15th Triennial World Congress, Barcelona, Spain

DISTURBANCE ATTENUATION FOR A CLASS OF NONLINEAR SYSTEMS VIA DISTURBANCE-OBSERVER-BASED APPROACH

Lei Guo and Wen-Hua Chen

Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK.
email:l.guo@lboro.ac.uk, w.chen@lboro.ac.uk

Abstract: Disturbance attenuation for a class of nonlinear systems under unknown disturbances is considered in this paper. Based on the Disturbance-Observer-Based Control (DOBC) concept, two design schemes are proposed for external disturbances generated by exogenous systems where the disturbances could be unbounded. One design scheme is obtained by augmenting the estimation of the disturbance into a state estimation and the other is to design a disturbance observer separately and then integrate it with a controller. It is shown that under both schemes, disturbances can be effectively rejected and global stability is guaranteed under certain conditions. Simulations on a flight control system show the efficiency of the approach.

Keywords: Disturbance attenuation; nonlinear systems; stability; observer design; optimization

1. INTRODUCTION

Analysis and synthesis of nonlinear control systems has been one of the most active research areas in the past decades. A lot of elegant approaches have been presented and applied in practical engineering. Many of them focus on the disturbance attenuation, for example, nonlinear output regulation theory (Isidori and Byrnes, 1990), stochastic nonlinear control theory (Basar and Bernhard, 1995), nonlinear H^∞ control (van der Schaft, 1992). In many cases only the stability of the nominal system without disturbances can be guaranteed. It is noticed that stability can be damaged in the presence of disturbances (Slotine and Li, 1991).

Some classical control approaches provide simple design methods to deal with disturbance but lack of sound theoretical justifications. Others are established on rigorous mathematical basis but are only suitable for nonlinear system with specific structures or have too large computation burden to use in practical engineering. For example, Partial Differential Equations (PDE's) have to be solved in the above mentioned methods.

DOBC appeared in the late of 80's and has found its application in many areas such as robot manipulators (Nakao *et al.*, 1987; Li and

Van Den Bosch, 1993; Chan, 1995; Kim *et al.*, 1996; Oh and Chung, 1999). However, only linear DOBC was concerned although these plants possess strong nonlinearity. That is, instead of designing a nonlinear control law to compensate for the nonlinearities in the dynamic systems, the nonlinearities are treated as a part of disturbances added on a linear plant. Obviously, for a nonlinear system, a nonlinear DOBC can improve the performance and robustness greatly against noises and unmodelled dynamics (Oh and Chung, 1999; Chen *et al.*, 2000). New nonlinear DOBC schemes have been proposed for robots with constant disturbance or for systems with harmonic disturbances(Chen *et al.*, 2000; Chen, 2001).

This paper addresses disturbance observer based control (DOBC) approach to disturbance attenuation for a class of nonlinear systems. After reformulating the DOBC design problem, two LMI-based schemes are proposed. Several restrictions on the existing nonlinear DOBC results are relaxed. Simulations on a flight control system show the efficiency of the approach.

2. PROBLEM STATEMENT

Consider a nonlinear dynamic system with unknown disturbances

$$\left\{ \begin{array}{rcl} \dot{x}(t) &=& A_0x(t) + F_{01}f_{01}(x,t) \\ && +B_0\left[u(t) + d(t)\right] \\ y(t) &=& C_0x(t) + F_{02}f_{02}(x,t) + D_0d(t) \end{array} \right. ,(1)$$

where $x \in \mathbb{R}^n$, $d \in \mathbb{R}^{m_1}$, $u \in \mathbb{R}^{m_2}$ and $y \in \mathbb{R}^{p_1}$ are the state, the unknown disturbance, the control input and the measurement output, respectively. Similar to the regulation theory, it is supposed that the unknown external disturbance d is generated by an exogenous system described by

$$\left\{ \begin{array}{rl} \dot{w}\ (t) &= Ww(t) \\ d(t) &= Vw(t) \end{array} \right. . \quad (2)$$

Many kinds of disturbances in engineering can be described by this model, for example, unknown constant and harmonics with unknown phase and magnitude. It should be noticed that most existing results in differential geometry and Lyapunov theory are restricted to bounded exogenous signals,.i.e, exogenous systems being neutral stable (Isidori and Byrnes, 1990; Zheng *et al.*, 2000). There is no such an assumption for DOBC.

Assumption 1. The mappings $f_{0i}(x,t)$ $(i = 1,2)$ are smooth nonlinear functions satisfying $f_{0i}(0,t) = 0$, $\|f_{0i}(x_1,t) - f_{0i}(x_2,t)\| \leq \|U_i(x_1 - x_2)\|$, for $i = 1,2$ and $t \geq 0$ where U_i are given constant weighting matrices.

A variety of nonlinear systems can be described by the model (1) satisfying Assumption 1. A system with weak nonlinearity, certainly, can be represented by this model. For a system with strong nonlinearity, after a nonlinear control technique like feedback linearization, dynamic inversion control and gain scheduling technique is applied, the majority of the nonlinearity has been cancelled and the resulting system can be described by this model. In the meantime, some nonlinear systems can be transferred into this form through equivalent transformations. Actually, the research on this model was mainly motivated from our work on applying disturbance observer based control on robotics where a nonlinear disturbance is integrated with Computed Torque Control (Chen *et al.*, 2000; Chen, 2000). The Computed Torque Control is designed to linearize a manipulator. However, due to the variation of the tip mass (load) of the manipulator, only the majority of the nonlinearity in dynamics is cancelled by Computed Torque Control. The remaining nonlinearity satisfies the bounded condition in Assumption (1). Therefore the manipulator under this control law can be well described by system (1) with Assumption 1.

The problem to be considered in the DOBC theory is to design an observer for the nonlinear system (1) to *estimate* the unknown disturbance d and then design a proper controller to *reject* the disturbance using its estimation. Two approaches to this problem will be developed in Section 3 and 4, respectively.

3. DOBC USING FULL-ORDER OBSERVERS

By augmenting the system's state equations (1) with the disturbance dynamics equations (2), the composite system is given by

$$\left\{ \begin{array}{rl} \dot{z}\ (t) &= Az(t) + F_1f_1(z,t) + Bu \\ y(t) &= Cz(t) + F_2f_2(z,t) \end{array} \right. \quad (3)$$

where

$$z(t) := \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, A := \begin{bmatrix} A_0 & B_0V \\ 0 & W \end{bmatrix},$$

$$F_1 := \begin{bmatrix} F_{01} \\ 0 \end{bmatrix}, f_1(z,t) = f_{01}(x,t)$$

$$F_2 = F_{02}, f_2(z,t) = f_{02}(x,t)$$
$$B := \begin{bmatrix} B_0 \\ 0 \end{bmatrix}, C := \begin{bmatrix} C_0 & D_0 V \end{bmatrix}$$

To make the control problem well posed, the following assumption is also necessary.

Assumption 2. (A, C) and $(W, B_0 V)$ is observable.

Its full-order observer is designed as

$$\begin{cases} \dot{\hat{z}}(t) &= A\hat{z}(t) + F_1 f_1(\hat{z}, t) + Bu + \\ & \quad + L(\hat{y} - y) \\ \hat{y}(t) &= C\hat{z}(t) + F_2 f_2(\hat{z}, t) \end{cases} \quad (4)$$

where

$$\hat{z}(t) := \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix}, L := \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$
$$f_1(\hat{z}, t) = f_{01}(\hat{x}, t), f_2(\hat{z}, t) = f_{02}(\hat{x}, t)$$

and L is the observer gain to be determined.

The estimation error

$$e := z - \hat{z} := \begin{bmatrix} e_x \\ e_w \end{bmatrix} := \begin{bmatrix} (x - \hat{x}) \\ (w - \hat{w}) \end{bmatrix} \quad (5)$$

is governed by

$$\begin{aligned} \dot{e} = &(A - LC)e + F_1[f_1(z,t) - f_1(\hat{z}, t)] \\ &+ LF_2[f_2(z,t) - f_2(\hat{z}, t)]. \end{aligned} \quad (6)$$

In the DOBC scheme, the control consists of two parts: one is to compensate for the disturbance d and the other is to stabilize the system and achieve performance specification. It is determined by

$$u = -\hat{d} + K\hat{x} \quad (7)$$

where the disturbance estimation is given by

$$\hat{d} = V\hat{w} = \begin{bmatrix} 0 & V \end{bmatrix} \hat{z}, \quad (8)$$

and K is the control gain to be determined.

Combining the estimation error equation (6) with the plant (1) yields

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_0 + B_0 K & \overline{B} \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} F_{01} & 0 & 0 \\ 0 & F_1 & LF_2 \end{bmatrix} \begin{bmatrix} f_1(z,t) \\ f_1(z,t) - f_1(\hat{z}, t) \\ f_2(z,t) - f_2(\hat{z}, t) \end{bmatrix} \quad (9)$$

where

$$\overline{B} := \begin{bmatrix} -B_0 K & B_0 V \end{bmatrix}. \quad (10)$$

It is noted that

$$\left\| \begin{bmatrix} f_1(z,t) \\ f_1(z,t) - f_1(\hat{z}, t) \\ f_2(z,t) - f_2(\hat{z}, t) \end{bmatrix} \right\| \leqslant \left\| \begin{bmatrix} U_1 & 0 & 0 \\ 0 & U_1 & 0 \\ 0 & U_2 & 0 \end{bmatrix} \begin{bmatrix} x \\ e_x \\ e_w \end{bmatrix} \right\|.$$

Our objective now is to *find L and K such that system (9) is exponentially stable.*

For shortening description, we denote

$$\overline{F}_1 := \begin{bmatrix} P_2 F_1 & R_2 F_2 \end{bmatrix},$$
$$\overline{U}_1 := \begin{bmatrix} U_1 & 0 \end{bmatrix}, \overline{U}_2 := \begin{bmatrix} U_2 & 0 \end{bmatrix}$$
$$\mathrm{sym}(M) := M + M^T.$$

Theorem 1. Consider the system (1) under the exogenous disturbance (2) satisfying Assumption 1 and 2. If there exist $\lambda_1 > 0$, $Q_1 > 0$ and R_1 satisfying

$$\begin{bmatrix} sym(A_0 Q_1 + B_0 R_1) + \lambda_1^2 F_{01} F_{01}^T & \frac{1}{\lambda_1} Q_1 U_1^T \\ \frac{1}{\lambda_1} U_1 Q_1 & -I \end{bmatrix} < 0, \quad (11)$$

and $\lambda_2 > 0$, $P_2 > 0$ and R_2 satisfying

$$\begin{bmatrix} sym(P_2 A - R_2 C) + \frac{1}{\lambda_2^2} \overline{U}_1^T \overline{U}_1 + \frac{1}{\lambda_2^2} \overline{U}_2^T \overline{U}_2 & \lambda_2 \overline{F}_1 \\ \lambda_2 \overline{F}_1^T & -I \end{bmatrix} < 0, \quad (12)$$

then the closed-loop system (9) under the DOBC consisting of the control law (7) with gain $K = R_1 Q_1^{-1}$ and the observer (4) with gain $L = P_2^{-1} R_2$ is exponential stable.

Proof: Let

$$V_1(x,t) = x^T Q_1^{-1} x + \frac{1}{\lambda_1^2} \int_0^t \left[\|U_1 x\|^2 - \|f_1(x, \tau)\|^2 \right] d\tau,$$

$$\begin{aligned} V_2(e,t) = &e^T P_2 e \\ &+ \frac{1}{\lambda_2^2} \int_0^t \left[\|U_1 e_x\|^2 - \|f_1(z,\tau) - f_1(\hat{z}, \tau)\|^2 \right] d\tau \\ &+ \frac{1}{\lambda_2^2} \int_0^t \left[\|U_2 e_x\|^2 - \|f_2(z,\tau) - f_2(\hat{z}, \tau)\|^2 \right] d\tau. \end{aligned}$$

Along with the trajectories of (9), firstly we have

$$\begin{aligned}\dot{V}_2\ (e,t) =& e^T(P_2(A-L_2C)+(A-L_2C)^TP_2)e\\ &+2e^TP_2\left[F_1(f_1(z,t)-f_1(\widehat{z},t))\right.\\ &\left.+LF_2(f_2(z,t)-f_2(\widehat{z},t)))\right]\\ &+\frac{1}{\lambda_2^2}\left[\|U_1e_x\|^2-\|f_1(z,t)-f_1(\widehat{z},t)\|^2\right]\\ &+\frac{1}{\lambda_2^2}\left[\|U_2e_x\|^2-\|f_2(z,t)-f_2(\widehat{z},t)\|^2\right]\\ \leq& 2e^T\left[P_2(A-L_2C)+(A-L_2C)^TP_2\right.\\ &+\lambda_2^2P_2F_1F_1^TP_2+\lambda_2^2P_2LF_2F_2^TL^TP_2\\ &+\frac{1}{\lambda_2^2}\overline{U}_1^T\overline{U}_1+\frac{1}{\lambda_2^2}\overline{U}_2^T\overline{U}_2)e\\ \leq& -\eta_1\|e\|^2\end{aligned}$$

which can be guaranteed by (12) using Schur complement, where $\eta_1 > 0$ is a proper constant.

Similarly, for (9) in the absence of the presence of e, based on (11) we can find a proper constant η_2 such that $(P_1 = Q_1^{-1})$

$$\begin{aligned}\dot{V}_1\ (x,t) \leq& x^T\left[P_1(A_0+B_0K)+(A_0+B_0K)^TP_1\right.\\ &\left.+\lambda_1^2P_1F_{01}F_{01}^TP_1+\frac{1}{\lambda_1^2}U_1^TU_1\right]x\\ \leq& -\eta_2\|x\|^2,\end{aligned}$$

which also means the pure stabilization problem for (1) without disturbances d is solvable. If (12) and (11) hold, then there exists $\eta_3 > 0$ involved in P_1 and $\overline{B}$ such that

$$2x^TP_1\overline{B}e \leq \eta_3\|x\|\|e\|.$$

Denote a Lyapunov function candidate for (9) as

$$V(x,e,t) = V_1(x,t) + \eta_0V_2(e,t),$$

where

$$\eta_0 = \frac{\eta_3^2}{4\eta_1\eta_2}.$$

Thus, along the closed-loop system (9), we have

$$\begin{aligned}\dot{V}\ (x,e,t) \leq& -\eta_0\eta_1\|e\|^2-\eta_2\|x\|^2+2x^TP_1\overline{B}e.\\ \leq& -\eta_0\eta_1\|e\|^2-\eta_2\|x\|^2+\eta_3\|x\|\|e\|\\ =& -(\sqrt{\eta_2}\|x\|+\frac{\eta_3}{2\sqrt{\eta_2}}\|e\|)^2\\ \leq& -\min\{\sqrt{\eta_2},\frac{\eta_3}{2\sqrt{\eta_2}}\}\left\|\begin{bmatrix}x\\ e\end{bmatrix}\right\|^2.\end{aligned}$$

Correspondingly the closed-loop system is asymptotically stable. Q.E.D

Remark 1. The observer and DOBC design which can reduces to LMI's can be shown as follows separately: (i) the observer gain L can be obtained via (12) together with P_2 and R_2. (ii) solve (11) to obtain Q_1, R_1 and K. Finally we can construct the control law as (7).

4. DISTURBANCE OBSERVERS

One of the features of the DOBC approach is the flexibility in the design of the disturbance observer, which can be used to enhance the disturbance attenuation ability of existing nonlinear or linear controllers that cannot deal with disturbance directly. In many cases, one may prefer to designing a disturbance observer separately from the controller design. When all states for a system are available like position and velocity in manipulators, it is unnecessary to estimate them. In the following, it is assumed that only the estimation of the disturbance is concerned and all the states are available.

In this section the disturbance observer is proposed as

$$\widehat{d} = V\widehat{w}, \widehat{w} = v - Lx$$

where v is the auxiliary variable satisfying

$$\begin{aligned}\dot{v} =& (W+LB_0V)(v-Lx)\\ &-L(-A_0x-B_0u-F_{01}f_{01}(x,t)).\end{aligned} \tag{13}$$

Comparing system dynamics (1) and the disturbance (2) with the observer (13) yields

$$\dot{e_w} = (W+LB_0V)e_w \tag{14}$$

where e_w is defined as in (5).

When the DOBC law $u = -\widehat{d}+Kx$ is applied, the composite system under the disturbance observer (13) and the DOBC is given by

$$\begin{aligned}\begin{bmatrix}\dot{x}\\ \dot{e_w}\end{bmatrix} =& \begin{bmatrix}A_0+B_0K & B_0V\\ 0 & W+LB_0V\end{bmatrix}\begin{bmatrix}x\\ e_w\end{bmatrix}\\ &+\begin{bmatrix}F_{01}\\ 0\end{bmatrix}f_{01}(x,t)\end{aligned} \tag{15}$$

It is obviously that the observer error dynamics is separated from the controller design.

Theorem 2. Consider the DOBC of the system (1) under disturbance (2). Suppose that there exist $\lambda > 0$, $Q_1 > 0$, $P_2 > 0$, R_1 and R_2 satisfying

$$\begin{bmatrix} \begin{array}{c} \text{sym}(A_0Q_1 + B_0R_1) \\ +\lambda^2 F_{01}F_{01}^T \end{array} & \dfrac{1}{\lambda}Q_1U_1^T \\ \dfrac{1}{\lambda}U_1Q_1 & -I \end{bmatrix} < 0, \quad (16)$$

and

$$\text{sym}(P_2W + R_2B_0V) < 0, \quad (17)$$

the closed-loop system (15) under the DOBC with gain $K = R_1Q_1^{-1}$ and the disturbance observer (4) with gain $L = P_2^{-1}R_2$ is exponentially stable.

Proof is omitted for brevity.

Remark 2. One of the features of the DOBC for linear systems is that the design of the disturbance observer is separated from the controller design. The approach discussed in this paper extends this feature from linear systems to a class of nonlinear systems.

5. AN EXAMPLE

The longitudinal dynamics of A4D at a flight condition of 15,000 ft altitude and 0.9 Mach can be given by (see (Petersen and Hollot, 1986) and (McRuer *et al.*, 1976, P259))

$$\dot{x}(t) = A_0x(t) + f(x,t) + B_0\left[u(t) + d(t)\right]$$

where x_1 is the forward velocity (ft s^{-1}), x_2 is the angle of attack (rad), x_3 is the pitching velocity (rad s^{-1}), x_4 is the pitching angle (rad), u is elevator deflection (deg) and

$$A_0 = \begin{bmatrix} -0.0605 & 32.37 & 0 & 32.2 \\ -0.00014 & -1.475 & 1 & 0 \\ -0.0111 & -34.72 & -2.793 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_0^T = \begin{bmatrix} 0 & -0.1064 & -33.8 & 0 \end{bmatrix}.$$

The eigenvalues of A_0 are $-2.1250 \pm j5.8510$ and -0.0039 ± 0.0896, which means the nominal system without nonlinearity is close to instability.

Since there may be a large degree of uncertainty in the $(3,2)$ entry of A_0, similar to Petersen and Hollot (1986), we suppose that

$$f(x,t) = F_{01}f_0(x,t),$$

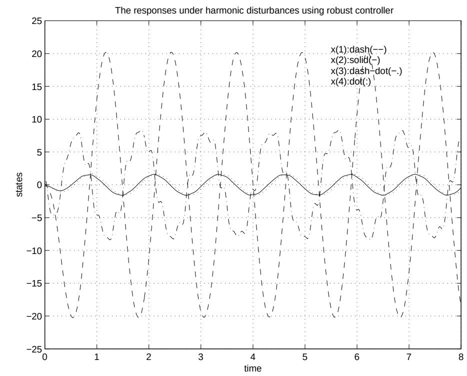

Fig. 1. Response of the states under disturbances using robust control

where

$$f_0(x,t) = \begin{bmatrix} 0 & \sin(10\pi t)x_2 & 0 & 0 \end{bmatrix}^T,$$

$$F_{01} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, U_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

and $\|f_0(x,t)\| \leq \|U_1x\|$.

$d(t)$ is assumed to be an unknown harmonic disturbance described by (2) with

$$W = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}, V = \begin{bmatrix} 25 & 0 \end{bmatrix}.$$

If the full states can be measured and apply the approach in Section 4, we can get

$$K = \begin{bmatrix} 2.3165 & 9.9455 & 4.0004 & 13.8525 \end{bmatrix},$$

$$L = \begin{bmatrix} 0 & 0.1255 & 0.0008 & 0 \\ 0 & 0.6470 & -0.0019 & 0 \end{bmatrix},$$

and the controller is $u = -\widehat{d} + Kx$.

Suppose that the initiate value

$$x(0) = \begin{bmatrix} 0.2 & -0.2 & 0.3 & -0.2 \end{bmatrix}$$

is taken. The robust control law obtained in (Petersen and Hollot, 1986) is firstly applied. Figure 1 shows that quite poor performance is yielded although the closed-loop system remains stable. Figure 2 shows that satisfactory performance and stability under the disturbance and unmodelled nonlinearity are achieved by the DOBC scheme developed in this paper.

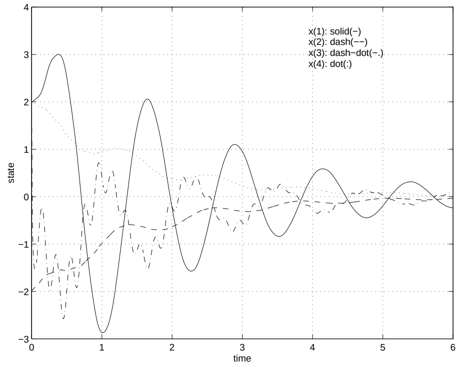

Fig. 2. Response of the states under disturbances using DOBC control

6. CONCLUSION

New Disturbance-Observer-Based (DOB) control approaches are presented for a class of nonlinear systems with possibly unbounded disturbances. Two disturbance observer design methods are provided which also have potential significances in some other areas like noise control. LMI-based design procedures are proposed. Based on the estimation of disturbances, the composited control law can guarantee the closed-loop systems globally stable at the presence of disturbances. Compared with other existing control methods for nonlinear systems with unknown disturbances, the DOBC approach has several advantages:

- No PDEs or PDIs has to be solved for controller design;
- Some restrictions on the existing nonlinear DOBC results are removed;
- Unknown disturbances are not necessary to be bounded;
- Global stability rather than local one might be achieved;
- Design procedure is relatively simple and easy to accept by engineers.

ACKNOWLEDGMENT

This part was supported by Engineering and Physical Science Research (EPSRC) under the grant GR/N31580.

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