15th Triennial World Congress, Barcelona, Spain

DISINFECTANT RESIDUAL CONTROL OF MULTIPLE NODES IN WATER DISTRIBUTION NETWORKS

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Abstract: Based on investigating the spatially distributed input-output relationship of disinfectant residual in water distribution networks, this paper formulates the water quality control problem of multiple nodes in an adaptive optimal control framework, with special consideration on the periodic variation of parameter uncertainty and imposed bounds on the control input. The periodic parametric uncertainty, which arises due to varying consumer demands, is represented by a Fourier series with on-line parameter estimation of the unknown coefficients. A modified indirect adaptive control scheme for a single-input is studied, and then is extended to the case of multiple disinfectant boosters. A simulation example is provided to illustrate the performance of the algorithm in a real water distribution network.

Keywords: Water distribution network, adaptive control, periodic uncertainty, process control.

1. INTRODUCTION

Chlorine is by far the most common disinfectant used in Drinking Water Distribution Networks (DWDN) to help free the water from a number of possible disease-causing organisms. The injection of appropriate amounts of chlorine at suitably selected nodes of DWDN is a key issue for maintaining high quality drinking water, known as the *water quality control problem.* Drinking water standards and regulations specify the minimum chlorine residual which must be present at points of water consumption to achieve pathogen control; on the other hand, chlorine residual can not be too large since reactions of chlorine with certain organic compounds may produce disinfectant byproducts (DBPs), some of which are suspected carcinogens (Bull and Kopfler, 1991). Therefore, the spatial distribution of chlorine concentration in a DWDN must be maintained within a certain range to guarantee the quality of drinking water and to limit formation of disinfectant byproducts.

Accurate and reliable control of chlorine residuals within a DWDN is a new and complex problem. Recently, the issue of modeling and controlling water quality is starting to attract significant attention (Polycarpou *et al.*, 2001). In a recent paper (Brdys *et al.*, 2000), a hierarchical two-level structure for the integrated quantity-quality control was proposed. At the upper level, the pump/valve and chlorine injection schedules are generated online using a repetitive control technique. The lower level is to adjust and maintain the chlorine concentration at the monitored nodes within some prescribed limits. A robust predictive controller was further developed in (Brdys *et al.*, 2001) to introduce a safety-zone based on output prediction techniques. By assuming that the parameters of the model are constant over the time slots, an off-line estimation method was applied to generate the bounds for the parameters. The safety zones for the output constraints

⋆ This research was partially supported by the American Water Works Association (AWWA) Research Foundation.

are proposed, and are iterated to maintain the chlorine within the prescribed limits under small set-bounded uncertainties. A general formulation for the water quality control problem based on parameter estimation methods and adaptive control was developed in (Polycarpou *et al.*, 2001).

In practice, it is impossible to maintain the chlorine concentration at all consumption nodes to their desired levels due to the decay of chlorine during transport time, unless a chlorine boosting station is placed at each node (which is clearly impractical). The objective of this paper is to formulate the global objective of maintaining the spatial distribution of chlorine residual at a set of monitored nodes to some corresponding levels, by balancing the allowed tracking errors at the various monitored nodes. This is achieved by selecting an appropriate cost function which is subject to input constraints. The underlining structure of the feedback scheme is based on an indirect adaptive control methodology with a periodic time-varying model. The presence of unknown and time-varying consumer demands gives rise to significant time-variation in model parameters, which is addressed by employing a Fourier series approximation of the parametric uncertainty. The coefficients of the Fourier series are updated on-line using parameter estimation techniques. Finally, a simulation example, based on a real water distribution network, is presented to illustrate the proposed methodology. Due to page limitations, the details of the design derivation and some analysis are omitted.

2. WATER QUALITY CONTROL FORMULATION

Due to the decay of chlorine, water utilities must balance between excessive disinfectant concentrations near the booster and loss of pathogen control at the network periphery. This balance is complicated further by the desire to minimize disinfectant dosage and contact time to reduce the formation of DBPs. The selection of actuator and sensor locations is crucial to achieving the control objectives. In general, sensors should be placed on various network paths that "cover" as many of the water consumption points as possible. Actuators need to be placed at locations that guarantee maximum coverage. Another consideration in selecting the boosting locations is to make the network as "decoupled" as possible since in that case it becomes easier to design a feedback controller.

Given a selected set of sensors and actuators, the input-output water quality dynamics can be modeled from a controls perspective. The input-output relationship for a DWDN with pipes can be written in the form (Zierolf *et al.*, 1998)

$$y(t) = \sum_{p \in \mathcal{P}(t)} \beta_p(t) u(t - d_p(t)),$$

where $\mathcal{P}(t)$ is the set of all paths from the input to the output; $\beta_p(t)$ is a path impact factor, and $d_p(t)$ is the transport delay associated with path p. The set of paths $\mathcal{P}(t)$ is time varying because the number of paths from an input node to an output node may change with time due to variation in the hydraulics. From a controls perspective, the transport delay $d_p(t)$ can be discretized so as to eliminate the time-variation in time delays:

$$y(k) = \sum_{i=\underline{d}}^{\overline{d}} \beta_i(k) u(k - i)$$

where $\beta_i(k)$ now reflect the sum of all path impact factors associated with each integer time delay, and $\underline{d}$ and $\overline{d}$ reflect the minimum and maximum transport delays over all possible paths between input and output. The above formulation transforms the time-varying nature of time delays into time-varying coefficients, making it more appropriate for control design.

Typical water distribution networks are comprised of a few tanks and possibly thousands of pipes. By combining the water quality dynamics in transport through pipes and storage tanks, a relationship between chlorine concentration at an injection point $u(k)$ and a monitored sensor node $y_i(k)$ $(i = 1, \ldots, m)$ can be derived as follows (Polycarpou *et al.*, 2001)

$$y_i(k) = \sum_{j=1}^{n_i} a_{ij}(k) y_i(k-j) + \sum_{j=\underline{d_i}}^{d_i} b_{ij}(k) u(k-j)$$

where n_i is the number of tanks that provide water to the i-th output; $\underline{d_i}, d_i$ are the lower and upper limits of transport delay between the input and the i-th output, respectively; $a_{ij}(k)$, $b_{ij}(k)$ are the corresponding time-varying coefficients.

By recursively using the above equation to replace $y_i(k-1) \ldots y_i(k - \underline{d_i})$, the following equivalent model is obtained

$$y_i(k) = \sum_{j=\underline{d_i}}^{q_i} \alpha_{ij}(k) y_i(k-j) + \sum_{j=\underline{d_i}}^{\overline{d_i}} \beta_{ij}(k) u(k-j)$$

where $\alpha_{ij}(k)$ and $\beta_{ij}(k)$ are new coefficients; $q_i = \underline{d_i} + n_i - 1$ and $\overline{d_i} = d_i + \underline{d_i} - 1$. The equivalent model is convenient for feedback control design in the sense that the controller will not need the future value of the sensored output.

Let $y_i^*(k + \underline{d_i})$ be the desired value of $y_i(k + \underline{d_i})$, and define $\tau_i \triangleq k + \underline{d_i}$. In general, it is not possible to achieve $y_i(\tau_i) = y_i^*(\tau_i)$ for all $i = 1, \ldots, m$ with only one control input. The main objective is to keep the chlorine concentration level at the monitored nodes as close as possible to the desired value and to maintain the outputs within the prescribed limits, in the presence of unknown and time-varying consumer demands. Therefore we seek a control input u that minimizes the weighted multi-objective cost function

$$J(u) = \sum_{i=1}^{m} \lambda_i (y_i(\tau_i) - y_i^*(\tau_i))^2, \qquad (1)$$

subject to

$$\underline{u} \leq u(k) \leq \overline{u}$$

where λ_i are the weight coefficients $(0 < \lambda_i \leq 1)$, which are used to represent the significance of tracking error at each monitored node; $\underline{u}$ and $\overline{u}$ are the lower and upper limits for $u(k)$, respectively.

Although the water quality model described is linear, the design of a feedback control systems presents some important challenges. First, the coefficients of the ARMA model are unknown and time-varying. Although the daily patterns of aggregate water demand may be known, the spatially distributed patterns at individual nodes are not (and thus neither is its effect on travel paths and time delays between particular inputs and outputs). Second, the presence of transport delays make the control design more difficult. Finally, it is important to note that the model may be subject to other modeling errors or disturbance type of uncertainties.

Next an indirect adaptive control scheme is designed based on discrete-time modeling methods (Tsakalis and Ioannou, 1993; Landau *et al.*, 1998).

3. ADAPTIVE OPTIMAL CONTROL SCHEME FOR WATER QUALITY

We first consider the design of feedback control scheme for the case of having only one chlorine injection booster. If the parameters in the system model are known and there is no input constraint, the problem can be solved directly by setting $\partial J/\partial u(k) = 0$, which gives

$$u(k) = \frac{1}{\sum_{i=1}^{m} \lambda_i \beta_{i\underline{d_i}}^2(\tau_i)} \sum_{i=1}^{m} \lambda_i \beta_{i\underline{d_i}}(\tau_i)(y_i^*(\tau_i) - \sum_{j=\underline{d_i}}^{q_i} \alpha_{ij}(\tau_i) y_i(\tau_i - j) - \sum_{j=\underline{d_i}+1}^{\overline{d_i}} \beta_{ij}(\tau_i) u(\tau_i - j)) \qquad (2)$$

From a controls perspective, various feedback control approaches can be used as design candidates, such as optimal control scheme, predictive control scheme, switching mode control scheme, etc. In this section, we propose a way for handling unknown large time variations of parameters, based on exploiting the fact that the parametric time-variations display a pronounced periodic pattern.

The unknown parametric variation is approximated by a Fourier series. We assume that the period is known but the periodic pattern is unknown. The assumption of a known period is quite realistic from studies in water hydraulics and water quality. Generally, the parametric time-variations have a periodic pattern with a typical period (denoted by T_p) of 24 hours due to the daily variations in water demand. We also assume that some lower and upper limits for the transport delay (denoted by $\underline{d_i}$ and $\overline{d_i}$ respectively) are available apriori (the limits on the transport delay can be estimated through hydraulic simulation).

Since the coefficients of the ARMA model are periodically varying , they are approximated by a Fourier series with finite number of terms:

$$f(k) = f_0 + \sum_{l=1}^{N} \left(f_l^s \sin(l\omega k) + f_l^c \cos(l\omega k) \right), (3)$$

where $\omega = 2\pi T/T_p$ (T is the sampling time), f denotes α_{ij} or β_{ij}, and f_0, f_l^s,f_l^c are unknown constant parameters based on the Fourier coefficients. In practice, N should not be needlessly large. Usually, the first few Fourier components are enough to approximate the periodic time variations.

For notational simplicity, $\sin(l\omega k)$ is denoted by s_k, $\cos(l\omega k)$ is denoted by c_k. From (3), the input/output relationship can be rewritten as

$$y_i(k) = \sum_{j=\underline{d_i}}^{q_i} \left(\alpha_{ij0} + \sum_{l=1}^{N} \left(\alpha_{ijl}^s s_k + \alpha_{ijl}^c c_k \right) \right) y_i(k-j)$$
$$+ \sum_{j=\underline{d_i}}^{\overline{d_i}} \left(\beta_{ij0} + \sum_{l=1}^{N} \left(\beta_{ijl}^s s_k + \beta_{ijl}^c c_k \right) \right) u(k-j), \qquad (4)$$

Based on (4), using parameter estimation techniques, the identification model is chosen as

$$\hat{y}_i(k) = \sum_{j=\underline{d_i}}^{q_i} \Bigg(\hat{\alpha}_{ij0}(k) + \sum_{l=1}^{N} \left(\hat{\alpha}_{ijl}^s(k) s_k \right.$$
$$\left. + \hat{\alpha}_{ijl}^c(k) c_k \right) \Bigg) y_i(k-j) + \sum_{j=\underline{d_i}}^{\overline{d_i}} \Bigg(\hat{\beta}_{ij0}(k)$$
$$+ \sum_{j=1}^{N} \left(\hat{\beta}_{ijl}^s(k) s_k + \hat{\beta}_{ijl}^c(k) c_k \right) \Bigg) u(k-j),$$

where $\hat{\alpha}_{ij0}(k)$, $\hat{\alpha}_{ijl}^s(k)$, $\hat{\alpha}_{ijl}^c(k)$, $\hat{\beta}_{ij0}(k)$, $\hat{\beta}_{ijl}^s(k)$, $\hat{\beta}_{ijl}^c(k)$ are the on-line parameter estimates of α_{ij0}, α_{ijl}^s, α_{ijl}^c, β_{ij0}, β_{ijl}^s, β_{ijl}^c respectively.

For notational compactness, we define the following vectors

$$\hat{\alpha}_{ij}(k) = \left[\hat{\alpha}_{ij0}(k) \ \cdots \ \hat{\alpha}_{ijN}^s(k) \ \hat{\alpha}_{ij1}^c(k) \ \cdots \ \hat{\alpha}_{ijN}^c(k) \right]$$

$$\hat{\beta}_{ij}(k) = \left[\hat{\beta}_{ij0}(k) \ \cdots \ \hat{\beta}_{ijN}^s(k) \ \hat{\beta}_{ij1}^c(k) \ \cdots \ \hat{\beta}_{ijN}^c(k) \right]$$

$$
\begin{aligned}
\bar{\alpha}_{ij} &= \left[\alpha_{ij0}\ \ \alpha^s_{ij1}\ \ \cdots\ \ \alpha^s_{ijN}\ \ \alpha^c_{ij1}\ \ \cdots\ \ \alpha^c_{ijN}\right] \\
\bar{\beta}_{ij} &= \left[\beta_{ij0}\ \ \beta^s_{ij1}\ \ \cdots\ \ \beta^s_{ijN}\ \ \beta^c_{ij1}\ \ \cdots\ \ \beta^c_{ijN}\right] \\
\theta_i &= \left[\bar{\alpha}_{i\underline{d_i}}\ \ \cdots\ \ \bar{\alpha}_{iq_i}\ \ \bar{\beta}_{i\underline{d_i}}\ \ \cdots\ \ \bar{\beta}_{i\overline{d_i}}\right]^\top \\
\hat{\theta}_i(k) &= \left[\hat{\alpha}_{i\underline{d_i}}(k)\ \cdots\ \hat{\alpha}_{iq_i}(k)\ \hat{\beta}_{i\underline{d_i}}(k)\ \cdots\ \hat{\beta}_{i\overline{d_i}}(k)\right]^\top \\
\bar{y}_i(k-j) &= y_i(k-j)\ \ [1\ \ \sin(\omega k)\ \ \cdots\ \sin(N\omega k) \\
&\quad \cos(\omega k)\ \ \cdots\ \cos(N\omega k)] \\
\bar{u}(k-j) &= u(k-j)\ \ [1\ \ \sin(\omega k)\ \ \cdots\ \sin(N\omega k) \\
&\quad \cos(\omega k)\ \ \cdots\ \cos(N\omega k)] \\
\zeta_i(k) &= \left[\bar{y}_i(k-\underline{d_i})...\bar{y}_i(k-q_i)\ \bar{u}(k-\underline{d_i})...\bar{u}(k-\overline{d_i})\right]^\top
\end{aligned}
$$

Let the identification error be defined as $e_i(k) \triangleq y_i(k) - \hat{y}_i(k)$, and the corresponding parameter estimation errors are defined as $\tilde{\theta}_i(k) \triangleq \hat{\theta}_i(k) - \theta_i$. Based on the network model (4) and the identification model it can be easily verified that $e_i(k)$ can be written in the form of

$$
e_i(k) = -\tilde{\theta}_i(k)^\top \zeta_i(k) \qquad (5)
$$

From (5), using techniques from adaptive control it is possible to derive an adaptive estimation law

$$
\hat{\theta}_i(k+1) = \hat{\theta}_i(k) + \frac{\gamma_0 e_i(k)\zeta_i(k)}{c_0 + ||\zeta_i(k)||^2} \qquad (6)
$$

where c_0 is a small positive constant, γ_0 corresponds to the adaptive gain, $0 < \gamma_0 < 2$, and

$$
||\zeta_i(k)||^2 = (N+1)\left(\sum_{j=\underline{d_i}}^{q_i} y_i(k-j)^2 + \sum_{j=\underline{d_i}}^{\overline{d_i}} u(k-j)^2\right)
$$

Based on techniques from adaptive literature, the adaptive law (6) guarantees that $\hat{\theta}_i(k)$ is uniformly bounded and for any finite d,

$$
\begin{aligned}
(i) &\quad \lim_{k\to\infty} \frac{e_i(k)}{(c_0 + ||\zeta_i(k)||^2)^{1/2}} = 0, \\
(ii) &\quad \lim_{k\to\infty} ||\hat{\theta}_i(k+d) - \hat{\theta}_i(k)|| = 0. \qquad (7)
\end{aligned}
$$

From (2), the future values $\alpha_{ij}(\tau_i)$ and $\beta_{ij}(\tau_i)$ of $\alpha_{ij}(k)$ and $\beta_{ij}(k)$ are needed to obtain the control input $u(k)$ because of the transport time delay. In the above estimation scheme, the estimate of $\hat{\theta}_i(k)$ is obtained based on the identification error $e_i(k)$, therefore, we can obtain the prediction value of the unknown coefficients based on these estimates. Let $\hat{\bar{y}}_i(\tau_i)$ be the prediction value of $y_i(k)$ based on the predicted coefficients. We first consider the optimal solution for the prediction $\hat{\bar{y}}_i(\tau_i)$ and then show the control law obtained is an asymptotical solution for the optimal control problem (1).

We define $\hat{\bar{\alpha}}_{ij}(\tau_i)$ and $\hat{\bar{\beta}}_{ij}(\tau_i)$ as the prediction of $\alpha_{ij}(k)$ and $\beta_{ij}(k)$ as follows:

$$
\hat{\bar{f}}(\tau_i) = \hat{f}_0(k) + \sum_{l=1}^{N}\left(\hat{f}^s_l(k)\sin(l\omega\tau_i) + \hat{f}^c_l(k)\cos(l\omega\tau_i)\right),
$$

where f denotes α_{ij} or β_{ij}.

Now, we consider the problem of minimizing

$$
J'(u) = \sum_{i=1}^{m} \lambda_i(\hat{\bar{y}}_i(\tau_i) - y^*_i(\tau_i))^2, \qquad (8)
$$

subject to $\underline{u} \le u(k) \le \overline{u}$, where

$$
\hat{\bar{y}}_i(\tau_i) = \sum_{j=\underline{d_i}}^{q_i} \hat{\bar{\alpha}}_{ij}(\tau_i)y_i(\tau_i - j) + \sum_{j=\underline{d_i}}^{\overline{d_i}} \hat{\bar{\beta}}_{ij}(\tau_i)u(\tau_i - j).
$$

In order to solve the optimal problem with box constraint, a saturation function $sat(u)$ is introduced to express the solution analytically:

$$
sat(u) = \begin{cases} \underline{u} & \text{if } u < \underline{u} \\ u & \text{if } \underline{u} \le u \le \overline{u} \\ \overline{u} & \text{if } u > \overline{u} \end{cases} \qquad (9)
$$

Similar to (2), from the identification model and (8), we obtain the control law

$$
u(k) = sat\left(\frac{1}{\sum_{i=1}^{m}\lambda_i \hat{\bar{\beta}}^2_{i\underline{d_i}}(\tau_i)} \sum_{i=1}^{m} \lambda_i \hat{\bar{\beta}}_{i\underline{d_i}}(\tau_i)(y^*_i(\tau_i) - \sum_{j=\underline{d_i}}^{q_i} \hat{\bar{\alpha}}_{ij}(\tau_i)y_i(\tau_i - j) - \sum_{j=\underline{d_i}+1}^{\overline{d_i}} \hat{\bar{\beta}}_{ij}(\tau_i)u(\tau_i - j))\right) (10)
$$

By comparing the definition of $\hat{\bar{y}}_i(\tau_i)$ and the identification model , we obtain

$$
\hat{y}_i(\tau_i) - \hat{\bar{y}}_i(\tau_i) = \left(\hat{\theta}_i(k + \underline{d_i}) - \hat{\theta}_i(k)\right)^\top \zeta_i(\tau_i) \ (11)
$$

Therefore, from (7), we obtain

$$
\begin{aligned}
&\lim_{k\to\infty} \hat{y}_i(\tau_i) - \hat{\bar{y}}_i(\tau_i) \\
&= \lim_{k\to\infty}\left(\hat{\theta}_i(k + \underline{d_i}) - \hat{\theta}_i(k)\right)^\top \zeta_i(\tau_i) = 0 \ (12)
\end{aligned}
$$

By applying the control law (10) to the input/output model (4), we know the input is bounded due to the saturation function, and the plant is a bounded input bounded output system, so the output $y_i(k)$ and $\zeta_i(k)$ are also bounded. Therefore, according to (7), $\lim_{k\to\infty} e_i(k) = 0$, i.e., $\hat{y}_i(k)$ converges to $y_i(k)$ asymptotically. From (12), $\hat{\bar{y}}_i(\tau_i)$ converges to $\hat{y}_i(\tau_i)$ asymptotically. Hence, $\hat{\bar{y}}_i(\tau_i)$ converges to $y_i(\tau_i)$ and $J'(u)$ converges to $J(u)$ asymptotically. We conclude that the control law (10) and the estimation law (6) guarantee that the tracking errors are minimized adaptively under the input constraint.

From water quality modeling in DWDN, $\beta_{i\underline{d}_i}(k)$ is related to the chlorine decay coefficient, which is non-negative. Therefore, a positive lower bound $\bar{\beta}_{i0}$ for $\beta_{i\underline{d}_i}(k)$ exists and is assumed to be known. In order to avoid the control input $u(k)$ (prior to saturation) from becoming unbounded, we need to keep $\hat{\bar{\beta}}_{i\underline{d}_i}(\tau_i)$ away from $\bar{\beta}_{i0}$. In this paper, we set $\hat{\bar{\beta}}_{i\underline{d}_i}(\tau_i)$ to $\bar{\beta}_{i0}$ if $\hat{\bar{\beta}}_{i\underline{d}_i}(\tau_i)$ drops below $\bar{\beta}_{i0}$. This is known as "projection modification" in the adaptive control literature.

To address the ignored modeling uncertainties and high Fourier series terms, we may use robust adaptive techniques, like dead-zone, leakage, parameter projection to modify the estimation laws and ensure the boundedness of the parameter estimates and the small-in-the-mean property of the tracking error in the presence of bounded disturbances. The robustness discussion is omitted here due to space limitations.

Extension to the case of multiple boosters Injecting chlorine at distinct locations throughout the distribution system may produce a more uniform disinfectant residual (in space and time) while lowering the required total chlorine dose. The general water quality model for a system with n boosters and m sensors is inherently a multiple-input, multiple-output (MIMO) interconnected system, which can be described by

$$y_i(k) = \sum_{j=1}^{n_i} \alpha_{ij}(k) y_i(k-j) + \sum_{l=1}^{n} \sum_{j=\underline{d}_{il}}^{\overline{d}_{il}} \beta_{ijl}(k) u_l(k-j),$$

where $i = 1, 2, ..., m$, $\underline{d}_{il}, \overline{d}_{il}$ are the lower and upper limits of delay respectively between the l-th input and the i-th output, $\alpha_{ij}(k)$, $\beta_{ijl}(k)$ are the coefficients of the multivariable ARMA model, and the integer n_i corresponds to the number of tanks that provide water to the i-th output. The adaptive control problem can be formulated in a similar way as the single input case.

The multivariable adaptive optimal control problem is more complex because of two kinds of coupling: coupling in space and coupling in time. Coupling in space occurs because the boosters and tanks are connected through multiple pipes. Coupling in time is present because the control inputs from different time steps are related through the model equation. If the model is known, the Tamura coordination method may be applied to solve the optimal control problem (Tamura, 1975; Brdys and Ulanicki, 1994). It leads to a three-layer optimization structure, and the objective is achieved by using Langrange multipliers, first for the spatial interaction and then for the time couplings. However, the relationship between the hydraulic dynamics and water quality dynamics in DWDN is extremely complex and the known model assumption is hard to justify.

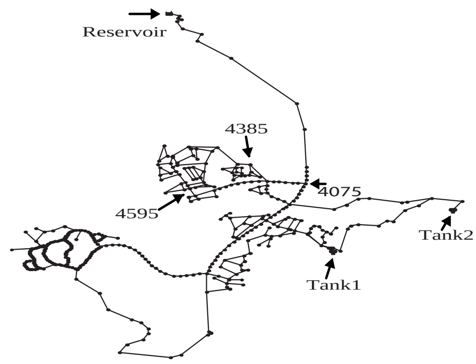

Fig. 1. Network configuration diagram and network hydraulics.

In practice, the interdependence among controllers and outputs may be taken as a periodic unknown external disturbance to the sub-network due to the daily periodic variation in water demand. Therefore we may decompose the whole network to sub-networks, and consider the decomposed sub-network based on the single-input model by adding an additional term to represent the chlorine from the rest of the network.

4. SIMULATION EXAMPLES

To illustrate the performance of the adaptive optimal control algorithm we performed several simulation studies using the network topology of a water utility in the western United States. The network configuration is shown on Figure 1. The flow velocity in the pipes and the draining/filling of the tanks are controlled by the water demand of the system and water supply from the reservoir, which is the only water source in the network. The periodicity of the water demand is 24 hours and most junctions share similar demand patterns.

The chlorine booster is at node 4075 and the monitored outputs are at nodes 4385 and 4595. The minimal transport delay from node 4075 to 4385 is about 1 hour; from 4075 to 4595 it is about 2.5 hours. The desired chlorine concentration for both output nodes is set to 0.6 mg/l and $\lambda_1 = \lambda_2 = 0.5$. The control objective is to minimize the tracking errors and to keep the input concentration within the bounds [0.6mg/l, 0.9mg/l].

In the first simulation, we ignore the output at node 4385 and just consider the control of the chlorine concentration at node 4595. An indirect adaptive control approach is employed with a Fourier series (the number of Fourier terms $N = 5$) approximating the unknown time-variation in water demands. The normalized gradient method is used with a learning rate $\gamma_0 = 0.2$ and the design variable $c_0 = 0.5$. The initial values for the parameters are set as $\hat{\alpha}_{i10} = 0.2$, $\hat{\beta}_{i\underline{d}_i0} = 0.5$, the initial values for other parameters are set to zero. The $\bar{\beta}_{i0}$ is set to 0.1 ($i = 1, 2$). The initial values

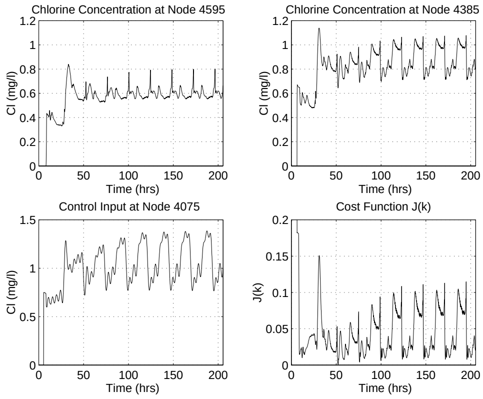

Fig. 2. Simulations with indirect adaptive control.

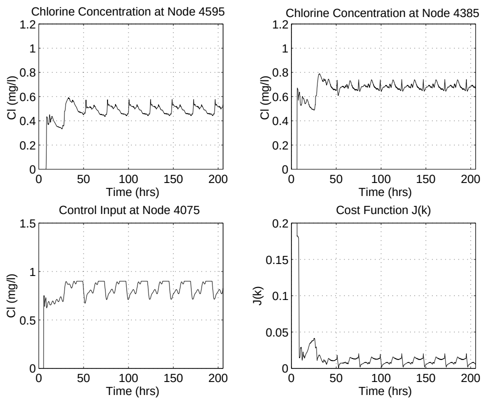

Fig. 3. Simulations with adaptive optimal control.

may affect the transition performance of the system. It is better to set these parameters to the values estimated based on experience and hydraulic simulations. However, from our simulations, the initial values do not make a big difference. Figure 2 shows the simulation result. To compare the result with the adaptive optimal result below, as shown in the figure, the output concentration at node 4385 is measured and displayed on the top right plot of the figure, and the cost function $J = \sum_{i=1}^{2} \lambda_i (y_i(\tau_i) - y_i^*(\tau_i))^2$ is shown on the bottom right plot of Figure 2. As can be seen from the figure, the tracking at node 4595 is reasonably good, however there are sharp overshoots. This is because we use only 5 Fourier terms and ignore the other uncertainties.

Next, under the same hydraulic dynamics, we perform the simulation with the adaptive optimal control. The input concentration at node 4075 is limited to be within the upper and lower bounds. We still use 5 Fourier terms to approximate the variation in parameters and use the same control parameter $(c_0, \gamma_0, \beta_{i0})$, and same initial conditions. The simulation result is shown on Figure 3. In comparing it with the indirect adaptive control result above, we see that the tracking errors are largely reduced and the input is within the prescribed set. The overall tracking performance at both monitored nodes is reasonably good.

5. CONCLUDING REMARKS

Regulating the spatio-temporal distribution of chlorine concentration is a crucial component of providing clean drinking water to consumers in spatially distributed water distribution networks where flow, temperature, and water quality variations occur on daily and seasonal cycles. This paper formulates the water quality control problem in an adaptive optimal control framework with special consideration on the case when the number of controllers is less than the sensors. The extended decomposed sub-network formulation for the control with multiple boosters can be taken as a special case of control with single booster by representing the coupling as a periodic external signal. The more general multi-level coordination optimization problem is under investigation.

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