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GENERATION OF OPTIMAL SCHEDULES FOR METRO LINES USING MODEL PREDICTIVE CONTROL

Wânderson O. Assis Basílio E. A. Milani

Faculdade de Engenharia Elétrica e de Computação, Universidade Estadual de Campinas, C.P. 6101, 13081-970 Campinas SP, Brazil.

Abstract: This paper presents an algorithm for automatic generation of train dispatches in metro lines using model predictive control (MPC) with receding horizon. Train trajetories are optimized with reduced computational effort according to a moving horizon scheme, allowing transition between periods with large variation of passenger demand. The model is based on linear programming. It considers all operational constraints and give a trade-off solution between operational costs and service quality to passengers. Piecewise-linear functions are used for directly or indirectly modelling of waiting time of passengers at stations, onboard passenger comfort, train trip duration and number of trains in service. The performance of the proposed methodology is illustrated using a metro line similar to North/South line of São Paulo underground. *Copyright © 2002 IFAC*

Keywords: Predictive control, Scheduling algorithms, Railways, Optimization problems, Linear programming.

1. INTRODUCTION

The generation of train time schedules in metro lines corresponds to obtaining the dwell time at stations, run time between stations and the dispatch time of trains according to the variation of passenger demand along train trajectories. It is important to rationalize the use of trains, according to the variation of passenger flow at stations during the day, still keeping the system flexible for recovery of disturbed situations. Train schedule must be a trade-off between operational costs and service quality offered to passengers.

The variation of passenger demand along the day and along the line makes the solution non trivial, requiring a large computational effort. Moreover, passenger demand variation along weeks, month and year, makes necessary repetitive solution of the problem.

A methodology for generation of optimal schedules for metro lines has been proposed in (Cury et al., 1980): a nonlinear optimal control formulation is used, solved by an hierarchycal multilevel decomposition method. A more simple approach has been proposed in (Bergamashi et al.,1982): the schedulling problem is formulated as a set of nonlinear equations solved using an iterative decomposition method. In both approaches, the operational aspects were not completely considered and they also require considerable computational effort due to nonlinear nature of the problem.

Assis et al. (2000) proposed a new methodology for generation of optimal time schedules using linear programming (LP). The proposed formulation consider all the operational constraints treated in (Cury et al.,1980), (Bergamashi et al.,1982) more an additional constraint correspondent to a control margin for traffic regulation during the comercial operation of the line, important to the practical use of train schedulles (Van Breusegem et al., 1991). The performance index use piecewise-linear functions for directly or indirectly modelling the waiting time of passengers at stations, onboard passenger comfort, train trip duration and number of trains in service. However, the approach is restricted to a small period of time where the passenger demand is assumed constant.

This paper presents a methodology for solution of the schedulling problem using a MPC approach (Bemporad et al.,2000) with receding control horizon for computation of train time schedules during a full day considering the continuous variation of the passenger flow along the day.

2. ANALYTICAL MODEL

Consider the metro line in Figure 1. The system is composed by two track segments joining the terminal stations, where the trains run in opposite directions. Train traffic model is composed by two set of dynamic equations: headway equations and passenger load equations. A set of constraints are also considered:

- safety margin and operational limits;
- guaranteed comfort level for passengers;
- guaranteed control margin for on-line traffic regulation;
- guaranteed onboard in the arriving train of all the passengers waiting at platforms;
- continuity and smoothness of train traffic along the line.

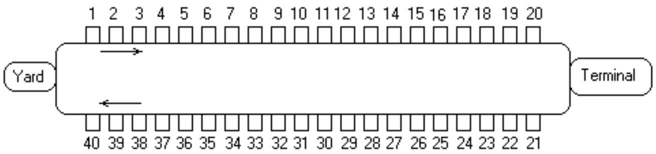

Fig. 1. North/South Line of S. Paulo Underground

Headway Equations

Headway is the interval between two consecutive trains along the line. For n trains and KT platforms (Cury et al., 1980) gives the following model:

$$x(k+1) = x(k) + Lup(k+1) + Lupl(k+1) \qquad (1)$$
$$\forall k = yard, 1, 2, \cdots, (KT-1), (KT), terminal, (KT+1), \cdots, (2KT-1), (2KT)$$

where: $x(k) \triangleq [x_1(k) \;\; x_2(k) \;\; \cdots \;\; x_n(k)]^T$
$up(k+1) \triangleq [up_0(k+1) \; up_1(k+1) \; \cdots \; up_n(k+1)]^T$
$upl(k+1) \triangleq [upl_0(k+1) \;\; \cdots \;\; upl_n(k+1)]^T$

$$L \triangleq \begin{bmatrix} -1 & 1 & 0 \cdots & 0 & 0 \\ 0 & -1 & 1 \cdots & 0 & 0 \\ \vdots & \vdots & \vdots \ddots & \vdots & \vdots \\ 0 & 0 & 0 \cdots & -1 & 1 \end{bmatrix}$$

where $x_i(k)$ is the waiting time of passengers of the train i at platform k, $up_i(k+1)$ is the dwell time of train i at platform k+1 and $upl_i(k+1)$ is the running time of train i between platforms k and k+1. Observe that $x_i(yard)$ is the headway between trains in the yard dispatching terminal (main element of train dispatch problem). For i=0 we get $up_0(k+1) + upl_0(k+1)$ is the difference between the openning time of platforms k and k+1 (known parameter).

Passenger Load Equations

Passenger load equations can be expressed as functions of passenger flows (Cury et al., 1980):

$$p(k+1) = p(k) + \nabla(k+1)x(k+1) - - \sum_{j=1}^{k} \nabla l(j,k+1)x(j) \qquad (2)$$

$p(k+1) \triangleq [p_1(k+1) \;\; p_2(k+1) \;\; \cdots \;\; p_n(k+1)]^T$
$\nabla(k+1) \triangleq diag(\alpha_1(k+1) \; \alpha_2(k+1) \; \cdots \; \alpha_n(k+1)$
$\nabla l(j,k+1) \triangleq diag(\alpha l_1(j,k+1) \;\; \cdots \;\; \alpha l_n(j,k+1))$

where $p_i(k)$ is the number of passengers in train i departing at platform k, $\alpha_i(k)$ is the flow of passengers boarding on train at platform k and $\alpha l_i(j,k)$ is the flow of passengers boarding on train at platform j and leaving train at platform k. These matrices correspond to statistical mean values of passenger flows during specific time periods of the day defined by origin-destination matrices (ODM). The boundary conditions are: $p(yard) = p(KT) = p(terminal) = p(2KT) = 0$.

Constraints

All the variables x(k), p(k), up(k) and upl(k) are constrained by upper and lower bounds imposed by the safety system, satisfaction of passengers demand, capacity of trains and operational range.

$$\begin{aligned} xmin_i(k) &\le x_i(k) \le xmax_i(k) \\ upmin_i(k) &\le up_i(k) \le upmax_i(k) \qquad (3) \\ uplmin_i(k) &\le upl_i(k) \le uplmax_i(k) \\ 0 &\le p_i(k) \le pmax_i(k) \end{aligned}$$

For guarantying that all the passengers waiting for a train will onboard in the next train i, the following constraint is formulated:

$$up_i(k) \ge \left(\frac{1 + MP_i(k)}{\beta_i(k) + MP_i(k)\alpha_i(k)} \right) \cdot . \left(\alpha_i(k)x_i(k) + \sum_{j=1}^{k-1} \alpha l_i(j,k)x_i(j) \right) \qquad (4)$$

where $MP_i(k)$ is a percentual control margin for on-line regulation and $\beta_i(k)$ is the flow of passengers boarding or leaving the train i. The percentual control margin is relative to the minimum dwell time and it can be ajusted by the designer.

3. DETERMINATION OF OPTIMAL SCHEDULES

For computation of optimal trajectories for **n** trains, the following performance index will be used:

$$J = \sum_{k=1}^{2KT} (T_1(k) + T_2(k) + T_3(k)) + T_4(k) + T_5(k) \quad (5)$$

Term $T_1(k)$, related to waiting time of passengers in the platforms, is given by piecewise linear approximation of a function obtained using queue theory (Assis et al., 2000):

$$T_1(k) = q\sum_{i=1}^{n} \varepsilon_i(k) \quad (6)$$

where q ≥ 0 is a weighting parameter and:

$$\begin{aligned} \varepsilon_i(k) &\geq \frac{\alpha_i(k)}{6}(xmax + 5xmin)x_i(k) \\ \varepsilon_i(k) &\geq \frac{\alpha_i(k)}{2}(xmax + xmin)x_i(k) - \\ &-\frac{\alpha_i(k)}{9}(xmax^2 + xmax.xmin - 2xmin^2) \\ \varepsilon_i(k) &\geq \frac{\alpha_i(k)}{6}(5xmax + xmin)x_i(k) - \\ &-\frac{\alpha_i(k)}{3}(xmax^2 - xmin^2) \end{aligned} \quad (7)$$

Term $T_2(k)$ represents a piecewise linear approximation of deviation between the actual number of passengers in the train and the desirable one (Assis et al., 2000):

$$T_2(k) = s\sum_{i=1}^{n} \rho_i(k) \quad (8)$$

where s ≥ 0 is a weighting parameter and $\rho_i(k)$ is:

$$\begin{aligned} \rho_i(k) &\geq -p_i(k) + p^r(k) \\ \rho_i(k) &\geq p_i(k) - p^r(k) \\ \rho_i(k) &\geq \delta p_i(k) + (1-\delta)pmaxx(k) - p^r(k) \end{aligned} \quad (9)$$

where $p^r(k)$ represents the desirable number of passengers at platform k, $pmaxx(k)$ is the passenger load at platform k, beyond which, passenger comfort is unsatisfactory. The parameter δ is a high weigthing factor adjusted to guarantee that $pmaxx(k)$ will not be exceeded.

The term $T_3(k)$ takes in account the train trip duration that depends on the control sequence of dwell times and running times of trains:

$$T_3(k) = r\sum_{i=1}^{n}(up_i(k) + upl_i(k)) \quad (10)$$

where r ≥ 0 is a weighting parameter.

Terms $T_1(k)$ and $T_3(k)$ represent a compromise between train trip duration and number of trains in service, which is related to operational cost.

The term $T_4(k)$ is related to traffic continuity. It is concerned with necessity of keeping constant train headways at line terminals.

$$T_4(k) = s_1\sigma_1 + s_2\sigma_2 \quad (11)$$

where $s_1 \geq 0$, $s_2 \geq 0$ are weighting parameters and:

$$\begin{aligned} -\sigma_1 &\leq x_i(yard) - x_{i-1}(2KT) \leq \sigma_1 \\ -\sigma_2 &\leq x_i(terminal) - x_i(KT) \leq \sigma_2 \\ \sigma_1 &\leq \sigma_1 max \quad , \quad \sigma_2 \leq \sigma_2 max \end{aligned} \quad (12)$$

Bounds $\sigma_1 max$ and $\sigma_2 max$ represent the maximum allowed variation of train headway in yard and terminal, respectively. If $\sigma_2 = 0$ the number of trains in service can be estimated:

$$nT_i = \sum_{k=1}^{2KT} \frac{up_i(k) + upl_i(k)}{x_i(yard)} \quad (13)$$

Finally, the term $T_5(k)$ is related to smoothness of traffic behaviour with respect to $up_i(k)$ and $upl_i(k)$.

$$T_5(k) = z(\zeta_1 + \zeta_2) \quad (14)$$

where z ≥ 0 is a weighting parameter and:

$$\begin{aligned} -\zeta_1 &\leq up_i(k) - up_{i-1}(k) \leq \zeta_1 \\ -\zeta_2 &\leq upl_i(k) - upl_{i-1}(k) \leq \zeta_2 \\ \zeta_1 &\leq \zeta_1 max \quad , \quad \zeta_2 \leq \zeta_2 max \end{aligned} \quad (15)$$

The problem of determination of optimal schedule can be stated as the linear programming problem:

$$\begin{aligned} \min(s_1\sigma_1 + s_2\sigma_2 + z\zeta_1 + z\zeta_2 + \sum_{i=1}^{n}\sum_{k=1}^{2KT}(q\varepsilon_i(k) + \\ +r(up_i(k) + upl_i(k)) + s\rho_i(k))) \end{aligned} \quad (16)$$

subject to constraints: (1), (2), (3), (4), (7), (9), (12) e (15).

Adjustable parameters q, r, s, s_1, s_2, z and δ, allow to get a trade-off solution between operational costs and service quality for passengers.

4. MODEL PREDICTIVE CONTROL

The LP problem (16) correspond the determination the trajectories of n trains along 2KT platforms. It can be verified that behaviour of each train depends exclusively of the train dispatched immediately before. So the following formulation can be done:

$$\min(\sum_{t=1}^{n}(P\nu(t) + Q\omega(t)))$$

subject to:

$$\begin{bmatrix} A_1 & 0 \\ A_2^t & B_1 \end{bmatrix} \begin{bmatrix} \nu(t+1) \\ \omega(t+1) \end{bmatrix} = \begin{bmatrix} C_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu(t) \\ \omega(t) \end{bmatrix}$$

$$\begin{bmatrix} D_1^t & 0 \\ D_2^t & E_1 \\ 0 & E_2 \\ D_3 & E_3 \\ D_4 & E_4 \end{bmatrix} \begin{bmatrix} \nu(t+1) \\ \omega(t+1) \end{bmatrix} \leq \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ F_1 & 0 \\ F_2 & 0 \end{bmatrix} \begin{bmatrix} \nu(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 0 \\ G_1^t \\ G_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\nu min \leq \nu(t) \leq \nu max \qquad \forall t = 1, \cdots, n$$

$$\omega min \leq \omega(t) \leq \omega max \qquad \forall t = 1, \cdots, n \quad (17)$$

$$\nu(t) \triangleq \begin{bmatrix} up_t(k) \\ upl_t(k) \\ x_t(k) \\ x_t(yard) \\ x_t(terminal) \end{bmatrix}; \omega(t) \triangleq \begin{bmatrix} p_t(k) \\ \rho_t(k) \\ \varepsilon_t(k) \\ \varepsilon_t(yard) \\ \varepsilon_t(terminal) \\ \sigma_1 \\ \sigma_2 \\ \zeta_1 \\ \zeta_2 \end{bmatrix}$$

$$\forall t = i = 1, \cdots, n \qquad (18)$$

$$\forall k = 1, 2, \cdots, KT, (KT+1), \cdots, 2KT$$

where P and Q are weighting matrices, A_1, B_1, C_1, D_3, D_4, E_1, E_2, E_3, E_4, F_1, F_2 and G_2 have constant terms and A_2^t, D_1^t, D_2^t and G_1^t depend on passenger flow variation. The LP problem (17) can be simplifyed as:

$$\min(\sum_{t=1}^{n}(P\nu(t) + Q\omega(t)))$$

subject to:

$$\begin{bmatrix} A^t & B \end{bmatrix} \begin{bmatrix} \nu(t+1) \\ \omega(t+1) \end{bmatrix} = C\nu(t)$$

$$\begin{bmatrix} D^t & E \end{bmatrix} \begin{bmatrix} \nu(t+1) \\ \omega(t+1) \end{bmatrix} \leq F\nu(t) + G^t$$

$$\nu min \leq \nu(t) \leq \nu max \qquad \forall t = 1, \cdots, n$$

$$\omega min \leq \omega(t) \leq \omega max \qquad \forall t = 1, \cdots, n \qquad (19)$$

It can be verified that variables $\omega(t)$ in spite of being included in the cost function, does not have direct influence in the next train $t+1$ behaviour. A MPC problem can be formulated as:

$$\min_{V,W} J'(V,W) = \sum_{k=1}^{Nt}(P\nu_{t+t_k|t} + Q\omega_{t+t_k|t})$$

subject to:

$$\nu min \leq \nu_{t+t_k|t} \leq \nu max \qquad t_k = 1, \cdots, Nt$$

$$\omega min \leq \omega_{t+t_k|t} \leq \omega max \qquad t_k = 1, \cdots, Nt \qquad (20)$$

$$v_{t|t} = v(t)$$

$$A^t\nu_{t+t_k+1|t} + B\omega_{t+t_k+1|t} = C\nu_{t+t_k|t}, \quad t_k \geq 0$$

$$D^t\nu_{t+t_k+1|t} + E\omega_{t+t_k+1|t} \leq F\nu_{t+t_k|t} + G^t, \ t_k \geq 0$$

where $V \triangleq \{\nu_t, \cdots, \nu_{t+Nt}\}$, $W \triangleq \{\omega_t, \cdots, \omega_{t+Nt}\}$. $\nu_{t+t_k|t}$ and $\omega_{t+t_k|t}$ denotes predicted vectors at time $(t+t_k)$ where $t_k = 1, \cdots, Nt$ denotes the number of trains considerated. So, for a *single-step* MPC, the problem is to obtain vectors $\nu_{t+Nt|t}$ and $\omega_{t+Nt|t}$ where ν_t is known and $Nt = 1$. If the optimization problem is repeated at time t+1, based on the known behaviour of ν_{t+1} and considering the variation of passenger flow we can get the optimal trajectory of the next dispatched train. Thus, the proposed model can be used for determination of optimal schedules along a full day assumming known the trajectory of the first train and daily origin-destination matrix. The proposed approach can also be used in the following situations:

- Execute the transition between periods with constant headways.
- Modify train time schedules on-line during comercial operation in the case of significative passenger flow disturbance.

In these situations the control strategy consists of two stages. In the first stage the MPC (20) is solved repeteadly using solution V_{t+1} as initial condition of the following step. In this way we get the trajectory for NP trains. For each step ($Step = 1, \cdots, NP$), ODM is giving by convex combination:

$$ODM = (1-\lambda)ODM_1 + \lambda ODM_2 \qquad (21)$$

where $\lambda = \frac{Step}{NP}$, $0 \leq \lambda \leq 1$ and the matrices ODM_1 e ODM_2 represent the expected matrices for the beginning and the end of transition period.

In the second stage it is proposed the following MPC problem with receding horizon:

$$\min_{V,W} J'(V,W) = \sum_{k=1}^{Nt-1}(P\nu_{t+t_k|t} + Q\omega_{t+t_k|t})$$

subject to:

$$min\{\nu(t), \nu(t+Nt)\} \leq \nu_{t+t_k|t} \leq$$

$$\leq max\{\nu(t), \nu(t+Nt)\} \qquad t_k = 1, \cdots, Nt$$

$$min\{\omega(t), \omega(t+Nt)\} \leq \omega_{t+t_k|t} \leq$$

$$\leq max\{\omega(t), \omega(t+Nt)\} \qquad t_k = 1, \cdots, Nt$$

$$v_{t|t} = v(t) \quad , \quad v_{t+Nt|t} = v(t+Nt) \qquad (22)$$

$$A^t\nu_{t+t_k+1|t} + B\omega_{t+t_k+1|t} = C\nu_{t+t_k|t}, \quad t_k \geq 0$$

$$D^t\nu_{t+t_k+1|t} + E\omega_{t+t_k+1|t} \leq F\nu_{t+t_k|t} + G^t, \ t_k \geq 0$$

where $\nu(t)$ and $\nu(t+Nt)$ denotes the known trajectory of the first and the last train and the passenger flow is consider constant for all trains. In this case, the trajectories of $(NPF-2)$ trains are obtained repeating the optimization problem, based in known behaviour of $\nu(t+1)$ and $\nu(t+Nt)$ with $Nt = (NPF - Step + 1)$ where $Step = 1, 2, \cdots, (NPF-2)$.

5. PRACTICAL APPLICATION AND SIMULATION RESULTS

Consider metro line in Figure 1, similar to North/South line of São Paulo underground.

Initially it is treated the generation of trajectories for n trains during the following periods in the morning:

- Beginning of the morning (05:00h to 06:00h);
- Period when the passenger flow fluctuates considerably (06:00h to 06:40h);
- Rush-hours in the morning (06:40h to 08:40h).

In the beginning of the morning one has the following statistical mean values of ODM passenger flows:

$$ODM_1 = \frac{1}{10000} * \left[\begin{matrix} 0 & 6 & 24 & 30 & 28 & 58 & 78 & 59 \\ 12 & 0 & 3 & 5 & 8 & 17 & 23 & 18 \\ 12 & 1 & 0 & 1 & 1 & 5 & 7 & 5 \\ 56 & 5 & 2 & 0 & 1 & 6 & 14 & 11 \\ 42 & 6 & 4 & 1 & 0 & 1 & 4 & 6 \\ 19 & 3 & 3 & 1 & 0 & 0 & 1 & 1 \\ 29 & 4 & 5 & 3 & 1 & 1 & 0 & 1 \\ 16 & 3 & 3 & 2 & 1 & 1 & 1 & 0 \\ 14 & 2 & 3 & 2 & 1 & 2 & 1 & 0 \\ 9 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 12 & 2 & 3 & 2 & 1 & 2 & 2 & 1 \\ 29 & 6 & 7 & 4 & 4 & 6 & 7 & 4 \\ 29 & 6 & 7 & 4 & 4 & 6 & 7 & 4 \\ 102 & 19 & 26 & 17 & 14 & 27 & 32 & 19 \\ 152 & 29 & 39 & 26 & 22 & 38 & 46 & 29 \\ 10 & 2 & 3 & 2 & 2 & 3 & 3 & 2 \\ 44 & 9 & 12 & 8 & 7 & 14 & 17 & 13 \\ 30 & 6 & 9 & 7 & 6 & 12 & 15 & 12 \\ 42 & 9 & 13 & 10 & 9 & 18 & 23 & 17 \\ 136 & 31 & 46 & 34 & 31 & 64 & 85 & 64 \end{matrix}\right.$$

$$\left.\begin{matrix} 87 & 43 & 22 & 79 & 79 & 237 & 216 & 39 & 101 & 32 & 14 & 51 \\ 29 & 14 & 8 & 28 & 28 & 83 & 76 & 14 & 37 & 12 & 6 & 21 \\ 9 & 5 & 3 & 9 & 9 & 29 & 27 & 5 & 13 & 4 & 2 & 8 \\ 19 & 11 & 6 & 23 & 23 & 75 & 69 & 13 & 36 & 13 & 6 & 24 \\ 11 & 6 & 3 & 15 & 15 & 51 & 47 & 9 & 25 & 9 & 4 & 17 \\ 3 & 2 & 1 & 6 & 6 & 20 & 18 & 3 & 11 & 4 & 2 & 8 \\ 2 & 2 & 2 & 7 & 7 & 28 & 24 & 5 & 14 & 6 & 3 & 12 \\ 1 & 1 & 1 & 3 & 3 & 12 & 11 & 2 & 8 & 3 & 2 & 7 \\ 0 & 0 & 0 & 2 & 2 & 9 & 9 & 2 & 7 & 3 & 2 & 6 \\ 0 & 0 & 0 & 1 & 1 & 6 & 6 & 1 & 4 & 2 & 1 & 4 \\ 1 & 0 & 0 & 1 & 1 & 7 & 7 & 2 & 7 & 4 & 2 & 7 \\ 5 & 1 & 0 & 0 & 0 & 8 & 16 & 5 & 19 & 10 & 4 & 22 \\ 5 & 1 & 0 & 0 & 0 & 8 & 16 & 5 & 19 & 10 & 4 & 22 \\ 26 & 11 & 5 & 9 & 9 & 0 & 11 & 7 & 49 & 38 & 21 & 93 \\ 41 & 20 & 9 & 31 & 31 & 17 & 0 & 4 & 39 & 37 & 33 & 147 \\ 3 & 2 & 1 & 3 & 3 & 4 & 1 & 0 & 1 & 2 & 2 & 9 \\ 19 & 10 & 6 & 22 & 22 & 50 & 24 & 2 & 0 & 3 & 5 & 43 \\ 20 & 11 & 6 & 25 & 25 & 85 & 49 & 6 & 7 & 0 & 2 & 22 \\ 30 & 17 & 10 & 41 & 41 & 144 & 140 & 19 & 33 & 5 & 0 & 12 \\ 102 & 59 & 36 & 154 & 154 & 579 & 563 & 102 & 262 & 62 & 11 & 0 \end{matrix}\right]$$

In the rush-hours in the morning one has the following statistical mean values of ODM passenger flows:

$$ODM_2 = \frac{1}{1000} * \left[\begin{matrix} 0 & 33 & 113 & 160 & 223 & 717 & 623 & 700 \\ 11 & 0 & 17 & 37 & 87 & 187 & 203 & 297 \\ 14 & 2 & 0 & 3 & 17 & 67 & 73 & 110 \\ 49 & 9 & 4 & 0 & 10 & 60 & 113 & 173 \\ 48 & 16 & 9 & 3 & 0 & 20 & 57 & 143 \\ 23 & 8 & 8 & 3 & 1 & 0 & 10 & 37 \\ 38 & 13 & 14 & 9 & 3 & 3 & 0 & 17 \\ 20 & 7 & 8 & 5 & 3 & 4 & 1 & 0 \\ 26 & 10 & 11 & 8 & 5 & 10 & 4 & 1 \\ 10 & 4 & 4 & 3 & 2 & 4 & 3 & 2 \\ 14 & 6 & 7 & 6 & 4 & 8 & 6 & 5 \\ 23 & 9 & 12 & 10 & 7 & 15 & 12 & 10 \\ 60 & 33 & 50 & 33 & 53 & 70 & 50 & 60 \\ 123 & 67 & 100 & 70 & 123 & 173 & 140 & 177 \\ 160 & 87 & 133 & 93 & 163 & 217 & 173 & 230 \\ 23 & 13 & 20 & 13 & 23 & 33 & 27 & 37 \\ 43 & 23 & 40 & 27 & 50 & 77 & 63 & 93 \\ 50 & 27 & 47 & 33 & 63 & 100 & 87 & 143 \\ 73 & 43 & 73 & 57 & 103 & 163 & 143 & 220 \\ 167 & 103 & 183 & 140 & 267 & 423 & 377 & 580 \end{matrix}\right.$$

$$\left.\begin{matrix} 583 & 300 & 220 & 427 & 427 & 1497 & 673 & 343 & 220 & 47 & 47 & 127 \\ 260 & 167 & 140 & 277 & 277 & 940 & 360 & 130 & 97 & 27 & 27 & 73 \\ 97 & 67 & 57 & 120 & 120 & 410 & 157 & 57 & 43 & 13 & 13 & 37 \\ 157 & 113 & 103 & 220 & 220 & 790 & 307 & 113 & 87 & 27 & 27 & 77 \\ 130 & 97 & 90 & 213 & 213 & 803 & 313 & 117 & 93 & 27 & 27 & 87 \\ 57 & 40 & 40 & 93 & 93 & 393 & 143 & 57 & 50 & 13 & 17 & 47 \\ 37 & 50 & 47 & 113 & 113 & 540 & 200 & 80 & 67 & 23 & 23 & 73 \\ 7 & 13 & 23 & 53 & 53 & 260 & 100 & 43 & 37 & 13 & 13 & 43 \\ 0 & 3 & 10 & 43 & 43 & 223 & 90 & 40 & 40 & 17 & 17 & 43 \\ 0 & 0 & 3 & 13 & 13 & 123 & 57 & 27 & 23 & 10 & 10 & 33 \\ 2 & 0 & 0 & 7 & 7 & 163 & 77 & 43 & 37 & 17 & 17 & 47 \\ 6 & 2 & 1 & 0 & 0 & 120 & 97 & 63 & 53 & 27 & 27 & 80 \\ 60 & 13 & 3 & 0 & 0 & 120 & 97 & 63 & 53 & 27 & 27 & 80 \\ 180 & 77 & 47 & 70 & 70 & 0 & 33 & 47 & 73 & 53 & 60 & 180 \\ 253 & 120 & 77 & 197 & 197 & 133 & 0 & 17 & 40 & 37 & 67 & 200 \\ 43 & 20 & 17 & 47 & 47 & 70 & 7 & 0 & 3 & 3 & 10 & 43 \\ 113 & 53 & 43 & 133 & 133 & 367 & 50 & 7 & 0 & 3 & 10 & 57 \\ 190 & 100 & 77 & 243 & 243 & 1010 & 163 & 30 & 10 & 0 & 7 & 6 \\ 297 & 160 & 130 & 417 & 417 & 1800 & 487 & 110 & 60 & 13 & 0 & 23 \\ 733 & 407 & 337 & 1143 & 1143 & 5223 & 1410 & 420 & 353 & 127 & 30 & 0 \end{matrix}\right]$$

The operational constraints bounds considered are:

$$uplmin_i = [\,28\ 107\ 99\ 79\ 88\ 110\ 100\ 93\ 84\ 74\ 87\ 82\ 73\ 71\ 82\ 66\ 74\ 111\ 78\ 88\ 118\ 74\ 79\ 111\ 71\ 72\ 84\ 76\ 67\ 82\ 83\ 73\ 74\ 91\ 102\ 111\ 85\ 80\ 96\ 107\,]$$

$$uplmax_i = [\,28\ 110\ 100\ 80\ 89\ 116\ 100\ 94\ 86\ 75\ 92\ 88\ 75\ 76\ 84\ 74\ 75\ 112\ 81\ 92\ 118\ 78\ 81\ 113\ 74\ 74\ 85\ 82\ 74\ 82\ 90\ 79\ 78\ 94\ 106\ 115\ 86\ 81\ 96\ 107\,]$$

$upmax_i(k)$=50 , $upmin_i(k)$=15 , $MP_i(k) = 0.2$
$xmax_i(k)$=300 , $xmin_i(k)$=90 , $\delta = 10$
$pmax_i(k)$=2800 , $pmaxx(k) = 2600$
$p^r(k) = 2400$, $\sigma_1 max$=40 , $\sigma_2 max$=10
$\zeta_1 max$=15 , $\zeta_2 max$=5

Figure 2 presents headway profile at yard for 40 trains in the morning (between 05:00h and 07:00h). For MPC1 the trajectory for the first train is solved using the optimization problem (16) assuming constant headway. The other trajectories were obtained solving the MPC problem (20). Figure 3 ilustrates the passenger load profile along stations of the 30° dispatched train. The following results were obtained:

$$x(yard) = 98.8547\ (s),\ x_{medium} = 97.9913\ (s)$$
$$nT = 40.8790 = 41\ (trains)$$
$$p_i(30) = 2578\ (pass.),\ \ p_{medium} = 1628\ (pass.)$$
$$up_{medium} = 15.7771\ (s),\ upl_{medium} = 85.2500\ (s)$$

Parameters used in all dispatches are: q=0.1; r=1; s=10; s_1=10; $s_2 = 10^6$; and z=1.

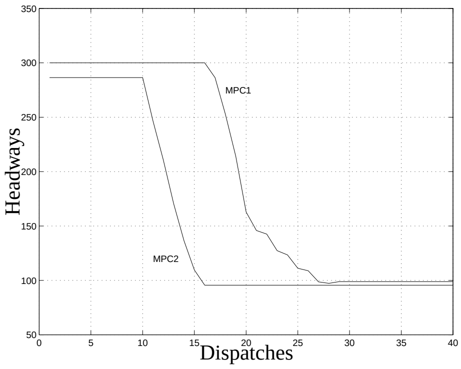

Fig. 2. Headway Profiles at Yard

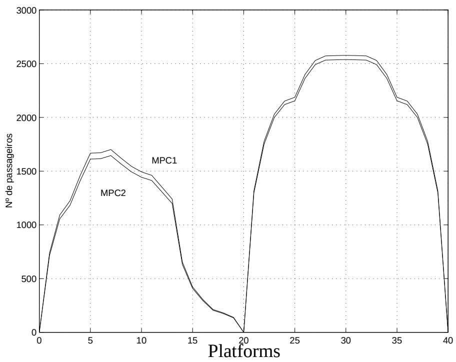

Fig. 3. Passenger Load Profiles of 30th Train

For MPC2 in Figure 2 on has the headway profiles using the optimization problem (16) assuming constant headway in the beginning of the morning and in the rush-hours. During the transition period one has NP steps as proposed in (20) and (21) and (NPF-2) steps using the model proposed in (22). In this case were used $NP = 16$, $NPF = 4$, q=100; r=10; s=0.1; s_1=10; $s_2 = 10^6$; and z=1. Figure 3 presents the passenger load profile at stations for the 30th train. The following results were obtained:

$$x(yard) = x_{medium} = 95.5729\ (s),\ nT = 42\ (trains)$$
$$p_i(30) = 2539\ (pass.),\ \ p_{medium} = 1593\ (pass.)$$
$$up_{medium} = 15.1016\ (s),\ upl_{medium} = 85.2500\ (s)$$

Comparing MPC1 and MPC2 results it can be noticed that MPC1 presents higher headway values than MPC2. Hence, MPC1 presents higher passenger load, reducing the number of trains in service during part of the morning. Moreover, the solution using MPC2 is computationally more expensive due to higher number of variables and equations along the NPF steps of the transition stage.

6. CONCLUSION

The proposed approach for generation of time schedules for metro lines is based on a model predictive control formulation with receding horizon and consider explicitly the operational constraints and control margin for on-line traffic regulation. The performance index using piecewise linear functions clearly contribute to computational effectiveness of proposed approach, allowing determination of train time schedules for all day long. The performance of the proposed approach was evaluated using a metro line similar to North/South line of São Paulo underground. The computational efficiency of the approach makes it applicable to on-line time schedule adaptation to disturbances in passenger flows during comercial operation.

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