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INTERVAL OBSERVERS WITH GUARANTEED CONFIDENCE LEVELS

APPLICATION TO THE ACTIVATED SLUDGE PROCESS

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Abstract. This paper deals with the design of robust observers for uncertain models, with application to the activated sludge process. We assume that the model is such that the bacterial growth rate is unknown, and the measurements and influent flow rates are disturbed. However we suppose that the upper and lower bounds of the uncertainties are known. Under appropriate hypotheses, we are able to build interval observers giving dynamic bounds containing the variables to estimate. Besides, if we know a priori the probability density of each uncertain parameter, we can synthesize interval observers with guaranteed confidence levels and provide a confidence density for the state. We apply this approach to the sludge process and compare the results to the probability density estimation obtained with Monte-Carlo simulations.

Keywords : uncertain models, non-linear estimation, interval observers, wastewater treatment, activated sludge process, biological models.

1. INTRODUCTION

One of the main limitations to the improvement of monitoring and optimization of bioreactors is probably due to the difficulty to measure chemical and biological variables. Indeed there are very few sensors which are at the same time cheap and reliable and that can be on-line used. The measurement of some biological variables (biomass, cellular quota, etc) is sometimes very difficult and can necessitate complicated and sophisticated operations.

The development of observers addresses this issue by estimating the internal state of a bioreactor. It relies both on a model of the system and on the available on-line measurements [1]. Nevertheless these methods are often disappointing when dealing with bioprocesses since observers are based on models which are often rough approximations and also because the used measurements are often corrupted by a high level of noise. As a consequence, it can be difficult to interpret the observer predictions.

To cope with the uncertainties which characterize the biological systems, a first class of observers was proposed by [2] on the principle of unknown input observers [3, 4]. The biological kinetics was considered as an unknown input and the observer construction did not use it. This approach provided more robustness to the observers, but managed less easily the uncertainty on the mass inputs or on the yield coefficients.

A complementary approach called "interval observers" was then proposed [5, 6, 7], based on the principle of cooperative systems [8]. This approach assumes a bounded uncertainty and provides tools to determine the bounds in which the state must lie. It is therefore a very robust approach, but it can be too conservative. In this paper, we use this approach to try to determine a more precise information related to the probability density of the state. The idea is to use statistical models on the probability densities of the unknowns (initial conditions, parameters, inputs, measurements) in order to estimate the probability density of the state. In general, this is a very difficult problem, and we propose therefore to approximate this probability density by a confidence level approach where we can give an upper bound for the probability of the state to lie in a given interval.

The paper is organized as follows. We fist recall the principles of the interval observers. Then we define the notion of confidence levels and we explain how to compute the intervals associated to a confidence level. We illustrate the method on a bioprocess used to process the wastewater: the activated sludge. We compare then the results with a Monte-Carlo approach and we show that the intervals associated to confidence levels provide a good estimate of the probability density of the state.

2. THE ACTIVATED SLUDGE PROCESS

The activated sludge process used for biological wastewater treatment consists of two tanks (FIG. 1). The main plant component, the aerator, is an aerobic biological reactor in which the substrate is biodegraded by suspended micro-organisms. This bioreactor is supposed to be continuously stirred, so that the substrate concentration is homogeneous. This substrate (organic matter) is consumed by a biomass which agglomerates into flocks: the activated sludge. We assume that the total solid part is separated by sedimentation of these flocks in the settler linked to the aerator. A fraction of the sludge collected in the settler is recycled to the aeration tank, whereas the remaining sludge is wasted to other sludge treatment units. The recycling increases the biomass concentration in the aeration tank, and extends the mean sludge residence time for the adaptation of micro-organisms to the available nutrients.

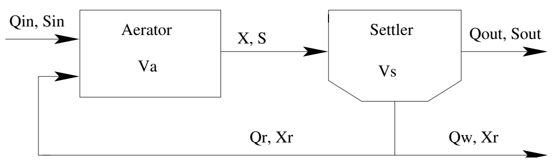

FIG. 1 – *Functional diagram of the activated sludge process.*

In the aerator, we consider that a single bacterial population *x* is growing on one limiting organic substrate *s*. We assume that it is the only biological reaction of the process. We also suppose that the organic matter does not settle in the sedimentation tank. The mass balance of the various constituents leads to the simplified model of activated sludge process [2, 5]:

$$\left\{\begin{array}{l} \dot{x} = \mu(\cdot)x - \big(1+q_r\big)D(t)x + q_r D(t)x_r \\ \dot{s} = -\dfrac{\mu(\cdot)x}{Y_s} - \big(1+q_r\big)D(t)s + D(t)s_{in}(t) \\ \dot{x_r} = w\big(1+q_r\big)D(t)x - w\big(q_w+q_r\big)D(t)x_r \end{array}\right. \quad (1)$$

with the following notations:

$$D(t) = \frac{Q_{in}}{V_a} \quad ; q_r = \frac{Q_r}{Q_{in}} \quad ; q_w = \frac{Q_w}{Q_{in}} \quad ; w = \frac{V_a}{V_s}$$

where *x*, *s* and x_r are the model state variables representing respectively the biomass, the substrate and the recycled biomass concentrations. Q_{in}, Q_{out}, Q_r, Q_w are respectively the influent, effluent, recycle and waste flow rates. V_a and V_s are the constant aerator and settler volumes. s_{in} represents the influent substrate concentration. Y_s corresponds to the yield coefficient of the growth of biomass on substrate. The initial conditions are respectively x_0, s_0, x_{r0}.

3. INTERVAL OBSERVERS

3.1. Recall on the interval observers [6]

We assume here that the disturbances and uncertainties are bounded, and that these bounds are known. We derive the dynamic bounds on the state variables from the bounds on the uncertainties. Thus, we compute two estimates bounding the state variables: an upper bound and a lower bound. Here, we limit our study to a very specific case of a linear system with uncertain input: it will suffice for our application to the sludge process. For a far more general setting, see [9].

We consider the system:

$$\mathcal{S}\left\{\ \dot{x} = Ax + B\phi(t); \quad x(t_0) = x_0\right.$$

with $x \in \mathcal{X} \subset \mathbb{R}^n$, $A \in \mathcal{M}^{n\times n}(\mathbb{R})$ $(n \geq 2)$ and $B \in \mathcal{M}^{n\times p}(\mathbb{R})$. Indeed, this system will correspond in the sequel to a reduced order observer, and $\phi \in \mathbb{R}^p$ will be the measured output of the whole system.

We suppose that the input is uncertain with known bounds ϕ^-, ϕ^+ such that:

$$\phi^-(t) \;\leq\; \phi(t) \;\leq\; \phi^+(t), \quad \forall t \in \mathbb{R}^+$$

Remark: The inequalities applied to vectors must be considered term by term.

Under this assumption, we build two asymptotic observers, using the detectability of the system [1, 2].

Definition 1 *Let us consider the system* (S). *The pair of systems* (S^-,S^+) *with:*

$$\mathcal{S}^-\left\{\begin{array}{l} \dot{x^-} = Ax^- + B^-(\phi^-(t),\phi^+(t)) \\ x^-(t_0) = x_0^- \end{array}\right.$$

$$\mathcal{S}^+\left\{\begin{array}{l} \dot{x^+} = Ax^+ + B^+(\phi^-(t),\phi^+(t)) \\ x^+(t_0) = x_0^+ \end{array}\right.$$

is an interval estimator for the system (S) *if for any compact set* $\mathcal{X}_0 \subset \mathcal{X}$, *the coupled system* (S,S^-,S^+) *verifies for any initial condition* $x(t_0) \in \mathcal{X}_0$:
$\forall t \geq t_0, \quad x^-(t) \;\leq\; x(t) \;\leq\; x^+(t)$

Function B^+ *(respectively* B^-*) is such that:*

$$B^-(\phi^-(t),\phi^+(t)) \leq B\phi(t) \leq B^+(\phi^-(t),\phi^+(t))$$

We remark that we need an estimate of x_0 and x_{r0} at initial time t_0, but this estimate can be very loose.

Let us define the upper error $E^+ = \hat{x}^+ - x$ and the lower error $E^- = x - \hat{x}^-$.

Lemma 1 *If the matrix* A *is cooperative (i.e. has positive off-diagonal elements), then:*

$$E^+(t_0) \geq 0 \Rightarrow E^+(t) \geq 0, \;\; \forall t \geq t_0.$$

We prove this lemma using the comparison theorem for cooperative systems [8]. More intuitively, it can be noticed that the vector field is repulsive on the boundaries. We have similar properties for the lower error, and consequently for the total error $E(t) = E^+(t) + E^-(t)$. The following theorem is a particular case of a theorem in [6].

Theorem 1 *If A is stable and cooperative, and if we have an initial estimation: $x_0^- \leq x(t_0) \leq x_0^+$, then system (S^-,S^+) is an interval estimator for the model (S).*
Moreover, if the total error is bounded by a positive vector M:

$$B^+(\phi^-(t),\phi^+(t)) - B^-(\phi^-(t),\phi^+(t)) \leq M$$

then the total error $E(t)$ is asymptotically lower (term by term) than the non-negative vector:

$$E_{max} = -A^{-1}M \tag{2}$$

The proof of this theorem is a consequence of Lemma 1, and (2) follows from the differential vector inequality between $\dot{E}$ and $\dot{E}_{max}$.

3.2. Application to the sludge process

We assume the following hypotheses for model (1):

– the specific growth rate $\mu(\cdot)$ is unknown.
– the substrate concentration *s* is the only measurable state variable, and the measurement is noisy: $y(t) = s(t) + b(t)$.
– we know the bounds on the uncertainty of measurement:

$$\forall\ t \geq t_0, \quad b^- \leq b(t) \leq b^+$$

which implies:

$$s^-(t) = y(t) - b^+ \leq s(t) \leq s^+(t) = y(t) - b^-$$

– the bounds on the inflow $s_{in}(t) = s_{in}^\star(t) + \delta s_{in}$ are known:

$$s_{in}^-(t) \leq s_{in}(t) \leq s_{in}^+(t)$$

– bounds on initial values x_0 and x_{r0} are known.

A similar case has already been discussed in [10] without any noise on measurements. We apply now the results presented in , with a slight difference due to the additional scalar term $D(t)$: it is easy to see that it does not change anything since this term is always positive.

Firstly, we build an asymptotic observer [2, 11] for the set of equations (1) in order to eliminate the unknown function $\mu(\cdot)$ by the following change of variable:

$$Z = X + \begin{bmatrix} Y_s \\ 0 \end{bmatrix} s \ \text{ with } Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, X = \begin{bmatrix} x \\ x_r \end{bmatrix} \tag{3}$$

and we obtain the reduced system:

$$\dot{Z} = D(t)[AZ + B(s,t)], \quad Z_0 = \begin{bmatrix} x_0 + Y_s.s_0 \\ x_{r0} \end{bmatrix} \tag{4}$$

$$A = \begin{bmatrix} -(1+q_r) & q_r \\ w(1+q_r) & -w(q_w+q_r) \end{bmatrix};$$

$$B(s,t) = \begin{bmatrix} Y_s.s_{in}(t) \\ -Y_s.w(1+q_r).s \end{bmatrix}$$

We build the two estimators for system (4):

$$\begin{cases} \dot{\widehat{Z}}^+ = D(t)[A\widehat{Z}^+ + B^+(s^-,s_{in}^+)]; \ \widehat{Z}_0^+ = X_0^+ + \begin{bmatrix} Y_s \\ 0 \end{bmatrix} s_0^+ \\ \dot{\widehat{Z}}^- = D(t)[A\widehat{Z}^- + B^-(s^+,s_{in}^-)]; \ \widehat{Z}_0^- = X_0^- + \begin{bmatrix} Y_s \\ 0 \end{bmatrix} s_0^- \\ X^+ = Z^+ - \begin{bmatrix} Y_s \\ 0 \end{bmatrix} s^- \\ X^- = Z^- - \begin{bmatrix} Y_s \\ 0 \end{bmatrix} s^+ \end{cases} \tag{3}$$

with

$$B^+(s^-,t) = \begin{bmatrix} s_{in}^+(t) \\ -w(1+q_r).s^-(t) \end{bmatrix} .Y_s$$

$$B^-(s^+,t) = \begin{bmatrix} s_{in}^-(t) \\ -w(1+q_r).s^+(t) \end{bmatrix} .Y_s$$

Matrix A is stable and cooperative, therefore hypotheses of Theorem 1 are fulfilled. As a result the estimators (5) define an interval observer for the system (1). On Figure 2 we obtain estimations for the unmeasured state variables *x* and x_r (FIG. 2).

The specific growth rate $\mu(\cdot)$ chosen for the simulation purpose follows the Monod law [12]:
$\mu(s) = \mu_{max}\dfrac{s}{k_M + s}$.
Besides, the numerical values used in the simulation are presented in (TAB. 1). We note $N(m, \sigma^2)$ the Gaussian distribution law with a mean m and a standard deviation σ.

4. INTERVAL OBSERVERS WITH GUARANTEED CONFIDENCE LEVELS

4.1. Confidence levels

Now, we assume that we know the probability density of the uncertainties and disturbances. Here, we consider that the probability densities for each parameter are independent. More precisely we assume that we know the probability density of the following quantities:

– The model parameters,
– The input disturbances,
– The disturbances associated to the measurements,
– The probability density associated to the initial condition.

For sake of brevity, we will talk about parameter uncertainties for these four types of uncertainties and disturbances.

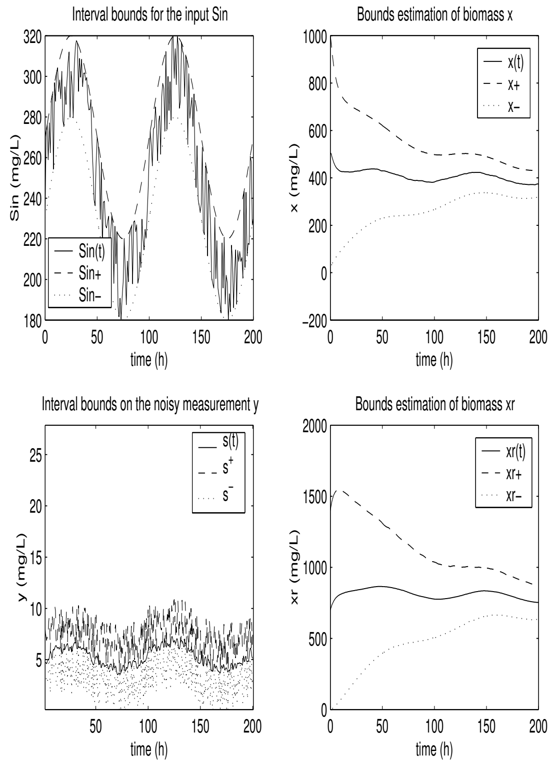

FIG. 2 – *Interval observer: bounds on the input, on the measurement and bound estimates of x and x_r.*

Param.	Units	Values
μ_{max}	h^{-1}	0.15
k_M	$mg.l^{-1}$	15
Y_s	-	0.65
w	-	1.00
q_r	-	0.60
q_w	-	0.20
D	$mg.l^{-1}$	0.1
$s_{in}^{\star}(t)$	$mg.l^{-1}$	$250 + 50\sin(\frac{2\pi t}{100})$
δs_{in}	$mg.l^{-1}$	$\bar{N}(0,10)$
s_{in}^{+}(t)	$mg.l^{-1}$	$s_{in}^{\star}(t) + 20$
s_{in}^{-}(t)	$mg.l^{-1}$	$s_{in}^{\star}(t) - 20$
b	$mg.l^{-1}$	$\bar{N}(0,5)$
b^+	$mg.l^{-1}$	10
b^-	$mg.l^{-1}$	-10
x_0	$mg.l^{-1}$	500
x_0^+	$mg.l^{-1}$	1000
x_0^-	$mg.l^{-1}$	0
x_{r0}	$mg.l^{-1}$	700
x_{r0}^+	$mg.l^{-1}$	1400
x_{r0}^-	$mg.l^{-1}$	0
s_0	$mg.l^{-1}$	40

TAB. 1 – *Parameters of the interval observer simulation. $\bar{N}(m,\sigma^2)$ denotes a normal distribution truncated between -2σ and $+2\sigma$.*

Note however that the experimental determination of the probability distribution of the uncertainties is often a difficult task requiring a high number of experiments. For illustration purpose we will consider Gaussian distributions, but the proposed algorithm can be used for any (unimodal) probability density function.

Assumption 1 *We assume that the uncertain parameters p_j, $j \in [1;k]$ have independent unimodal probability densities f_{p_j} on k intervals $\mathcal{I}_J$. Let us note $P_j = P(p_j \in [p_j^-;p_j^+])$ the probability for each parameter to be in a given interval $[p_j^-;p_j^+]$.*

Now we will index the intervals $[p^-;p^+]$ by a confidence level χ corresponding to the probability for p to be in this interval. This confidence level is thus defined as follows:

$$\chi = \prod_{j=1}^{k} P_j = P(p \in [p^-;p^+])$$

We will see that χ is a lower bound for the probability of the state x to lie in an interval $[x^- ; x^+]$. As a consequence, χ will be referred as a "guaranteed confidence level". We will consider the particular interval $[x^- ; x^+]$ provided by the interval observer, and we will therefore estimate the probability $P(x(t) \in [x^-(p^-,p^+,t);x^+(p^-,p^+,t)])$. This computation is complicated; and we will only provide a lower bound of this probability. Indeed, if the uncertainties p_j are in the interval $[p_j^-;p_j^+]$, then the previous section ensures that:

$$x(t) \in [x^-(p^-,p^+,t);x^+(p^-,p^+,t)]$$

It follows that we have the following property:

$$P(x(t) \in [x^-(p^-,p^+,t);x^+(p^-,p^+,t)]) \geq \chi$$

We will thus propose a way to choose a set of bounds p^- and p^+, associated to a confidence level χ. Of course, the choice of the bounds associated to a confidence level is not unique and other bounds could be considered.

Definition 2 *Under Assumption 1, we can choose the (finite) bounds ${p_j}^-(\chi)$ and ${p_j}^+(\chi)$ of the interval associated to the confidence level χ as follows:*
i)

$$\sqrt[k]{\chi} = \int_{{p_1}^-}^{{p_1}^+} f_{p_1}(p_1).dp_1 = \cdots = \int_{{p_k}^-}^{{p_k}^+} f_{p_k}(p_k).dp_k$$

ii)

$$\forall j \in [1;k], \quad f_{p_j}({p_j}^-) = f_{p_j}({p_j}^+)$$

Definition 3 *Under these assumptions and those inherent to the synthesis of interval observers, we can define a new class of observers: the interval observers with guaranteed confidence level χ.*

Choosing a confidence level χ gives us k fixed bounds ${p_j}^-(\chi)$ and ${p_j}^+(\chi)$ associated with the k parameters p_j. We will now consider several possible values χ_i of χ, and for each of these r values we will build an interval observer $[x^-(p^-(\chi_i),p^+(\chi_i),t);x^+(p^-(\chi_i),p^+(\chi_i),t)]$

based on the bounds $[p^-(\chi_i); p^+(\chi_i)]$. The probability to have p in $[p^-; p^+]$ is χ, and therefore, the probability that $x(t)$ is in $[x^-(p^-(\chi_i),p^+(\chi_i),t); x^+(p^-(\chi_i),p^+(\chi_i),t)]$ is larger than χ_i.

If r is large enough, we use $x^-(p^-(\chi_i),p^+(\chi_i),t)$ and $x^+(p^-(\chi_i),p^+(\chi_i),t)]$ to estimate the so called confidence density function.

We will see in the next section that (after a renormalization to ensure that the total probability is 1) the confidence density function can be interpreted as a probability density function.

4.2. Application to the activated sludge process

We apply these interval observers with guaranteed confidence levels to the activated sludge process under the same operating conditions as before. Besides, we suppose that uncertainty on the model is mainly due to four parameters: an offset δs_{in} on the input s_{in}, a noise $b(t)$ on the measurement of substrate s and the initial conditions on biomasses: x_0 and x_{r0}. We assume that the uncertainties are characterized by Gaussian distributions (TAB. 2).

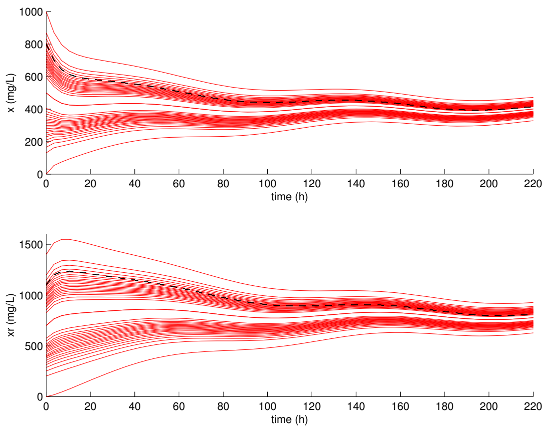

FIG. 3 – *The* $r = 21$ *interval observers with guaranteed confidence levels.*

We run the simulation for $r = 21$ values of χ, from $\chi = 0$ to $\chi = 0.99$, with a step of 0.05. Thus we build $r = 21$ interval observers with guaranteed confidence levels (FIG. 4). Then we interpolate the results of these r interval observers with confidence levels to obtain the estimations of the biomasses bounds with various guaranteed confidence levels (FIG. 5).

Thus, at any time, we get the confidence density for the unmeasured variables x and x_r.

The parameters used in these simulations are presented in (TAB. 2).

5. COMPARISON

In order to compare the confidence density computed from the interval observers to the actual probability

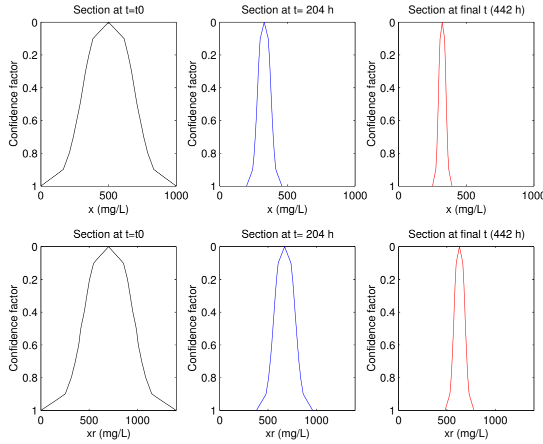

FIG. 4 – *Interval bounds on estimated biomasses* x *and* x_r *at different times, with respect to the* r *confidence levels.*

Param.	Units	Values
b	$mg.l^{-1}$	$N(0,1)$
x_0	$mg.l^{-1}$	$N(500,150)$
x_{r0}	$mg.l^{-1}$	$N(700,200)$
δs_{in}	$mg.l^{-1}$	$N(0,20)$
$s^\star_{in}(t)$	$mg.l^{-1}$	$250 + 50\sin(\frac{2\pi t}{100})$

TAB. 2 – *Parameter distribution used to build the interval observers with guaranteed confidence levels.*

density, we perform a Monte Carlo analysis. Thus, we run 30000 simulations associated to 30000 different values for the set of parameters δs_{in}, x_0 and x_{r0}, according to their distribution (TAB. 2). Note that the disturbance on the output does not intervene here.

Thus we can compare the results of the two computations: on (FIG. 6) for the Monte Carlo computation, on (FIG. 7) for the interval observers with guaranteed confidence levels.

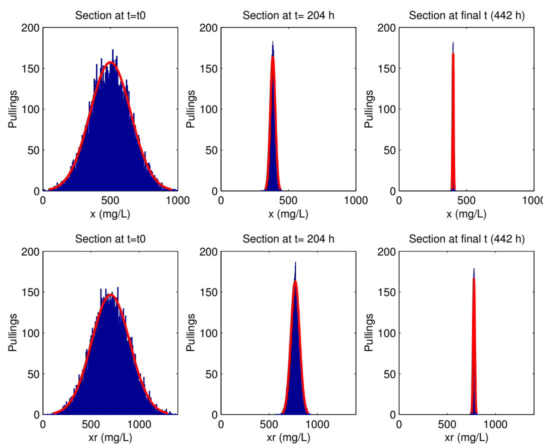

FIG. 5 – *Probability density estimated by Monte Carlo computation.*

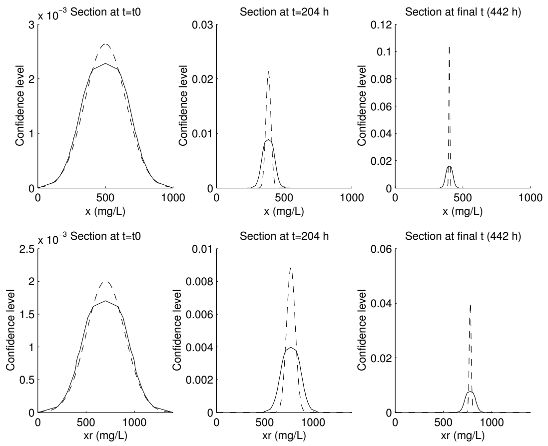

FIG. 6 – *Unmeasured variables estimated by interval observers with guaranteed confidence levels (continuous) and directly computed with Monte Carlo (dashed).*

The comparison between the average value and standard deviations issued from the Monte Carlo analysis and from the interval observers with guaranteed confidence levels computation shows that the results are very close (same average, but different standard deviations). It ensures us that the two computations lead roughly to the same results: the interval observers with confidence levels give a good approximation of the probability densities for the variables to estimate. The computation of the set of interval observers is much faster.

However, the interval observers with guaranteed confidence levels computation provide a distribution which is more spread, because it is based on a worst case approach.

6. CONCLUSION

We have used a set of interval observers, which are based upon the deterministic bounds on uncertain parameters, by using the knowledge of their probability density to build confidence levels. Let us emphasize the following points:

- It is necessary to provide a probability density for each unknown parameter. They are supposed to result from an experimental analysis. Here we have assumed Gaussian distributions, but any kind of distribution law could be chosen.
- These observers cope with measurements noise, even if the noise distribution is not Gaussian.
- In this paper, we guarantee only an asymptotic rate of observer convergence, but in some cases, it is possible to tune this rate [9].
- These observers could improve monitoring of bioreactors since they characterize the spread of the state estimate.

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