t-\tau) + 2b_d\lambda_{max}(P)\|\tilde{x}\|\\ &\quad +\tilde{x}^T(t-\tau)(4(K_2C)^T(K_2C) - R)\tilde{x}(t-\tau)\end{aligned} \tag{18}$$

where $I \in R^{n\times n}$ is an identity matrix. For a $Q = Q^T > 0$, there exists a unique $P = P^T > 0$ satisfying following equation

$$(A-LC)^T P + P(A-LC) + R + \Gamma + 2PP = -Q, \tag{19}$$

and let

$$B^T B \le \Gamma, \qquad 4K_1^T K_1 \le I, \qquad 4(K_2C)^T(K_2C) \le R, \tag{20}$$

according to (Khalil 1996), then equation (18) can be simplified as

$$\begin{aligned}\dot{V} &\le -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\|\\ &= -\mu\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\|\\ &= -(1-\theta)\mu\|\tilde{x}\|^2 - \theta\mu\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\|, \quad 0 < \theta < 1\\ &\le -(1-\theta)\mu\|\tilde{x}\|^2, \quad \forall\ \|\tilde{x}\| \ge (2b_d\lambda_{max}(P))/(\theta\mu),\end{aligned} \tag{21}$$

where

$$\mu = \lambda_{min}(Q) - 2\eta_1\lambda_{max}(P) > 0, \tag{22}$$

This concludes the proof.

Remark 2. In fact, $\dot{\tilde{x}}$ can be proved bounded, to this end, let $z = \dot{\tilde{x}}$, and differentiate state estimation error (11) to obtain

$$\dot{z} = (A-LC)z + s + Bz(t-t_h) + \dot{d}(t) - \dot{v}(t) \tag{23}$$

where $\dot{v}(t) = K_1\dot{v}(t-\tau) + K_2Cz(t-\tau)$ and $s = \frac{d}{dt}(\Phi(x,u) - \Phi(\hat{x},u)) = (\frac{\partial\Phi}{\partial x}(x,u)\dot{x} - \frac{\partial\Phi}{\partial x}(\hat{x},u)\dot{\hat{x}}) + (\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u})$. Assumptions A1, A2 and A3 can guarantee the boundedness of $\dot{x}$ and

$$\begin{aligned}\|s\| &\le \|(\frac{\partial\Phi}{\partial x}(x,u)\dot{x} - \frac{\partial\Phi}{\partial x}(\hat{x},u)\dot{\hat{x}})\| + \|\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u}\|\\ &\le \|\frac{\partial\Phi}{\partial x}(x,u) - \frac{\partial\Phi}{\partial x}(\hat{x},u)\|\|\dot{x}\| + \|\frac{\partial\Phi}{\partial x}(\hat{x},u)\|\|z\| + \eta_3\|\tilde{x}\|\\ &\le r_1 + r_2\|z\|\end{aligned} \tag{24}$$

where r_1 and r_2 are two positive constants.
Using an analysis similar to that used in the analysis of the estimation error dynamics, one can conclude that $\|z\|$ is bounded.

Remark 3. If (11) is stable, then the estimation error $\tilde{x}$ is bounded, and thus from remark 2 $\dot{\tilde{x}}$ is also bounded. Accordingly, $-v(t) + d(t)$ is bounded, thereby, the ILO input $v(t)$ can estimate or reconstruct the disturbance signal $d(t)$. This will be illustrated in the simulation study. On the other hand, the boundedness of $-v(t) + d(t)$ also explains that the robustness of ILO results from ILO input $v(t)$. It is $v(t)$ that compensates the effect of disturbance $d(t)$ on estimate error dynamics.

3.2 *Output Disturbance Attenuation by ILO*

In this subsection, we discuss output disturbance attenuation issue by considering equation (4). To this end, we have the following assumptions:

A4: The variation of $d(t)$ is bounded with a known bound

$$\|d(t) - d(t-\tau)\| \le l_d \tag{25}$$

where τ is the sampling interval in a sampled-data system. Apparently, that $l_d \ll b_d$.

A5: Consider that $W(\tilde{x}) = \tilde{x}^T(t)P\tilde{x}(t)$ is a positive definite function, where $P = P^T > 0$ satisfies equation (37) and $\tilde{x}(t) = x(t) - \hat{x}(t)$. Assume that $W(\tilde{x}(t-\tau)) \leq q^2 W(\tilde{x}(t)), q > 1$, then $\|\tilde{x}(t-\tau)\| \leq q\rho\|\tilde{x}(t)\|$, where $\tau > 0$, the sampling time interval and $\rho = (\lambda_{max}(P)/\lambda_{min}(P))^{1/2}$.

Remark 4. Assumption A5 is based on the stability theorem of Razumikhin (Hale 1977).

Assumption 4 is a key assumption in output disturbance attenuation, because if $d(t)$ varies slowly, its variation would be very small, or $l_d \ll d_{max}$. We will use this property to attenuate the effect of output disturbances on observer's error dynamics.
To cope with the output disturbance, consider the following ILO

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \Phi(\hat{x}, u) + B\hat{x}(t-t_h) + K_1(y-\hat{y}) + v(t) \\ v(t) &= K_2 v(t-\tau) - K_1[y(t-\tau) - \hat{y}(t-\tau)]. \end{aligned} \tag{26}$$

The observer's error dynamics is then given by

$$\begin{aligned} &\dot{\tilde{x}} = (A - K_1C)\tilde{x} + [\Phi(x,u) - \Phi(\hat{x},u)] + B\tilde{x}(t-t_h) + \\ &K_1C\tilde{x}(t-\tau) - K_1[d(t) - d(t-\tau)] - K_2 v(t-\tau). \end{aligned} \tag{27}$$

Remark 5. Observing the error dynamics (27), we can see that the effect of disturbance $d(t)$ is attenuated by its immediate past sampling value that results from measurable output $y(t-\tau)$ in ILO input $v(t)$. Since it is assumed that the $d(t)$ varies slowly, $d(t) - d(t-\tau)$ is guaranteed to be very small. Therefore, even if it is multiplied by a high gain K, its effect on the error dynamics will be negligable. In addition, $K_1(d(t)-d(t-\tau))$ can be further compensated by $v(t)$ if the error dynamics (27) is stable.

Remark 6. In (Busawon and Kabore 2001), the observer gain K is guaranteed not to amplify the effect of $d(t)$, but the disturbance $d(t)$ itself still effects the error dynamics. In this ILO approach, the effect of disturbance $d(t)$ on error dynamics is further reduced. This is important in FDI applications, because minimizing the effect of the disturbance can improve the robustness of the fault detection scheme.

Before stating the main theorem of this section, consider the following lemma and the subsequent development.

Lemma 2. Consider ILO update law $v(t) = K_2 v(t-\tau) - K_1(y(t-\tau) - \hat{y}(t-\tau))$, if assumption A5 holds, then $\|v(t)\| \leq l_n\|\tilde{x}(t-\tau)\| + b_n$, where l_n and b_n are two positive constants.

proof. Omitted for brevity.

To derive the stability condition of estimation error equation (27), we choose a Lyapunov function candidate as follows

$$\begin{aligned} V \;=\; & \tilde{x}^T P\tilde{x} + \int_{t-\tau}^{t} \tilde{x}^T(\theta)R\tilde{x}(\theta)d\theta + \int_{t-2\tau}^{t} \tilde{x}^T(\gamma)S\tilde{x}(\gamma)d\gamma \\ & + \int_{t-t_h}^{t} \tilde{x}^T(\beta)\Gamma\tilde{x}(\beta)d\beta \end{aligned} \tag{28}$$

where P, R, S and Γ are symmetric positive definite matrices. Substituting estimate error equation (27) into the derivative of Lyapunov function V, we have

$$\begin{aligned} \dot{V} \;=\; & \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T(t)R\tilde{x}(t) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) \\ & + \tilde{x}^T(t)\Gamma\tilde{x}(t) - \tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) + \tilde{x}^T(t)S\tilde{x}(t) \\ & - \tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau) \\ =\; & \tilde{x}^T((A-K_1C)^T P + P(A-K_1C) + R + \Gamma + S)\tilde{x} \\ & + 2\tilde{x}^T P(\Phi(x,u) - \Phi(\hat{x},u)) + 2\tilde{x}^T PB\tilde{x}(t-t_h) \\ & + 2\tilde{x}^T PK_1C\tilde{x}(t-\tau) - 2\tilde{x}^T PK_2 v(t-\tau) \\ & - 2\tilde{x}^T PK_1(d(t) - d(t-\tau)) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) \\ & - \tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau) - \tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h). \end{aligned} \tag{29}$$

Considering following inequalities

$$2\tilde{x}^T PK_1C\tilde{x}(t-\tau) \leq \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-\tau)(K_1C)^T(K_1C)\tilde{x}(t-\tau), \tag{30}$$

$$2\tilde{x}^T PB\tilde{x}(t-t_h) \leq \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-t_h)B^T B\tilde{x}(t-t_h). \tag{31}$$

and equation (7) of assumption A3, equation (29) can be further extended as

$$\begin{aligned} \dot{V} \;\leq\; & \tilde{x}^T((A-K_1C)^T P + P(A-K_1C) + R + \Gamma + S + 2PP)\tilde{x} \\ & + 2\eta_1\|P\|\|\tilde{x}\|^2 + \tilde{x}^T(t-t_h)(B^T B - \Gamma)\tilde{x}(t-t_h) \\ & + \tilde{x}^T(t-\tau)((K_1C)^T(K_1C) - R)\tilde{x}(t-\tau) \\ & + 2\|K_2\|\|\tilde{x}^T P\|\|v(t-\tau)\| + 2l_d\|K_1\|\|P\|\|\tilde{x}\| \\ & - \tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau) \end{aligned} \tag{32}$$

Applying lemma 2 to the equation above, we obtain

$$\begin{aligned} \dot{V} \;\leq\; & \tilde{x}^T((A-K_1C)^T P + P(A-K_1C) + R + \Gamma + S + 2PP)\tilde{x} \\ & + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + \tilde{x}^T(t-t_h)(B^T B - \Gamma)\tilde{x}(t-t_h) \\ & + \tilde{x}^T(t-\tau)((K_1C)^T(K_1C) - R)\tilde{x}(t-\tau) \\ & + 2l_{n-1}\|K_2\|\|\tilde{x}^T P\|\|\tilde{x}(t-2\tau)\| + 2b_{n-1}\lambda_{max}(P)\|K_2\|\|\tilde{x}\| \\ & + 2l_d\lambda_{max}(P)\|K_1\|\|\tilde{x}\| - \tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau) \end{aligned} \tag{33}$$

Since

$$2\|\tilde{x}^T P\|\|\tilde{x}(t-2\tau)\| \leq \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-2\tau)\tilde{x}(t-2\tau) \tag{34}$$

then

$$\begin{aligned} \dot{V} \;\leq\; & \tilde{x}^T((A-K_1C)^T P + P(A-K_1C) + R + \Gamma + S + 2PP \\ & + 2l_{n-1}\|K_2\|PP)\tilde{x} + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 \\ & + \tilde{x}^T(t-t_h)(B^T B - \Gamma)\tilde{x}(t-t_h) \\ & + \tilde{x}^T(t-\tau)((K_1C)^T(K_1C) - R)\tilde{x}(t-\tau) \\ & + 2b_{n-1}\lambda_{max}(P)\|K_2\|\|\tilde{x}\| + 2l_d\lambda_{max}(P)\|K_1\|\|\tilde{x}\| \\ & + \tilde{x}^T(t-2\tau)(2l_{n-1}\|K_2\|I - S)\tilde{x}(t-2\tau) \end{aligned} \tag{35}$$

Let

$$(K_1C)^T(K_1C) - R \leq 0, \qquad (B)^T(B) - \Gamma \leq 0,$$

$$2l_{n-1}\|K_2\|I - S \leq 0, \tag{36}$$

and for a positive definite symmetric matrix Q there exists a positive definite symmetric matrix P in

$$(A - K_1C)^T P + P(A - K_1C) + R + \Gamma + S + 2PP$$
$$+2l_{n-1}\|K_2\|PP = -Q, \tag{37}$$

then

$$\begin{aligned}\dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\lambda_{max}(P)\|K_1\|l_d\|\tilde{x}\| \\ &\quad +2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + 2b_{n-1}\lambda_{max}(P)\|K_2\|\|\tilde{x}\| \\ &\leq -(\lambda_{min}(Q) - 2\eta_1\lambda_{max}(P))\|\tilde{x}\|^2 \\ &\quad +2(\lambda_{max}(P)\|K_1\|l_d + b_{n-1}\lambda_{max}(P)\|K_2\|)\|\tilde{x}\|\end{aligned} \tag{38}$$

where $\lambda_{min}(Q) > 2\eta_1\lambda_{max}(P)$.
Therefore, if $\|\tilde{x}\| \geq \frac{2\lambda_{max}(P)\|K_1\|l_d + 2b_{n-1}\lambda_{max}(P)\|K_2\|}{\lambda_{min}(Q) - 2\eta_1\lambda_{max}(P)}$, then $\dot{V} \leq 0$.

Theorem 2. Consider equation (4) and assumptions A3, A4 and A5. If equations (36) and (37) hold, then, estimate error dynamics (27) is stable.

Remark 7. From theorem 2, we could say that $-K_1(d(t) - d(t-\tau)) - K_2v(t-\tau)$ is bounded because equation (27) is stable and similarly, one can prove that $\dot{\tilde{x}}$ is also bounded. The boundedness of $-K_1(d(t) - d(t-\tau)) - K_2v(t-\tau)$ implies that ILO input $v(t)$ can compensate the effect of $(d(t) - d(t-\tau))$ on error dynamics, which demonstrates the effectiveness of output disturbance attenuation by this ILO.

4. APPLICATION EXAMPLE

In this section, we apply the above proposed ILO to detect and estimate actuator faults in an automotive engine described by a second-order nonlinear engine model that involves intake to torque production delay and unmeasurable time varying disturbances. The model is that of (Stotsky A. and Eriksson 2000):

$$\begin{aligned}J_e\dot{w} &= a_2k\frac{p(t-t_h)}{p_0}(cos(-b+u_2))^{2.875} - \\ &\quad T_f - T_d - T_p \\ \frac{\dot{p}}{p_0} &= k_1(a_1p_ru_1 - kw\frac{p}{p_0}).\end{aligned} \tag{39}$$

where p is the manifold pressure and w is the engine speed. u_1 is the throttle input, whereas u_2 controls the spark influence. For the definition of the remaining engine parameters, consult (Stotsky A. and Eriksson 2000). For convenience, letting $x_1(t) = w, x_2(t) = p$, in the model of (Stotsky A. and Eriksson 2000), we get the following model

$$\begin{aligned}\dot{x}_1 &= 576.65x_2(t-t_h) - 76 - 0.112x_1 - 2.148\times10^{-4}x_1^2 \\ &\quad -7.84\times10^{-4}(1-x_2) + f_{actuator} \\ \dot{x}_2 &= 69.498p_ru_1 - 3.114\times10^{-2}x_1x_2 \\ y &= [x_1 \quad x_2]^T.\end{aligned} \tag{40}$$

Based on the equation above, the ILO is constructed according to equation (10) as follows:

$$\begin{aligned}\dot{\hat{x}}_1 &= 576.65\hat{x}_2(t-t_h) - 76 - 0.112\hat{x}_1 \\ &\quad -2.148\times10^{-4}\hat{x}_1^2 - 7.84\times10^{-4}(1-\hat{x}_2) \\ &\quad +2e_{y1}(t) - 0.0001e_{y2}(t) + v(t) \\ \dot{\hat{x}}_2 &= 69.498\hat{p}_ru_1 - 3.114\times10^{-2}\hat{x}_1\hat{x}_2 - \\ &\quad 0.0001e_{y1}(t) + 3e_{y2}(t) \\ v(t) &= v(t-\tau) + 0.4e_{y1}(t-\tau) - 0.0001e_{y2}(t-\tau) \\ y &= [\hat{x}_1 \quad \hat{x}_2]^T.\end{aligned} \tag{41}$$

The ILO will detect and estimate actuator fault $f_{actuator}$. We assume in the following fault detection that the healthy system has a fault $f_{actuator} = 0$. In the following simulation study, sampling time interval $\tau = 0.01$ and $t_h \simeq \tau$. Also assume $d(t) = 0.15sin(5t)$.

Figures 1 and 2 show system and observer trajectories without actuator faults. Here ILO input $v(t)$ can compensate and estimate system disturbance $d(t)$ in equation (1). In Figure 1, ILO states asymptotically converge to system states. Figure 2 demonstrates the system disturbance estimate by ILO input $v(t)$. We can see from the zoomed plot of $v(t)$ and $d(t)$ that after some transients in $v(t)$, it can reconstruct $d(t)$ very accurately.
Figures 3 and 4 indicate system and observer trajectories with an actuator fault. An actuator fault occurs at $t = 15$, however, observing engine speed diagram in Figure 3 we see that ILO state can still track the varied engine speed. In Figure 4, it can be seen that the ILO input $v(t)$ can be chosen as a residual because it can estimate actuator fault very accurately. Actually, $v(t)$ has been estimating system disturbance $d(t)$, after the occurrence of an actuator fault, $v(t)$ jump to a higher value, which is the real fault value. Meanwhile, $v(t)$ varies according to disturbance $d(t)$, therefore, $v(t)$ can estimate both actuator fault and system disturbance at the same time (see Figure 5). This is the key reason for the ILO ability to track post-fault system model.
Figures 6 and 7 describe output disturbance attenuation. Figure 6 describes state errors between the state of the actual system and the Luenberger observer estimate. Figure 7 describes state errors between the state of the actual system and ILO. Clearly the estimation errors in Figure 7 are much smaller than those in Figure 6. This demonstrates the effectiveness of ILO-based disturbance attenuation.

5. CONCLUSIONS

An Iterative Learning Observer was presented for fault detection and estimation applications. It was shown that the proposed ILO can not only compensate the effect of disturbances and actuator faults, but also attenuates slow varying measurement disturbances. In addition, it can successfully estimate the disturbance and fault, allowing the ILO to follow the post-fault system model.

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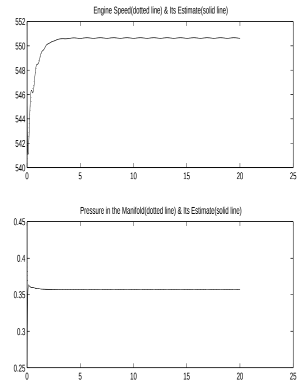

Fig. 1. System states and their estimates.

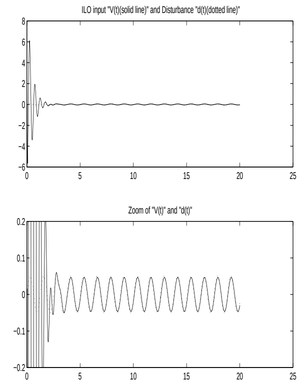

Fig. 2. Disturbance estimate by V(t).

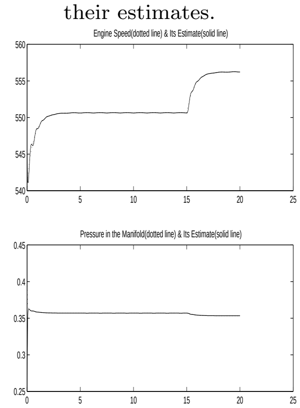

Fig. 3. Post-fault system states.

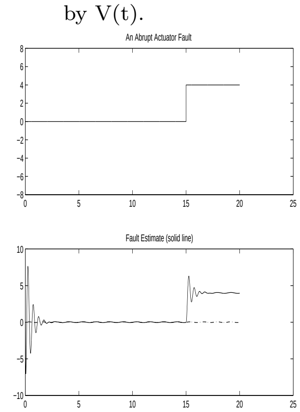

Fig. 4. Fault and its estimate.

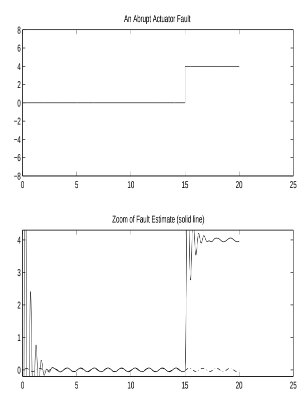

Fig. 5. Fault and its estimate.

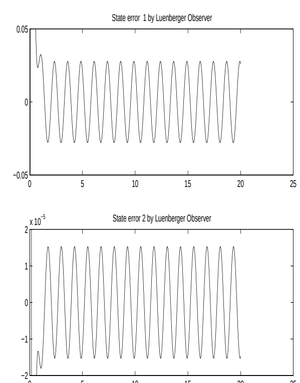

Fig. 6. Error dynamics between system and Luenberger observer.

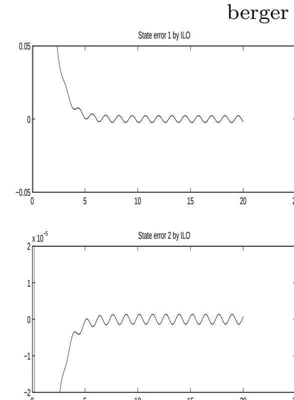

Fig. 7. Error dynamics be-