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FAULT DETECTION IN MULTIRATE SAMPLED-DATA SYSTEMS WITH TIME-DELAYS

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Abstract: In this paper, an approach is proposed to the fault detection in multirate sampled-data (MSD) systems with multiple time-delays in both input and output channels. The background of our study is the increasing demands for fault detection in complex, distributed process control systems, where the plant, controllers, sensors and actuators are networked by standardized bus systems. The core of the approach proposed is a) to derive parity relations of the MSD system with time-delays while taking the different sampling rates and time-delays into account; b) to take the intersample behaviour of the continuous-time disturbances and faults into consideration with the help of operators. *Copyright ©2002 IFAC*

Keywords: Fault detection, multirate, sampled-data systems, time-delay, robustness.

1. INTRODUCTION

In recent years, model based fault detection and isolation (FDI) technology is receiving more and more attention. Implementation of observer based FDI schemes, parity space approach and parameter estimation based fault identification on a computer system is the state of the art (Gertler, 1998; Chen and Patton, 1999; Frank *et al.*, 2000).

Fig.1 sketches a typical application of an FDI system in a process control system. The process under consideration is a continuous-time process. Both the controller and the FDI system are discrete-time systems which are implemented on a computer system. The process output signals are discretized by A/D converters and then fed to the controller as well as to the FDI system. The D/A converters convert the discrete-time control input signals into continuous-time signals. Since both continuous-time and discrete-time signals exist in the system, the system design should be indeed considered from the viewpoint of a sampled-data (SD) system.

During the last decade, the topic on SD system control has been intensively studied. The achieved results show a significant improvement in control performance when the so-called direct design of digital controller for continuous-time process is adopted (Chen and Francis, 1995; Rosenwasser and Lampe, 2000). Consequently, on account of the intimate relationship between the control and FDI problems, research and practical realisation of FDI in SD systems increasingly receive attention.

Recently, Zhang *et al.* (2001) have formulated the FDI problem for SD systems and demonstrated

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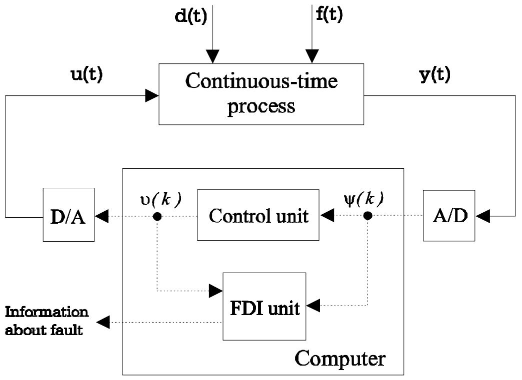

Fig. 1. Schematic description of the application of an FDI system in a process control system

that indirect design of fault detection systems for the SD system, i.e. *analog design and SD implementation* or *discrete-time design based on a discretization of the process model*, may not achieve the desired performance because of the approximation in the design phase. Motivated by this knowledge, a direct design approach is then proposed, whose key is the introduction of an operator in order to describe the sampling effect.

In industrial applications, modern complex control systems may be distributedly structured, where the plant, controller, sensors and actuators are networked by standardized bus systems (Zhang and Branicky, 2001). Often, the A/D and D/A converters in different input and output channels have different working frequencies (Patton *et al.*, 1995). Furthermore, the data transmission to and from the central control and supervision station may cause time-delays. All these require an extension of the existing FDI theory and technology in order to solve FDI problems in such kinds of processes. With this background, an approach is proposed in this paper to deal with the fault detection problem for the so-called multirate sampled-data (MSD) systems with multiple time-delays in both input and output channels.

2. PROBLEM FORMULATION

2.1 System description

The system under consideration consists of three parts:

(1) continuous linear time-invariant (LTI) process

$$\dot{x}(t) = Ax(t) + Bu(t) + E_d d(t) + E_f f(t)$$
$$y(t) = Cx(t) \qquad (1)$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^p$ the vector of control inputs, $d \in \mathbf{R}^{k_d}$ the vector of unknown disturbances and $f \in \mathbf{R}^{k_f}$ the vector of faults to be detected, $y \in \mathbf{R}^m$ the vector of process outputs. Under the assumption that model (1) describes the process dynamics including the dynamics of the anti-aliasing (low-pass) filter before the sampler, without loss of generality, it is assumed that the process model under consideration is strictly proper.

(2) A/D converters and data transmission in output channels

$$\psi_l(k^l) = y_l(k^l T_{y,l} - \tau_{y,l})$$
$$l = 1, 2, \cdots, m; \; k^l = 0, 1, 2, \cdots \qquad (2)$$

where y_l is the l-th process output, ψ_l is the sampled version of y_l, $T_{y,l}$ and $\tau_{y,l}$ are the sampling period and time-delay in the l-th process output channel respectively, $k^1, k^2, \cdots, k^m$ are used to denote the different discrete time sets due to the different sampling rates.

(3) D/A converters and data transmission in input channels

$$u_j(t) = \upsilon_j(k^j), \; j = 1, 2, \cdots, p; \;\; k^j = 0, 1, 2, \cdots$$
$$k^j T_{u,j} + \tau_{u,j} \le t < (k^j + 1)T_{u,j} + \tau_{u,j} \qquad (3)$$

where υ_j is the j-th discrete-time control input sequence given out by the computer, u_j is the j-th continuous-time control input fed to the process, $T_{u,j}$ and $\tau_{u,j}$ are the period and time-delay in the j-th control input channel respectively.

For the MSD system with time-delays, the fault detection problem can be formulated as: Design an optimal discrete-time FD system, which makes use of the control input sequences υ_j ($j = 1, 2, \cdots, p$) and the sampled process output sequences ψ_l ($l = 1, 2, \cdots, m$), so that it is robust to disturbances $d(t)$ while sensitive to faults $f(t)$.

2.2 A motivating example

In this subsection, it is shown through a simple but illustrative example that, not to mention the time-delays, the performance of the FD systems may be strongly influenced if the different sampling rates are not taken into consideration.

Given a process model in the form of (1) with

$$A = \begin{bmatrix} -1 & 5 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, E_d = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix} \qquad (4)$$
$$E_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The sampling periods in the first output channel, the second output channel and the input channel are $0.5s$, $1s$ and $0.5s$, respectively. All the simulations are made under the same conditions: $d(t)$ is white noise with noise power being 1; $f(t)$ is a step function with the step time at the 60th second and amplitude 10.

Since the sampled values in both input and output channels are available at every $1s$. It seems natural to design the FD systems in the following two ways:

Indirect approach I: First a continuous-time FD system is designed based on the continuous-time process model (4), which yields

$$\dot{\zeta}(t) = G\zeta(t) + Hu(t) + Ly(t) \quad (5)$$
$$r(t) = -q_\zeta \zeta(t) + q_u u(t) + q_y y(t) \in \mathbf{R}$$

with

$$G = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}, H = \begin{bmatrix} -0.02 \\ 0.001 \end{bmatrix}$$
$$L = \begin{bmatrix} -0.2 & 1.0 \\ -0.19 & -0.03 \end{bmatrix}, q_u = 0$$
$$q_y = \begin{bmatrix} -0.01 & 0.001 \end{bmatrix}, q_\zeta = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Note that for the continuous-time process (4), (5) is an ideal FD system because the transfer function from $d(s)$ to $r(s)$ is zero, i.e. the residual generated by (5) is perfectly decoupled from disturbances, see also Fig.2.

Doing a discretization of the resulting FD system (5) with sampling period $1s$ yields

$$\zeta(k+1) = G_d\zeta(k) + H_d u(k) + L_d y(k) \quad (6)$$
$$r(k) = -q_\zeta \zeta(k) + q_u u(k) + q_y y(k) \in \mathbf{R}$$

with

$$G_d = \begin{bmatrix} 0.66 & -0.53 \\ 0.53 & 0.13 \end{bmatrix}, H_d = \begin{bmatrix} -0.02 \\ 0.006 \end{bmatrix}$$
$$L_d = \begin{bmatrix} -0.11 & 0.86 \\ -0.17 & -0.31 \end{bmatrix}$$

Indirect approach II: First the process model (4) is discretized with sampling period $1s$, which yields

$$x(k+1) = A_{d2}x(k) + B_{d2}u(k) + E_{dd2}d(k) + E_{fd2}f(k)$$
$$y(k) = Cx(k) \quad (7)$$

with

$$A_{d2} = \begin{bmatrix} 0.37 & 1.16 \\ 0 & 0.14 \end{bmatrix}, B_{d2} = \begin{bmatrix} 1.00 \\ 0.43 \end{bmatrix}$$
$$E_{dd2} = \begin{bmatrix} 1.06 \\ 0.43 \end{bmatrix}, E_{fd2} = \begin{bmatrix} 1.00 \\ 0.43 \end{bmatrix}$$

Based on (7), a discrete-time FD system is designed. Set $s = 2$ and apply the standard parity space method. As a result, we get the optimal parity vector

$$w_s = \begin{bmatrix} 0.0 & -0.0 & 0.13 & 0.30 & -0.36 & 0.88 \end{bmatrix}$$

which allows us to construct residual generator as

$$r_k = w_s\left(\begin{bmatrix} y(k-2) \\ y(k-1) \\ y(k) \end{bmatrix} - H_{u,s} \begin{bmatrix} u(k-2) \\ u(k-1) \\ u(k) \end{bmatrix} \right) \quad (8)$$

with

$$H_{u,s} = \begin{bmatrix} O & O & O & O \\ CB_{d2} & O & O & O \\ CA_{d2}B_{d2} & CB_{d2} & O & O \\ CA_{d2}^2 B_{d2} & CA_{d2}B_{d2} & CB_{d2} & O \end{bmatrix}$$

Note that

$$w_s \begin{bmatrix} O & O & O & O \\ CE_{dd2} & O & O & O \\ CA_{d2}E_{dd2} & CE_{dd2} & O & O \\ CA_{d2}^2 E_{dd2} & CA_{d2}E_{dd2} & CE_{dd2} & O \end{bmatrix} = 0$$

so for the purely discrete-time system (7), the discrete-time FD system (8) achieves a perfect decoupling from disturbances, see also Fig.3.

To check the performance of the discrete-time FD systems designed above in the MSD system, (6) and (8) are applied to the MSD system respectively. Unfortunately, neither (6) nor (8) can detect the fault, as shown in Fig. 4 and 5.

The above observation motivates us to look for a new design approach.

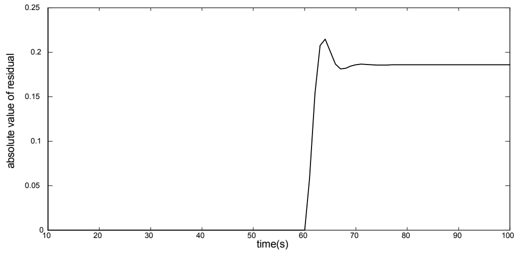

Fig. 2. Simulation result of applying (5) to (4)

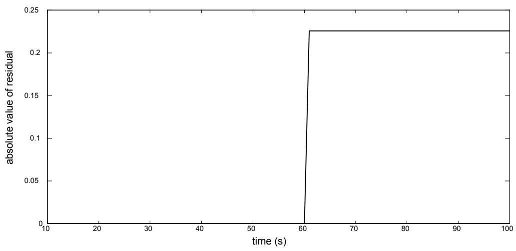

Fig. 3. Simulation result of applying (8) to (7)

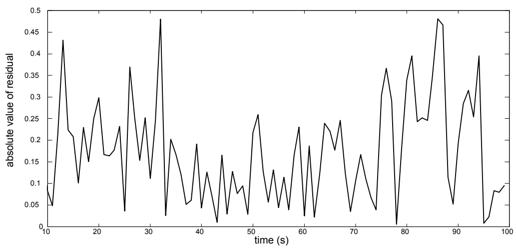

Fig. 4. Simulation result of applying (6) to the MSD system

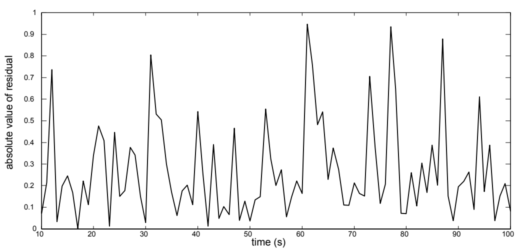

Fig. 5. Simulation result of applying (8) to the MSD system

3. A NEW DESIGN APPROACH

The MSD system with time-delays is a periodic time-varying system. Its least period, denoted by T, is the least common multiple of the periods of all A/D and D/A converters in the system. For convenience, define

$$\bar{\alpha}_j = T/T_{u,j},\ \bar{\beta}_l = T/T_{y,l} \tag{9}$$

It is assumed that the detection instants are kT ($k = 0, 1, \cdots$) and that the values of ψ_l ($l = 1, 2, \cdots, m$) during the period from $kT - sh$ to kT will be used for fault detection.

Let h be the greatest common divisor of all the periods $T_{u,j}$, $T_{y,l}$ and time-delays $\tau_{u,j}$ and $\tau_{y,l}$. Define

$$\underline{\alpha}_j = T_{u,j}/h,\ \underline{\beta}_l = T_{y,l}/h,\ \vartheta = T/h \tag{10}$$

$$\sigma_j = \tau_{u,j}/h,\ \varepsilon_l = \tau_{y,l}/h \tag{11}$$

where $j = 1, \cdots, p;\ \ l = 1, 2, \cdots, m$. Let $\varepsilon_{\max}$ and $\varepsilon_{\min}$ denote the maximal and minimal value in the set $\{\varepsilon_1, \varepsilon_2 \cdots, \varepsilon_m\}$ respectively and define

$$\begin{aligned} k_{st} &= k\vartheta - s - \varepsilon_{\max},\ k_{end} = k\vartheta - \varepsilon_{\min} \\ \delta_s &= k_{end} - k_{st} \end{aligned} \tag{12}$$

At the time instants kh, the dynamics of the continuous-time process (1) is exactly described by

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) + \bar{d}(k) + \bar{f}(k) \\ y(k) &= Cx(k) \end{aligned} \tag{13}$$

where

$$x(k) = x(kh), u(k) = u(kh), y(k) = y(kh)$$

$$A_d = e^{Ah},\ B_d = \int_0^h e^{At} B dt$$

$$\bar{d}(k) = \int_0^h e^{A(h-t)} E_d d(kh+t) dt$$

$$\bar{f}(k) = \int_0^h e^{A(h-t)} E_f f(kh+t) dt$$

Let $k = k_{st}, k_{st}+1, \cdots, k_{end}$. Then a group of input-output relations are obtained as

$$\begin{aligned} y_{\delta_s}(k_{end}) &= H_{o,\delta_s} x(k_{st}) + H_{u,\delta_s} u_{\delta_s}(k_{end}) \\ &\quad + H_{\delta_s}(\bar{d}_{\delta_s}(k_{end}) + \bar{f}_{\delta_s}(k_{end})) \end{aligned} \tag{14}$$

where

$$y_{\delta_s}(k_{end}) = \left[\, y'(k_{st}) \ \cdots \ y'(k_{end}) \,\right]'$$

$$u_{\delta_s}(k_{end}) = \left[\, u'(k_{st}) \ \cdots \ u'(k_{end}) \,\right]'$$

$$\bar{d}_{\delta_s}(k_{end}) = \left[\, \bar{d}'(k_{st}) \ \cdots \ \bar{d}'(k_{end}) \,\right]'$$

$$\bar{f}_{\delta_s}(k_{end}) = \left[\, \bar{f}'(k_{st}) \ \cdots \ \bar{f}'(k_{end}) \,\right]'$$

$$H_{o,\delta_s} = \left[\, C' \ A_d'C' \ \cdots \ (A_d^{\delta_s})'C' \,\right]'$$

$$H_{u,\delta_s} = \begin{bmatrix} O & O & \cdots & O \\ CB_d & O & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ CA_d^{\delta_s-1}B_d & \cdots & CB_d & O \end{bmatrix}$$

$$H_{\delta_s} = \begin{bmatrix} O & O & \cdots & O \\ C & O & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ CA_d^{\delta_s-1} & \cdots & C & O \end{bmatrix}$$

However, (14) can not be directly used for the construction of residual generator, because

- due to the different sampling rates, not all the components in the vector $y_{\delta_s}(k_{end})$ are available;
- due to the multiple time-delays in input and output channels, the relative order of the components in the vectors $y_{\delta_s}(k_{end})$ and $u_{\delta_s}(k_{end})$ will change.

To treat the different sampling rates and time-delays in the output channels, the first step is to find out which components in the vector $y_{\delta_s}(k_{end})$ have available sampled values. To this end, define some subscript sets as

$$\begin{aligned} \Omega_i = \{\, l \mid 1 \le l \le m,\ l \in \mathbf{N},\ & \frac{s + \varepsilon_{\max} - i - \varepsilon_l}{\underline{\beta}_l} \\ & \in \mathbf{Z} \text{ and } \varepsilon_{\max} - i \le \varepsilon_l \le s + \varepsilon_{\max} - i \,\} \\ i = 0, 1, \cdots \delta_s & \end{aligned} \tag{15}$$

Assume that the set Ω_i has a total of μ_i components which are denoted as $\rho_{i1}, \rho_{i2}, \cdots, \rho_{i\mu_i}$ in ascending order, i.e. $1 \le \rho_{i1} < \rho_{i2} < \cdots < \rho_{i\mu_i} \le m$. Let

$$y_{\Omega_i}(k_{st}+i) = [\, y_{\rho_{i1}}(k_{st}+i) \ \cdots \ y_{\rho_{i\mu_i}}(k_{st}+i) \,]'$$

where y_l denotes the l-th process output. Further, let

$$\hat{y}_{\delta_s}(k_{end}) = [\, y'_{\Omega_0}(k_{st}) \ \cdots \ y'_{\Omega_{\delta_s}}(k_{end}) \,]'$$

$\hat{y}_{\delta_s}(k_{end})$ consists of only those components in $y_{\delta_s}(k_{end})$ with available sampled values. To pick out the corresponding equations in (14), define the matrices C_{Ω_i} as

$$C_{\Omega_i} = [\, c'_{\rho_{i1}} \ \cdots \ c'_{\rho_{i\mu_i}} \,]',\ i = 0, 1, \cdots \delta_s \tag{16}$$

where c_l is the l-th row of the matrix C.

The second step is to determine the relationship between $\hat{y}_{\delta_s}(k_{end})$ and the sampled process output sequences $\psi_l(k^l)$ ($l = 1, \cdots, m;\ k^l = 0, 1, \cdots$). From (2), there is

$$y_l(k_{st}+i) = \psi_l\left(\frac{k_{st}+i+\varepsilon_l}{\underline{\beta}_l}\right)$$

thus

$$y_{\Omega_i}(k_{st}+i) = \begin{bmatrix} \psi_{\rho_{i1}}\left(\dfrac{k_{st}+i+\varepsilon_{\rho_{i1}}}{\underline{\beta}_{\rho_{i1}}}\right) \\ \vdots \\ \psi_{\rho_{i\mu_i}}\left(\dfrac{k_{st}+i+\varepsilon_{\rho_{i\mu_i}}}{\underline{\beta}_{\rho_{i\mu_i}}}\right) \end{bmatrix} \tag{17}$$

Denote the vector on the right side of (17) as ψ_{Ω_i} and define

$$\psi_{k,\delta_s} = [\, \psi'_{\Omega_0} \ \cdots \ \psi'_{\Omega_{\delta_s}} \,]' \tag{18}$$

Apparently, there is

$$\hat{y}_{\delta_s}(k_{end}) = \psi_{k,\delta_s} \tag{19}$$

To treat the different periods and time-delays in the input channels, note that from (3) there is

$$u_j(k_{st}+i) = \upsilon_j(k_i^j), \ j = 1, 2, \cdots, p \tag{20}$$

where $k_i^j \in \mathbf{Z} \cap ((k_{st}+i-\sigma_j)/\underline{\alpha}_j - 1, (k_{st}+i-\sigma_j)/\underline{\alpha}_j]$. Thus

$$u(k_{st}+i) = [\, \upsilon_1(k_i^1) \ \cdots \ \upsilon_p(k_i^p) \,]' \tag{21}$$

Denote the vector on the right side of (21) as υ_i and define

$$\upsilon_{k,\delta_s} = [\, \upsilon'_0 \ \cdots \ \upsilon'_{\delta_s} \,]' \tag{22}$$

Then the vector $u_{\delta_s}(k_{end})$ in (14) can be expressed by the control input sequences $\upsilon_j(k^j)$ ($j = 1, \cdots, p$; $k^j = 0, 1, \cdots$) as

$$u_{\delta_s}(k_{end}) = \upsilon_{k,\delta_s} \tag{23}$$

Based on (16), (19) and (23), those rows of (14) with available sampled values are picked out and reorganized into a new equation group in the form of

$$\begin{aligned} \psi_{k,\delta_s} = {} & \hat{H}_{o,\delta_s} x(k_{st}) + \hat{H}_{u,\delta_s} \upsilon_{k,\delta_s} \\ & + \hat{H}_{\delta_s}(\bar{d}_{\delta_s}(k_{end}) + \bar{f}_{\delta_s}(k_{end})) \end{aligned} \tag{24}$$

where $\hat{H}_{o,\delta_s}$, $\hat{H}_{u,\delta_s}$ and $\hat{H}_{\delta_s}$ are composed of some rows picked out from H_{o,δ_s}, H_{u,δ_s} and H_{δ_s}, respectively, as

$$\hat{H}_{o,\delta_s} = \begin{bmatrix} C'_{\Omega_0} & A'_d C'_{\Omega_1} & \cdots & (A_d^{\delta_s})' C'_{\Omega_{\delta_s}} \end{bmatrix}'$$

$$\hat{H}_{u,\delta_s} = \begin{bmatrix} O & O & \cdots & O \\ C_{\Omega_1} B_d & O & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_{\Omega_{\delta_s}} A_d^{\delta_s - 1} B_d & \cdots & C_{\Omega_{\delta_s}} B_d & O \end{bmatrix}$$

$$\hat{H}_{\delta_s} = \begin{bmatrix} O & O & \cdots & O \\ C_{\Omega_1} & O & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_{\Omega_{\delta_s}} A_d^{\delta_s - 1} & \cdots & C_{\Omega_{\delta_s}} & O \end{bmatrix}$$

Based on (24), a parity relation based residual generator can be constructed as

$$r_k = w_{\delta_s}(\psi_{k,\delta_s} - \hat{H}_{u,\delta_s} \upsilon_{k,\delta_s}), \ w_{\delta_s} \in P_{\delta_s} \tag{25}$$

where w_{δ_s} is the parity vector to be selected and P_{δ_s} is the parity space defined as

$$P_{\delta_s} = \{ w_{\delta_s} \mid w_{\delta_s} \hat{H}_{o,\delta_s} = 0, w_{\delta_s} \in \mathbf{R}^{1 \times (\mu_0 + \cdots + \mu_{\delta_s})} \}$$

The dynamics of residual generator (25) is governed by

$$r_k = w_{\delta_s} \hat{H}_{\delta_s} (\bar{d}_{\delta_s}(k_{end}) + \bar{f}_{\delta_s}(k_{end})) \tag{26}$$

Bearing in mind that the main objective of designing residual generators is to enhance the robustness of the FD system to disturbances $d(t)$ without loss of the sensitivity to faults $f(t)$, two operators Γ_{E_d} and Γ_{E_f} are introduced in order to describe and analyse the influence of $d(t)$ and $f(t)$ on the residual r_k quantitatively. Define

$$\begin{aligned} \bar{d}_{\delta_s}(k_{end}) &= \Gamma_{E_d} d_{k,\delta_s}(t) \\ \bar{f}_{\delta_s}(k_{end}) &= \Gamma_{E_f} f_{k,\delta_s}(t) \end{aligned} \tag{27}$$

Γ_{E_d} and Γ_{E_f} reflect the mapping relationship from the continuous-time signals $d(t)$ and $f(t)$ over a finite horizon $[k_{st}h, k_{end}h)$ to the discrete-time ones respectively (Zhang *et al.*, 2001). The dynamics (26) of residual generator can then be re-written as

$$r_k = w_{\delta_s} \hat{H}_{\delta_s} (\Gamma_{E_d} d_{k,\delta_s}(t) + \Gamma_{E_f} f_{k,\delta_s}(t)) \tag{28}$$

Thus the optimal selection of design parameter w_{δ_s} can be formulated as an optimization problem

$$\min_{w_{\delta_s} \in P_{\delta_s}} J = \min_{w_{\delta_s} \in P_{\delta_s}} \frac{w_{\delta_s} \hat{H}_{\delta_s} \Gamma_{E_d} \Gamma^*_{E_d} \hat{H}'_{\delta_s} w'_{\delta_s}}{w_{\delta_s} \hat{H}_{\delta_s} \Gamma_{E_f} \Gamma^*_{E_f} \hat{H}'_{\delta_s} w'_{\delta_s}} \tag{29}$$

Because there exist always matrices $\bar{E}_d$ and $\bar{E}_f$ so that (Chen and Francis, 1995; Zhang *et al.*, 2001)

$$\begin{aligned} \bar{E}_d \bar{E}'_d &= \int_0^h e^{A\tau} E_d E'_d e^{\tau A'} d\tau \\ \bar{E}_f \bar{E}'_f &= \int_0^h e^{A\tau} E_f E'_f e^{\tau A'} d\tau \\ \Gamma_{E_d} \Gamma^*_{E_d} &= diag(\bar{E}_d \bar{E}'_d, \cdots, \bar{E}_d \bar{E}'_d) \\ \Gamma_{E_f} \Gamma^*_{E_f} &= diag(\bar{E}_f \bar{E}'_f, \cdots, \bar{E}_f \bar{E}'_f) \end{aligned} \tag{30}$$

the optimization problem (29) is equivalent to

$$\min_{w_{\delta_s} \in P_{\delta_s}} J = \min_{w_{\delta_s} \in P_{\delta_s}} \frac{w_{\delta_s} \hat{H}_{dd,\delta_s} \hat{H}'_{dd,\delta_s} w'_{\delta_s}}{w_{\delta_s} \hat{H}_{df,\delta_s} \hat{H}'_{df,\delta_s} w'_{\delta_s}} \tag{31}$$

where

$$\hat{H}_{dd,\delta_s} = \begin{bmatrix} O & O & \cdots & O \\ C_{\Omega_1} \bar{E}_d & O & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_{\Omega_{\delta_s}} A_d^{\delta_s - 1} \bar{E}_d & \cdots & C_{\Omega_{\delta_s}} \bar{E}_d & O \end{bmatrix}$$

$$\hat{H}_{df,\delta_s} = \begin{bmatrix} O & O & \cdots & O \\ C_{\Omega_1} \bar{E}_f & O & \ddots & \vdots \\ \vdots & \ddots & \ddots & O \\ C_{\Omega_{\delta_s}} A_d^{\delta_s - 1} \bar{E}_f & \cdots & C_{\Omega_{\delta_s}} \bar{E}_f & O \end{bmatrix}$$

The solution to (31) and also to (29) is

$$w_{\delta_s} = p_{\delta_s} N_{basis} \tag{32}$$

where N_{basis} is the basis matrix of parity space P_{δ_s}, p_{δ_s} is the eigenvector corresponding to the minimal eigenvalue of the generalized eigenvalue-eigenvector problem (Ding *et al.*, 2000)

$$\begin{aligned} p_{\delta_s}(& N_{basis} \hat{H}_{dd,\delta_s} \hat{H}'_{dd,\delta_s} N'_{basis} - \\ & \lambda_{\min} N_{basis} \hat{H}_{df,\delta_s} \hat{H}'_{df,\delta_s} N'_{basis}) = 0 \end{aligned} \tag{33}$$

Algorithm 1 Optimal design of discrete-time FD system for the MSD system with time-delays described by (1)-(3):

- Compute the period T of the whole system.
- Determine $h, \vartheta, \underline{\alpha}_j, \bar{\alpha}_j, \underline{\beta}_l, \bar{\beta}_l, \sigma_j, \varepsilon_l$.
- Compute A_d, B_d, $\bar{E}_d$ and $\bar{E}_f$.
- Determine the subscript sets Ω_i and the matrices C_{Ω_i} for $i = 0, 1, \cdots, \delta_s$.
- Determine the matrices $\hat{H}_{o,\delta_s}$, N_{basis}, $\hat{H}_{u,\delta_s}$, $\hat{H}_{dd,\delta_s}$ and $\hat{H}_{df,\delta_s}$.
- Solve (33) to get the optimal parity vector w_{δ_s}.
- Construct the residual generator (25) with ψ_{k,δ_s} and υ_{k,δ_s} constructed by (17)-(18) and (20)-(22) respectively.

4. SIMULATION EXAMPLE

Given the same process model (4) as in Section 2.2 and suppose the sampling periods and time-delays are $T_{y,1} = 0.5s$, $T_{y,2} = 1s$, $T_u = 0.5s$ and $\tau_{y,1} = 0.5s$, $\tau_{y,2} = 2s$, $\tau_u = 1s$ respectively. A discrete-time residual generator is designed with the approach proposed in this paper.

Apparently the period of the system is $T = 1s$ and $h = 0.5s$, $\vartheta = 2$, $\underline{\alpha} = 1$, $\bar{\alpha} = 2$, $\underline{\beta}_1 = 1$, $\bar{\beta}_1 = 2$, $\underline{\beta}_2 = 2$, $\bar{\beta}_2 = 1$, $\sigma = 2$, $\varepsilon_1 = 1$, $\varepsilon_2 = 4$ and

$$A_d = \begin{bmatrix} 0.61 & 1.19 \\ 0 & 0.37 \end{bmatrix}, B_d = \begin{bmatrix} 0.39 \\ 0.32 \end{bmatrix}$$

$$\bar{E}_d = \begin{bmatrix} 0.65 & 0 \\ 0.37 & 0.28 \end{bmatrix}, \bar{E}_f = \begin{bmatrix} 0.60 & 0 \\ 0.36 & 0.30 \end{bmatrix}$$

Set $s = 4$. Then $k_{st} = 2k - 8$, $k_{end} = 2k - 1$, $\delta_s = 7$. The subscript sets are $\Omega_0 = \Omega_2 = \{2\}$, $\Omega_1 = \emptyset$, $\Omega_3 = \Omega_5 = \Omega_6 = \Omega_7 = \{1\}$, $\Omega_4 = \{1, 2\}$ and correspondingly, $C_{\Omega_0} = C_{\Omega_2} = c_2$, $C_{\Omega_1} = \emptyset$, $C_{\Omega_3} = C_{\Omega_5} = C_{\Omega_6} = C_{\Omega_7} = c_1$, $C_{\Omega_4} = C$. Solving (33) yields the optimal parity vector

$$w_{\delta_s} = [\,0\ \ -0.20\ \ -0.90\ \ 1.49\ \ -3.31\ \ 0\ \ 0\ \ 0\,]$$

Construct the residual generator according to (25) as

$$r_k = w_{\delta_s}(\psi_{k,\delta_s} - \hat{H}_{u,\delta_s}\upsilon_{k,\delta_s}) \in \mathbf{R} \qquad (34)$$

with

$$\begin{aligned}\psi_{k,\delta_s} = [\,&\psi_2(k-2)\ \psi_2(k-1)\ \psi_1(2k-4) \\ &\psi_1(2k-3)\ \psi_2(k)\ \psi_1(2k-2) \\ &\psi_1(2k-1)\ \psi_1(2k)\,]' \\ \upsilon_{k,\delta_s} = [\,&\upsilon(2k-10)\ \upsilon(2k-9)\ \cdots\ \upsilon(2k-3)\,]'\end{aligned}$$

Apply the resulting residual generator (34) to the MSD system with time-delays and do the simulation under the same conditions as in Section 2.2. Figure 6 shows that the fault can be successfully detected and demonstrates the effectiveness of the design approach given in Algorithm 1.

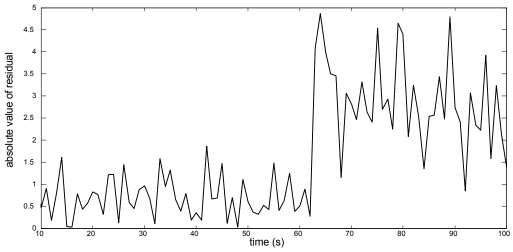

Fig. 6. Simulation result of applying (34) to the MSD system with time-delays

5. CONCLUSIONS

In this paper, an approach is proposed to the fault detection for MSD systems with multiple time-delays. The different sampling rates and multiple time-delays are taken into account during the derivation of parity relations. Moreover, the inter-sample behaviour of continuous-time disturbances and faults is taken into consideration with the help of operators. In this way all available information can be exploited to the largest extent and no approximation is made during the design phase. This ensures a better handling of effects due to the sampling and times-delays and thus results in an improvement in FD performance.

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