

A PROBABILISTIC APPROACH TO COLLISION RISK ESTIMATION FOR PASSENGER VEHICLES

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Abstract: Active safety and collision avoidance (CA) is a growing field within the automotive industry. The aim of CA systems is to prevent or mitigate collisions by active interventions i.e. warning, braking and steering. For many reasons, such as driver acceptance of the system and legal requirement that the system itself must not cause hazards, the decision making is a crucial part of the system. This paper presents a method for risk estimation on which to base the decision making. We will show how one can form criterions for decision making in terms of probability of collision. This criterion handles the noisy sensor data and process noise (driver behavior) in a natural way, using existing tracking theory. The method is illustrated by simulation results as well as test result from a prototype vehicle. Simulations and tests are examples of a system which performs autonomous braking actuation at imminent collision.

Keywords: Collision Avoidance, Collision Mitigation, Tracking, Decision Making, Collision Probability

1. INTRODUCTION

The passive safety of passenger cars has significantly improved over the last decades. As existing safety technologies grow mature many automotive manufacturers have begun looking at alternative ways to improve the passenger safety. One alternative is to introduce driver support systems that help the driver avoiding collision by active intervention (i.e. warning, braking, steering etc.). The developments of these systems are driven by the fact that driver errors cause a majority of all car accidents (Treat *et al.* 1977). In this paper we will discuss one type of driver support system which will be called a collision mitigation by braking (CMBB) system. The system uses sensors to monitor the environment directly in front of the vehicle. The aim of the system is to avoid or mitigate collisions by letting the vehicle autonomously apply the brakes as the risk for collision becomes to high. Other actions that could be considered are warning and steering manoeuvres. Examples of sensors that can be used to perceive other objects are radar, laser radar (lidar) and cameras. All of these sensors are already being installed in production vehicles to realize

automatic cruise control functions.

The two main desired properties on a collision avoidance system are:

- (1) Avoid all collisions
- (2) Make no faulty interventions

A faulty intervention is defined as an intervention that occurs when the driver would have avoided collision without the help of the system. These two properties are in contradiction and the decision making algorithm has to be a good trade off between them. For CMBB systems the tolerance for faulty intervention is very low. The focus for the decision making algorithm presented in this paper will thus be on avoiding faulty interventions.

The decision when to intervene, depends on several factors. Some of the most important factors are sensor uncertainty and driver behavior. Below we will present a method for collision risk estimation, which will form a base for the decision making.

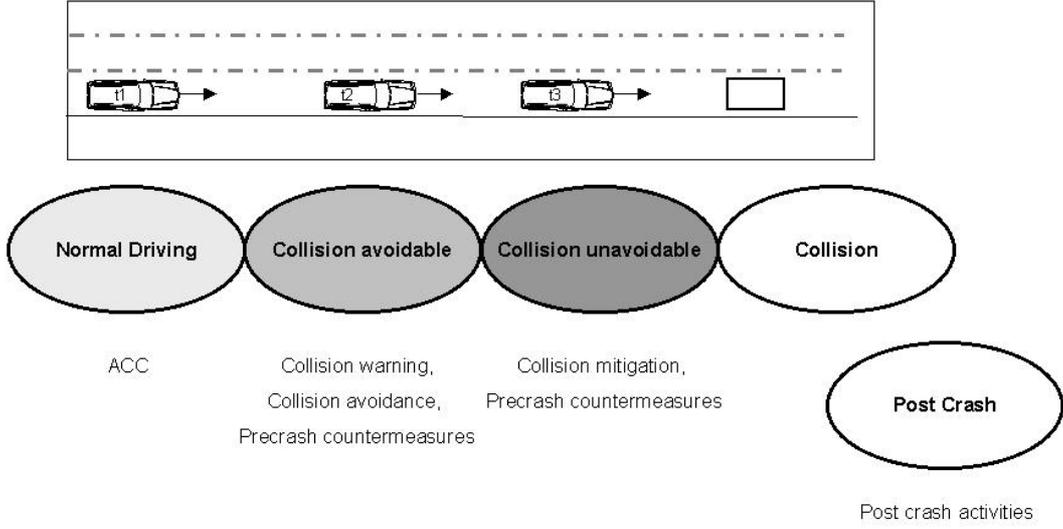


Fig. 1. Driving states of a vehicle approaching a stationary obstacle

2. DECISION MAKING

To simplify the analysis of decision making we divide driving into five states:

- (1) Normal driving
- (2) Collision avoidable
- (3) Collision unavoidable
- (4) Collision
- (5) Post collision state.

An example of these states are given in Figure 1. As we have already stated the tolerance for faulty interventions is low in a CMBB system. Therefore brake maneuvers are normally initiated close to the border between the collision avoidable and unavoidable states. Considering measurement and modeling errors, the intervention cannot be initiated before the collision unavoidable state has been entered, if one wants to be certain not to make a faulty intervention.

Many algorithms suggest to use metrics based on longitudinal motion such as time to collision, relative speeds and distances and deceleration for decision making ((Tamura *et al.* 2001), (Seiler *et al.* n.d.) and (Doi *et al.* 1994)). The underlying assumption in these systems is that the observed vehicle is in the same lane. These metrics although intuitive and easy to understand, might be difficult to use for complex driving situations. To determine if a collision is unavoidable or not, one generally has to consider motion both in the longitudinal and the lateral direction.

We here propose to use the probability of collision ($P_t(\text{collision})$) as a metric for decision making. The probability of collision can be calculated according to equation (1).

$$\begin{aligned}
 P_t(\text{collision}) &= P_t(P_{POV} - P_{host} \in D) \\
 &= \iiint_{\delta p_x, \Delta p_y, \Delta \psi \in D} p_t(\Delta p_x, \Delta p_y, \Delta \psi | Y_t) dx dy d\psi
 \end{aligned} \tag{1}$$

Here:

P_{host} = Position of the host vehicle

P_{POV} = Position of the principal other vehicle (POV)

D = The area which corresponds to a collision (i.e. the area that corresponds to physical overlap of the two vehicles.

Δp_x = relative position in the x coordinate

Δp_y = relative position in the y coordinate

$\Delta \psi$ = relative heading angle

The probability density function (PDF) of the vehicle's relative position to each other $p_t(\Delta p_x, \Delta p_y, \Delta \psi | Y_t)$ is obtained directly from the Bayesian solution to the tracking problem, if we choose relative coordinates. If we use ground fixed coordinates we first have to calculate the density in order to calculate the collision probability. Assuming that the states x_{host} and x_{POV} are independent the density of $\Delta x = x_{host} - x_{POV}$ is given by the convolution in Equation (2).

$$p_{\Delta x, t}(\Delta x) = \int_{-\infty}^{\infty} p_{x_{POV}}(x) p_{x_{host}}(x - \Delta x) dx \tag{2}$$

In the case were x_{host} and x_{POV} are Gaussian and independent

$$\begin{aligned}
 x_{POV} - x_{host} &\in \mathcal{N}(\hat{x}_{host} - \hat{x}_{POV} \\
 &\quad , \text{Cov}(x_{host}) + \text{Cov}(x_{POV})) \\
 &= \mathcal{N}(\Delta \hat{x}, \text{Cov}(\Delta x)).
 \end{aligned} \tag{3}$$

Such estimate typically comes from a Kalman filter, which also provides the desired covariance matrices. In the next section we will discuss in more detail how to calculate an approximation of the PDF using extended Kalman filtering.

3. TRACKING

The decision making algorithms rely on the fact that information about surrounding objects states is available. It is the task of the tracking system to provide this information. In automotive application radars, laser radars, and vision systems are used for detecting and tracking surrounding objects.

In general a CMBB systems will use more than one tracking sensor. We are faced with a sensor fusion problem, that involves synchronization in time and space. In this paper we will use (extended) Kalman filtering to solve this problem. We will work with systems that can be described by Equation (4) and (5).

$$x_{t+1} = A_t x_t + v_t, \text{Cov}(v_t) = Q_t \quad (4)$$

$$y_t = C_t x_t + e_t, \text{Cov}(e_t) = R_t \quad (5)$$

The state vector x_t consist of x and y coordinate position (p_x and p_y), x and y velocity (v_x and v_y) and vehicle yaw rate (ω) as displayed in Equation (6).

$$x = \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \\ \omega \end{bmatrix} \quad (6)$$

To model the dynamics of the host vehicles and other vehicles we use the model in Equation (7).

$$A_t = \begin{bmatrix} 1 & 0 & \frac{\sin(\omega)T}{\omega} & \frac{1 - \cos(\omega)T}{\omega} & 0 \\ 0 & 1 & -\frac{1 - \cos(\omega)T}{\omega} & \frac{\sin(\omega)T}{\omega} & 0 \\ 0 & 0 & \cos(\omega)T & \sin(\omega)T & 0 \\ 0 & 0 & -\sin(\omega)T & \cos(\omega)T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

This model is often called the nearly coordinated turn model. It has been widely used in aircraft tracking, typically in applications where one is interested in tracking manoeuvring targets. The assumption of the model is that the tracked objects moves in straight lines or on circle segments. We will assume that the noise (v_t) that models the transitions between segments is Gaussian and has a known covariance Q_t . The main contributions of the process noise comes from the driver. Q_t is thus used to model what maneuvers the car/driver physically can perform. The measurement provided by the sensors are range, range rate and azimuth angle.

$$y = \begin{bmatrix} R \\ \dot{R} \\ \alpha \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta p_x^2 + \Delta p_y^2} \\ \frac{\Delta p_x \Delta v_x + \Delta p_y \Delta v_y}{(\Delta p_x^2 + \Delta p_y^2)^{3/2}} \\ \arctan\left(\frac{\Delta p_y}{\Delta p_x}\right) \end{bmatrix} \quad (8)$$

Linearizing Equation (8) we get Equation (9).

$$C_t^T = \frac{\partial y_t}{\partial x_t}^T = \begin{bmatrix} \Delta p_x R^{-1} & \Delta v_x R^{-1} - a \Delta p_x R^{-\frac{3}{2}} & -\Delta p_y (\Delta p_x b)^{-1} \\ \Delta p_y R^{-1} & \Delta v_y R^{-1} - a \Delta p_y R^{-\frac{3}{2}} & (\Delta p_x b)^{-1} \\ 0 & \Delta p_x R^{-1} & 0 \\ 0 & \Delta p_y R^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (9)$$

Where:

$$a = \Delta p_y \Delta v_y + \Delta p_x \Delta v_x$$

$b = 1 + \frac{\Delta p_y^2}{\Delta p_x^2}$ With a system described by the above equations we can use the time varying Kalman filter given by Equations (10 -13).

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + P_{t|t-1} C_t^T (C_t P_{t|t-1} C_t^T + R_t)^{-1} \times (y_t - C_t \hat{x}_{t|t-1}) \quad (10)$$

$$P_{t|t} = P_{t|t-1} + P_{t|t-1} C_t^T \times (C_t P_{t|t-1} C_t^T + R_t)^{-1} C_t P_{t|t-1} \quad (11)$$

$$\hat{x}_{t+1|t} = A_t \hat{x}_{t|t} \quad (12)$$

$$P_{t+1|t} = A_t P_t A_t^T + Q_t \quad (13)$$

Under the assumption that $v_t \in \mathcal{N}(0, \sigma_{v_t})$ and $e_t \in \mathcal{N}(0, \sigma_{e_t})$ then also $x_{t+1} \in \mathcal{N}(0, \sqrt{\text{Cov}(x_{t+1})})$. The Bayesian solution needed to calculate the probability of collision is thus given by the EKF. The PDF is given by Equation (14).

$$p(x_{t+1}|y_t) = \frac{1}{\sqrt{(2\pi)^p \det(\text{Cov}(x_{t+1}))}} \times e^{-\frac{(x_{t+1} - \hat{x}_{t+1})^T \det(\text{Cov}(x_{t+1}))^{-1} (x_{t+1} - \hat{x}_{t+1})}{2}} \quad (14)$$

The covariance of the state estimate $\text{Cov}(x) = E(x - \hat{x})(x - \hat{x})^T$ and the point estimate \hat{x}_t is obtained from the time update equation of the Kalman filtering Equations 12 and 13.

4. SYSTEM OVERVIEW

The proposed algorithm will of course only give an approximation of the true probability of collision. This is partly due to the fact that both the model of the vehicle dynamics and the process noise model are approximations. To test what performance one can achieve using commercial sensors both a simulation environment and a prototype vehicle have been built. An overview of the CMBB systems' architecture is given in Figure 2. The prototype vehicle is equipped with one radar and one lidar to detect and track objects. Both the radar and the lidar has a maximum range larger than 100 m, the range resolution is less than 1 m, the angular resolution is less than 1 degrees and the field of view is 12 degrees. The simulation environment also uses two sensor models with similar characteristics as the two physical sensors. For collision tests with the prototype

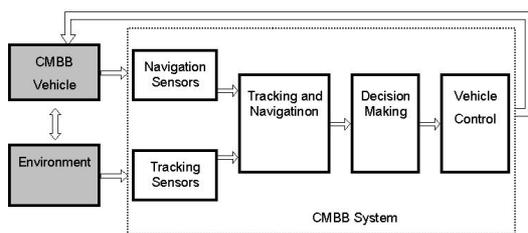


Fig. 2. Overview of the architecture of a CMBB System

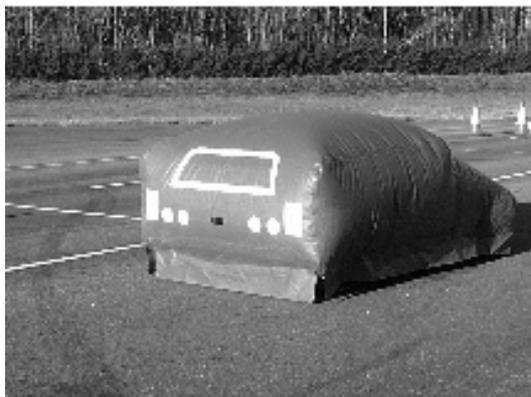


Fig. 3. Inflatable car

vehicle an inflatable car (Figure ??) was used. The strategy is to perform full braking as soon as possible when the collision unavoidable state has been entered. To determine when this event occurs the maximum probability of collision is calculated at each time instant. Maximizing the probability over time is done by calculating $P_{t+1}(\text{collision}) \dots P_{t+N}(\text{collision})$ and simply picking the maximum value. Performance of the system has been tested both in simulated scenarios as well as in physical tests. Some results from the testing is presented in the following sections.

5. SIMULATION RESULTS

The evaluation of the system can be divided into two parts. First we are interested to see if any faulty intervention occurs. Secondly we are interested in system performance in situations where an intervention is desired. Several different scenarios have been simulated to evaluate the system. In this section we will present a selection of the simulation results. Scenarios 1-3 were designed to provoke faulty interventions. These scenarios are used for system design, the threshold for intervention (probability of collision) is set sufficiently high not to have an intervention in these scenarios. All the scenarios were simulated 10 times at each speed (speeds [10 20 30 ... 150] km/h). In scenario 1-3 there are no faulty interventions (since this is how the probability threshold is chosen).

Scenario 1: Head to Head, the driver of the CMBB vehicle turns right at the last moment. Scenario 1 is displayed in Figure 4.

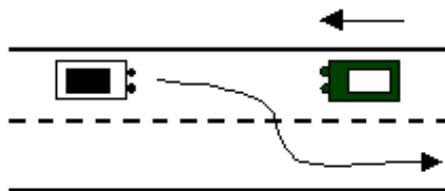


Fig. 4. Scenario 1

Scenario 2: Straight roadway, the principal other vehicle (POV) is traveling in the same lane as the CMBB vehicle. Suddenly the POV brakes hard and then turns hard. The POV just clears the path of the CMBB vehicle. Scenario 2 is displayed in Figure 5.

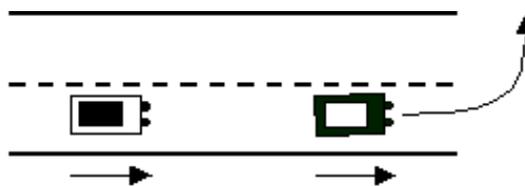


Fig. 5. Scenario 2

Scenario 3: The CMBB vehicle changes from right to left lane (both lanes in the same direction) at the same time as it meets another vehicle in the opposite lane. Scenario 3 is displayed in Figure 6.

Scenarios 4-6 presented below are used to see what performance the system achieves in situation where an intervention is appropriate. In all the scenarios we assume that the driver continues the same maneuver (steering wheel angle etc.) he was doing before the system applies the brakes.

Scenario 4: Straight roadway, the POV is traveling in the same lane as the CMBB vehicle. Suddenly the POV brakes hard (deceleration 7 m/s^2). The headway is 15 m. The initial speed, prior to the POV brake maneuver, is the same for both vehicles. Scenario 4 is displayed in Figure 7.

Result: In Figure 8 the relative speed at the collision moment is plotted as a function of the initial speed. One can see that for low speeds the relative velocity at impact is reduced 10-20 km/h. For higher speeds the system response time is too long to be able to reduce the collision speed.

Scenario 5: This scenario is the same as scenario 4

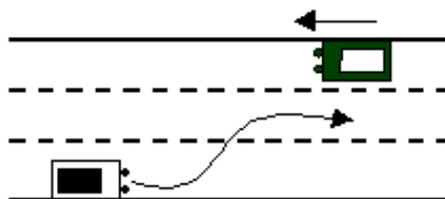


Fig. 6. Scenario 3

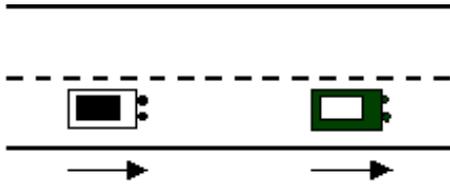


Fig. 7. Scenario 4

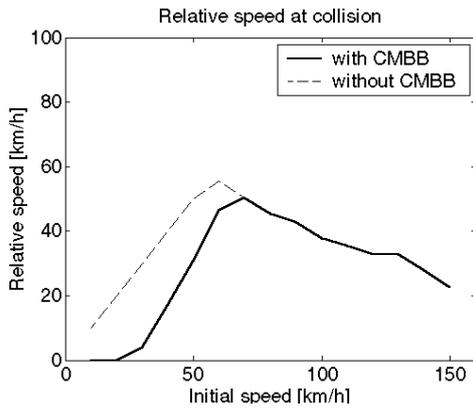


Fig. 8. Simulation results for scenario 4, relative speed at collision with and without the CMBB system with the exception that there is a lateral offset of 0.5 m between the CMBB vehicle and the POV. Scenario 5 is displayed in Figure 9.

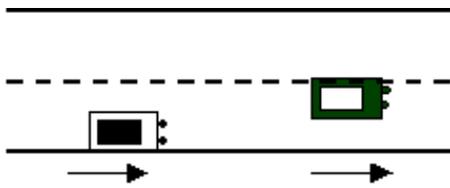


Fig. 9. Scenario 5

Result: Again the relative speed of the vehicles at impact is plotted as a function of the initial speed (in Figure 10). For comparison the results from the previous scenario has been plotted in Figure 10. In the figure one can see that there is no intervention for low speeds. The reason for this is that before the decision is made the target moves out of the sensors' field of view.

Scenario 6: An object on the side of the road suddenly jumps into the path 10 m in front of the CMBB vehicle. Scenario 6 is displayed in Figure 11. Result: The performance for scenario 6 is plotted as a phase diagram in Figure 12. This shows impact speed and at what distance the system intervenes.

6. TEST RESULTS

The test vehicle is a Volvo V70 equipped with a millimeter wavelength radar and a laser radar. The sensor update rate is 10 Hz for both sensors. The sensor fusion, data association and decision making algorithms

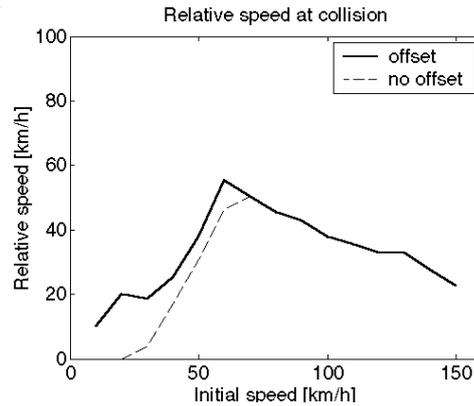


Fig. 10. Simulation results for scenario 5, the no offset result from the previous scenario is plotted for comparison

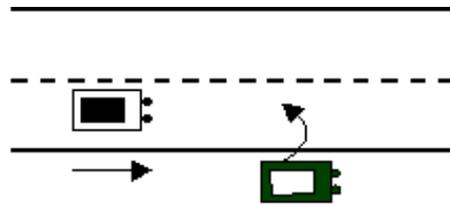


Fig. 11. Scenario 6

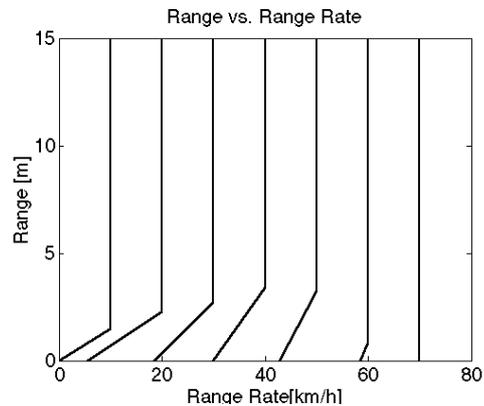


Fig. 12. Simulation results for scenario 6, Range and Range rate between the host vehicle and the POV

are executed on an on-board processing unit which is also connected to the vehicles' braking system. It normally takes about 0.3 seconds for the brake system to rise to full brake pressure (ABS-braking).

The purpose of having two sensors is to try to discriminate targets that are not valid (i.e spurious reflection and background clutter). For example a millimeter wavelength radar can receive a strong echo from a tin can. To check if the algorithm makes faulty decisions the prototype system has been driven in real traffic (urban and highway traffic) with braking disabled. We found that some faulty interventions do occur. These interventions mainly occur at low speeds when there are a lot of potential targets/obstacles close to the sensors (i.e. in front of the car). However we did not find any case where it was obvious that the algorithm made an erroneous decision based on a target that it

had been tracking for a longer period of time. All the faulty interventions seem to come from erroneous measurements, false targets or from bad initialization of obstacles.

To evaluate collision mitigation performance, collision tests against a stationary obstacle have been performed. As an obstacle, the inflatable car in Figure ?? is used. Performance for the head-on scenario displayed in Figure 1 is plotted in Figure 13. We here plot range between host vehicle and obstacle vs. range rate, the plotted result is an average from 5-20 collisions at each speed. As can be seen in the figure, speed was reduced 10-17 km/h for initial speeds ranging from 30-70 km/h.

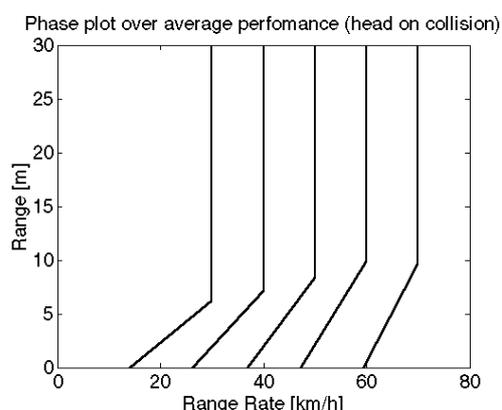


Fig. 13. Results from head on collisions with inflatable car

7. DISCUSSION

In this paper a method for decision making in collision avoidance applications has been presented. The main advantages of the method are: The use of existing tracking theory which in a straight forward way incorporates measurement and process uncertainty in the decision making process. Motion in two dimensions is considered.

The prototype system presented in this paper significantly reduces the impact speed in frontal collisions. As can be seen in Figure 13 interventions typically occur when the obstacle is closer than 20 m away from the CMBB vehicle (more than 90 percent of all rear end collisions occur at relative speeds below 100 km/h (Zhu 2001)).

A sensor with a shorter detection range but a larger field of view might be more appropriate for collision mitigation purposes. Further work on the sensors and the sensor fusion is needed to have a system with zero faulty interventions. The sensors used in this system only has rudimentary target classification capabilities. It would be desirable to have a sensor which provides better target classification.

Factors that limit the system performance are: Measurement uncertainties, system response time (computational time and sensor measurement rate) and system modeling errors. Another factor that limits the system performance is the time to build up brake pressure and maximum achieved pressure. The brake system on the test vehicle here achieved decelerations between 5-7 m/s^2 . A brake system that quickly gives a deceleration of 10 m/s^2 potentially gives an additional speed reduction of 10 km/h to the test results plotted in figure 17.

Some specific problems with the system presented here where that the laser radar and millimeter radar could not be synchronized. This of course causes some discrepancy between laser and radar measurement. Both sensors loose the target at close range (less than 10 m) because their narrow field of view. This causes deteriorated performance at low speeds. The laser radar seems to have problems detecting the inflatable car which at some occasions caused missed interventions, since both sensors are required to detect the obstacle in order to have an intervention.

To design a good collision avoidance system we need to solve two issues. One is the risk estimation discussed in this paper. The other issue, that has not been addressed here, is that of object recognition. This is a matter of the sensing capabilities of the sensors but also a matter of how to do the sensor fusion. For correct decision making accurate target classification and feature extraction is imperative.

8. REFERENCES

- Doi, A., T. Butsuen, T. Niibe, T. Takagi, Y. Yamamoto and H. Seni (1994). Development of a rear-end collision avoidance system with automatic brake control. In: *JSAE Review*. Vol. 15. pp. 335-340.
- Seiler, P., B. Song and J. K. Hedrick (n.d.). Development of a collision avoidance system. In: *SAE Congress 1998*.
- Tamura, M., H. Inoue, T. Wtanabe and N. Maruko (2001). Research on a brake assist system with a preview function. In: *SAE paper no 2001-01-0357*.
- Treat, J.R., N.S. McDonald, D. Shinar, R.D Hume, R.E Mayer, R.L. Stansifer and N.j. Castellan (1977). Tri-level study of the causes of traffic accidents. Technical report. NHTSA.
- Zhu, Bo (2001). Potential effects on accidents from forward collision warning/avoidance system. Master's thesis. Linköping University.