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PUBLIC TRANSPORTATION SCHEDULE BASED ON MODULAR CONTROLLED STOCHASTIC PETRI NETS

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Abstract : This paper introduces a Modular Controlled Stochastic Petri Nets (MCSPN)-based approach in order to deal with connection systems. This work aims at improving the timetable of public transportation networks those are based on the hub principle. Decision variables of this problem are departure-times of means of transports those serve the most important junctions.

The MCSPN is an adaptation of the Continuous-Time Stochastic Petri Net (CTSPN). This adaptation aims at reducing the number of states of the studied system. Using the MCSPN-approach, the departure-time problem of a local company is optimized. This approach provides us a schedule with the minimum connection delays and the minimum expected operation cost. As a result this approach helps the schedule stuffs to provide a high level of quality of service at a reasonable future cost.

Keywords: Public transportation planning, hub-based network, Scheduling, Controlled Stochastic Petri Nets.

1 INTRODUCTION

In response to the increasing demand for transportation, several efforts have been centered on the improving of the public transportation networks. The modern management of such networks, according to the hub principle, crates more and more connections in order to optimize links and to attract potential customers. Regional traffic is thus optimally linked with Inter-city and express traffic. However, to provide a high quality of service for passengers of such networks, the travelling time between the most important junctions and the connection times must be decreased. Besides, it is required to warrant reliable connections. In the public transportation research literature (Toth and Vigo, 1998), there is a trend towards maximizing the junctions and finding the periodic timetables that minimize the total passenger waiting times (Isaai and Singh 2000). The passengers should thus be afforded maximal connection and minimal transfer times.

If the public transportation network design is based on the hub principle, transfer times and especially connection times are very significant criteria of service. Indeed, often a long connection time exhausts passengers, complicates the management of stations and thus penalizes the company's public image. Then, besides scheduling the timetable that provides minimum connection times, during the transportation system operation, dynamic connection monitoring tasks are required. Theses aim at facing problems caused by traffic hazardous. The most important ones are hub congestions and the unreliability of connections, caused by the lateness of the upstream line.

Scheduling departure times requires several simulations of timetables. Theses ones are done after computing a set of solutions. Straightforwardly, schedulers chose the departure-times which minimize recourses to connection monitoring tasks. Afterward, they try to ameliorate the chosen solution manually. However, this scheduling process presents the drawback that it is not rigorous.

The public transportation planning must provides the minimal connection delay at a reasonable future cost. To face this challenge, this paper deals with connection systems in probabilistic universe. It uses the Markov Chain theories in order to fix departure-times of means of transport of core lines. More precisely, the MCSPN-based approach is introduced for this problem. The MCSPN is an adaptation of the well-known Controlled Stochastic Petri Net CSPN (Meer and Düsterhöft, 1997) where the CTSPN is decomposed judiciously into set of elementary modules. This decomposition allows to deal with a reduced number of states of the Markov Chain. As we show in the following, the MCSPN-approach provides schedulers with an optimized departure-times of the hub-based lines.

This paper is organized as follow: the following section introduces the necessary background and definitions. Then, the MCSPN and the Modular Markov Decision Process (MMDP) are briefly presented. The analytical MCSPN model for an elementary connection system is given in the section IV. As an application, we deal with a local company network. This network is thus described and its timetable is improved. Conclusion and prospects are presented in the last section.

2 BACKGROUND AND DEFINITIONS

2.1 Public transportation planning process

The process of public transport planning deals with the determination of routes between an origin and a destination and the assignment of the necessary resources with regard to the future. Due to the wide range of criteria and the complexity of the public transportation system, in the early days of ground public transport management, this process followed a stringent order (Busiek, *et al*, 1996). Thus, generally speaking, the first step is the estimation of the future volume of traffic. The subsequent step, called line planning, consists in determining the lines and the cycle time for the regular routes (Patz, 1925; Busiek, *et al*, 1996; ABBAS-TURKI, *et al*, 2001). At the third step which is the core of this paper, all the departure-times are fixed with respect to the cycle time of the line plan (Nachtigall, 1996). This raw timetable is refined by including the operational constraints and the temporal variations, at the schedule planning step. The engines, the coaches and the personnel are finally distributed in order to equip each trips.

2.2 Hub based network particularity

If the public transportation network design is based on the hub principle, the most important junctions are linked with routes called transferred lines. These lines must be speed and reliable. Due to the traffic hazardous and to the high frequency of transfer lines, a daily monitoring of connection delays and a daily management of the traffic congestion at the must important junctions are required. These daily activities are costly if departure times of core lines are not optimized. Dealing with the connection issue in probabilistic universe is then required in order to fix the optimal departure-times.

Hence, in order to fix departure times of the hub-based line, we take into consideration the following points:

- The capacity of connection stations. Connection stations can not receive more than a restricted number of means of transport.
- The probably lateness of upstream means of transport. Theses lateness complicate the daily connection management.
- At this step of the public transportation planning process we have just the first rough draft of the final timetable. Then, for the performance evaluation, it is more rigorous to take into consideration the eventual modifications of this temporary raw schedule.

2.3 Departure-time problem formulation

To model the departure-time problem, we use the following notation:

Let a set of core lines $l_i = (r_i, \varphi_i, s_v, \Gamma_i) \in L \subset R \times Z_+ \times R_+^{v_i} \times R_+^{\varphi_i}$, which consists of known route $r_i \in R$, frequency $\varphi_i \in Z_+$, stop times S_{v_i} at the set of junctions V_i and unknown departure-time $\Gamma \in R_+^{\varphi_i}$ at the must important junctions. The route r_i is a path or cycle in the supply network $G(V, E)$, where the set of nodes V represents the stations and the set of edge E represents the connecting routes of adjacent stations. The frequency φ_i denotes the number of means of transport that serve the line within a considered time interval $(0, ..., \tau]$. The timetable $\Gamma_i \in R_+^{\varphi_i}$ denotes departure times of means of transport to serve the line l_i. The arrival times at the station can be straightforwardly computed from departure times and stop times. Departure times $\Gamma_i \in R_+^{\varphi_i}$ must be computed in order to minimize the expected expenditure having the weighted sum of the following cost rates:

- C_{ij} : The financial weight per unit of time of the connection delay between two lines l_i and l_j.
- CS_v: The cost rate of the stop time at the connection stations v.

Besides these two cost rates, it is possible to considerate other cost rates if we have more information concerning transportation expenditures or the behavior of passengers.

Unknowing the definitive timetable, we assume that the temporal distance between all the stations and the stop times are exponentially distributed random variable. Hence we use the Markov decision process and more precisely the MMDP theory and the dynamic programming in order to deal with the departure-time problem and to minimize expected operation costs.

3 MCSPN MODEL

The Petri Net (PN) allows modeling of sequential and concurrent actions including phenomena such as contention and synchronization. The additional structure in the Stochastic PN (SPN) allows the extraction of additional performance information about the modeled system (Molloy, 1981). Indeed, the reachability graph of the Continues Time SPN (CTSPN) with exponentially distributed delays is isomorphic to homogeneous Continues Time Markov Chain (CTMC). This open-up an area of analysis for performance measures such as average delay and throughput. From Stochastic Reward Nets (SRN) model, where a real-valued reward rate is associated to each marking of the reachability graph, we obtain a Markov Reward Process (MRP) that allows the pondered evaluation of performances.

Let $Z = \{Z(t), t \geq 0\}$ denotes a CTMC with finite state space Ω. Define the infinitesimal generator matrix $Q = \lfloor q_{ij} \rfloor$ consisting of direct transition rates from state i to j and diagonal entries, defined as: $q_{ii} = -\sum_{j, j \neq i} q_{ij}$. A homogeneous CTMC can be completely described by its infinitesimal generator matrix Q and its initial probability vector $p(0)$. The transient probability vector at time $t > 0$ for a CTMC, $p(t)$, is obtained by solving the equation

$\frac{dp(t)}{dt} = p(t) \cdot Q$ (1) written as $p(t) = p(0) \cdot H(t)$ whose solution can be formally $H(t) = e^{Qt}$.

Let to each state $i \in \Omega$ a real-valued reward rate r_i is associated. If the Markov chain stays in state i for duration t, a reward $r_i \cdot t_i$ is gained. Let $\Re(t)$ represents the random variable corresponding to the instantaneous reward rate. The expected value of the reward rate as a function of time can be computed as $X(t) = E[\Re(t)] = \sum_{i \in \Omega} r_i \cdot p_i(t)$. In the time $[0,...t)$ the accumulated reward is defined by $Y(t) = \int_0^t \Re(u) du$. The expected value of the accumulated reward $E[Y(t)]$ can be computed as: $Y(t) = \int_0^t X(u) du = \sum_{i \in \Omega} r_i \int_0^1 p_i(u) du$.

The MCSPN consists of a set of CTSPN modules. The decomposition of the global CTSPN model aims at reducing the number of states. It also facilitates considerably, for the departure-time timetabling, the mathematic formulation of the deterministic control of the dealt stochastic process. After, the decomposition of the CTSPN it is necessary to aggregate resulting CTMC for performance analysis. Fortunately, the proposed CTSPN model for the dealt problem can be decomposed straightforwardly in such a way that we can aggregate Markov chains with an exact method.

Decision variables of the proposed MCSPN are the determination of:

- The CTSPN module that must be aggregated with the current MCSPN.
- The moment of this aggregation.

Hence, we have a variable MMDP that is consist in aggregated MRP modules. We define decision arcs which denote the option of the MRP module selection. This module is the one that will be aggregated to the current MMDP. At every point of time a set of decision arcs are possible. A strategy $S(T)$ comprises a set of done selection for all options of the model at particular points of time $t_0, t_1 \ldots t_n$ with $0 \leq t_0 < t_1 \ldots < t_n \leq T$. A strategy $\hat{S}(T)$ is considered optimal if the performability measure under strategy $\hat{S}(T)$ is greater equal than the performability measure under any other strategy $S(T)$.

4 ELEMENTARY CONNECTION SYSTEM EXAMPLE

4.1 Elementary connection system

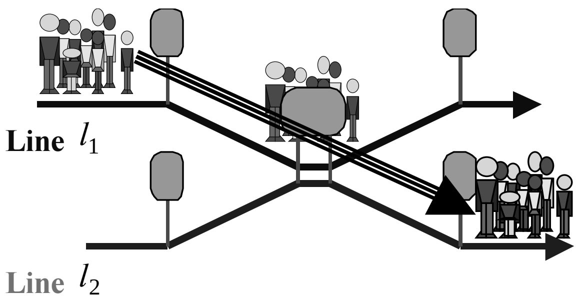


Fig 1. Elementary connection system

This part deals with the departure-time problem of an elementary public transportation connection system (figure 1). We show the straightforward interest of using MCSPN by comparing the result of our approach with the deterministic one. The elementary MCSPN model can be thus extended to deal with a real size system.

Let public transportation that is compound of two core lines l_1 and $l_2 \in L$. The system operates within the interval $(0,..., \tau]$. The number of available means of transport of each line is fixed. The lines l_1 and l_2 serve the station v_1. Several passengers P_{CS} take the line l_1 to go to the station v_1. Once there, they take the line l_2 that connects between the junction v_1 and their destinations. Hence, to improve the quality of service, it is required to minimize the connection times at the station v_1.

4.2 Numerical application

For the numerical application, we assume that the system operates within the interval of one hour. The frequency of the two lines are as follow: $\varphi_1 = 1$ and $\varphi_2 = 2$. In order to simplify the example we suppose that both of the lines l_1 and l_2 serve just three stations: departure stations DS_1 and DS_2, the connection station v and finally arrival stations AS_1 and AS_2. The average of temporal distances between those stations are presented on the table 1. The problem is the minimization of connection times and the maximization of the probability of serving all stations within an interval of one hour.

Each line l_i is modeled by a CTSPN module called up $CTSPN_{ij}$ (figure 2) with *j* denotes the number of the en route means of transport. The aggregated model is called up $MCSPN(n_1, n_2): n_i$ denotes the number of the en route means of transport of the line l_i. To model the connection an additional CTSPN model $CTSPN_C$ must be aggregated with $MCSPN(n_1, n_2)$ if n_1 and $n_2 \geq 1$. The departure time of the line l is called t. The decision variables are thus simply deduced. They consist in $\Delta t_1 = t_{11} - t_{21}$ and $\Delta t_2 = t_{21} - t_{22}$.

Table 1. System parameters

Parameter	Meaning	line	Time (min)
$1/\mu_{iD\to v}$	The average of the temporal distance between the stations DS_i and v	1	15
		2	10
$1/\mu_{i \to A}$	The average of the temporal distance between the stations v and AS_i	1	10
		2	10
s_{vi}	The stop time	1	1
		2	1

Using dynamic programming, we solve the stochastic problem. The deterministic timetable is straightforwardly computed. Both of stochastic and deterministic optimal timetables are represented in table 2.

From this table, we observe that in the case of stochastic approach the two passages of the line l_2 are planned after the passage of the line $_1$ at the connection station. This solution makes the connection more reliable. Indeed, remember from table 4, that the temporal distances between DS_i and v of lines l_1 and l_2 are 15min and 10min respectively. Hence, during the system operation, if we chose the deterministic solution, we have to delay the line l_2 whenever the line l_1 is late. This problem is avoided using the stochastic approach and consequently the recourse to future monitoring activities is minimized.

The stochastic timetable is nearer to the intuitive solution than the deterministic one. This could avoid manual adaptations of the deterministic timetable.

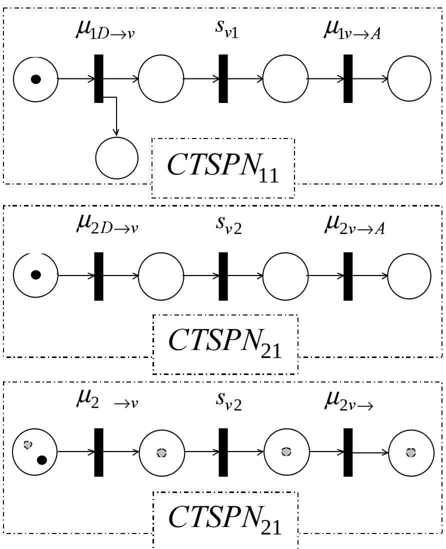
Table 2. Line timetables

	Deterministic		Stochastic	
Line	l_1	l_2	l_1	l_2
Γ_i	0	0	0	8 min
		5 min		13 min

5 APPLICATION TO A LOCAL TRANSPORT COMPANY NETWORK

Except some mini-buses, buses are the main means of transport of the company's fleet. The company's network has two important junctions, denoted (As) and (Ts) (see figure 3). The first one (As) is located in the town center and the second one (Ts) is located in the center of the suburb. Besides existing low lines (l_2 and l_3), a line with a high speed and frequency connects between these two important stations. This line is denoted by D. D provides the minimal transfer time between these two stations.

There are sixteen lines in the company network. The line D connects between four lines which serve the station Ts and eleven ones which serve the station As.



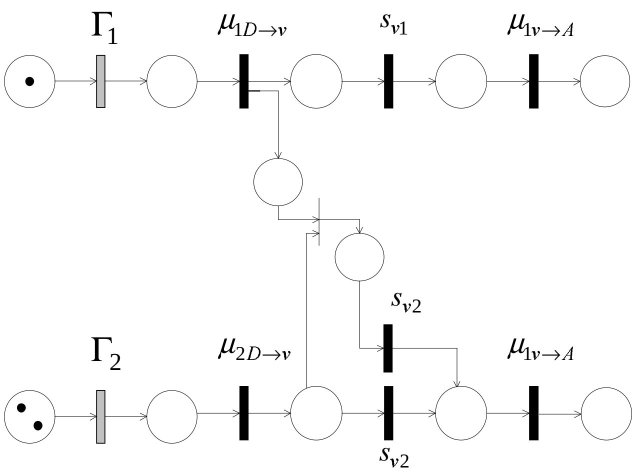


Fig 2. MCSPN model for the elementary connection timetabling

The studied connection system consists of a set of seven core lines, besides the line D.

Even if the hub principle provides an economical line planning of the network, connections require additional tasks of dynamic monitoring. Indeed, due to the traffic hazardous, problems of station congestions and of the unreliability of connection systems occur. The local company uses the fuzzy logic based tool as a base for the line D connection monitoring. The MCSPN approach is applied in order to minimize the recourse to the monitoring. As it was shown previously, this is possible by providing a judicious timetable.

First we used the approach, presented in (ABBAS-TURKI, *et al*, 2001), in order to compute the line frequencies. Afterward, the departure times are calculated with respect to the cycle time of the lines (frequencies). The MCSPN approach is applied in order to improve the departure-times in terms of connection costs and delays between company's core lines. The studied connection system consists of a set of seven core lines, besides the line D. Aggregating the CTSPN model for the elementary connection system, the set of itineraries is modeled. Cost rates are evaluated. The reward of the system is zeroed at the initial time. The optimal control of the system is thus computed solving the equation (1) and using the dynamic programming.

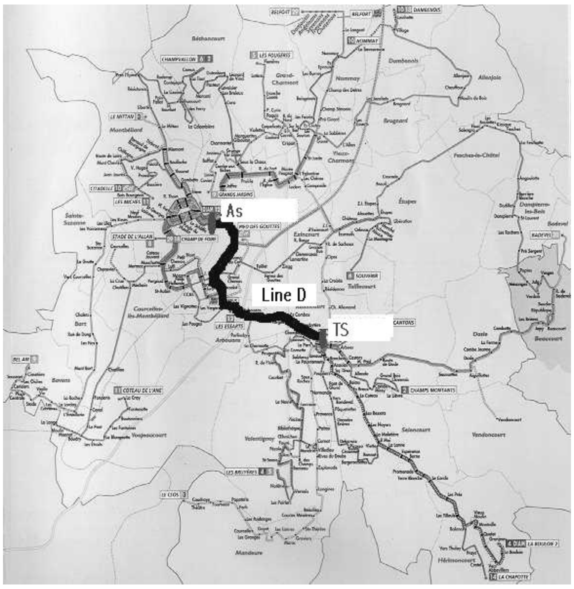


Fig. 3 The local company network.

The objective function is the weighted minimization of connection times and of the As and the Ts congestions. A sample of results of this application is depicted in the figure 4. This figure shows percentage variations of the quality improvement in term of expected operation costs and connection delays. One can observe that connection times decrease significantly by raising the given weight. This quality improvement does not influence the operation costs which still relatively stable. As a result, the use of MCSPN approach for the departure-time problem has three main advantages:

1. Providing a better quality of service at a reasonable cost.
2. Improving significantly departure-times in term of connection costs.
3. Minimizing significantly the connection time.

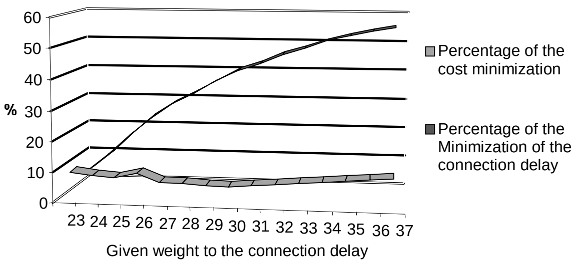


Fig.4. The improvement of the timetable

6 CONCLUSION

We have proposed in this paper an approach to schedule public transportation hub-based lines, using the MCSPN and MMDP theories. This new approach in the public transportation field, allows improving of first basic timetables in term of connection costs.

Indeed Knowing the importance of the connections, the companies has to manage dynamically the traffic at the most important connection stations. Due to the traffic hazardous and to the insecurity, daily tasks are done in order to minimize the connection delay and the station congestion. Due to the traffic hazards, the daily monitoring is very complex. For this reason, it is wiser to take into account these costly activities at the timetable design phase. This can minimize the occurrence of the undesired cases or at least facilitates monitoring tasks.

The proposed approach may be generalized to deal with the daily monitoring of the traffic hazards at the connection stations. We expect to adapt, in a near future, the MCSPN model for the daily optimization of the connection using other random functions.

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